Atmosphere Modeling I: Dynamics

Atmosphere component in CESM = CAM = Community Atmosphere Model

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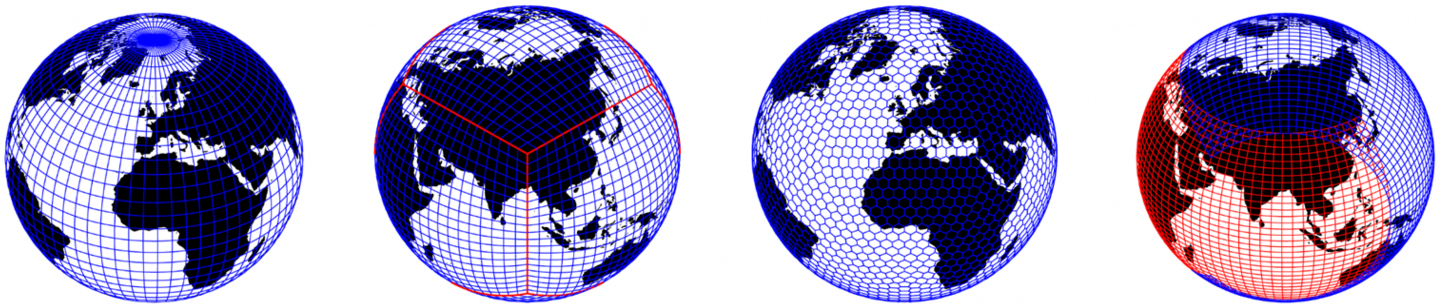
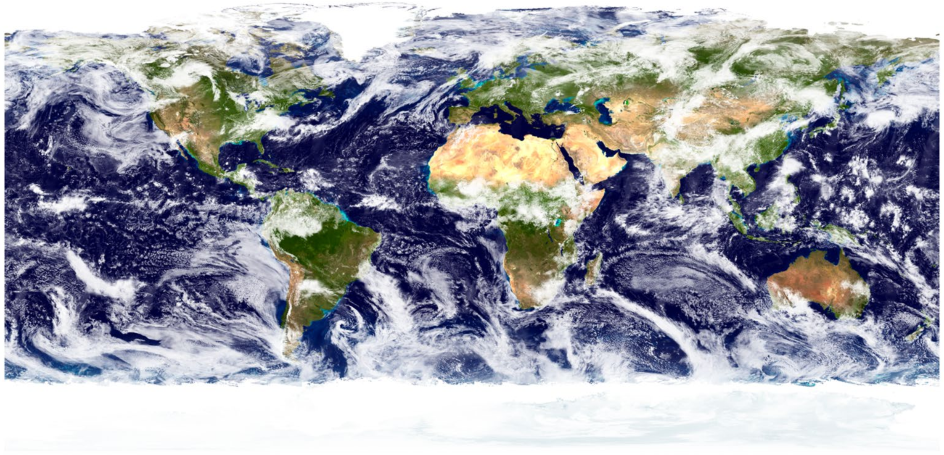


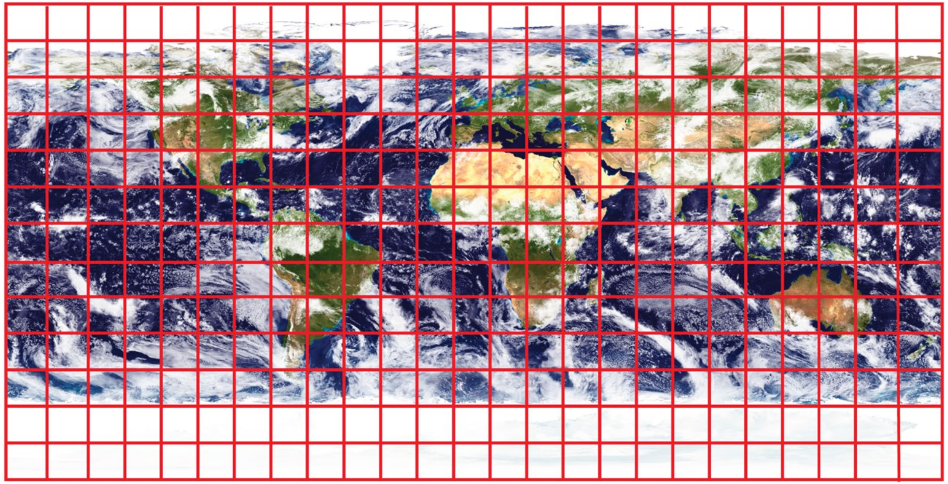
Some figures and slides adapted from Adam Herrington's DCMIP talk

Domain



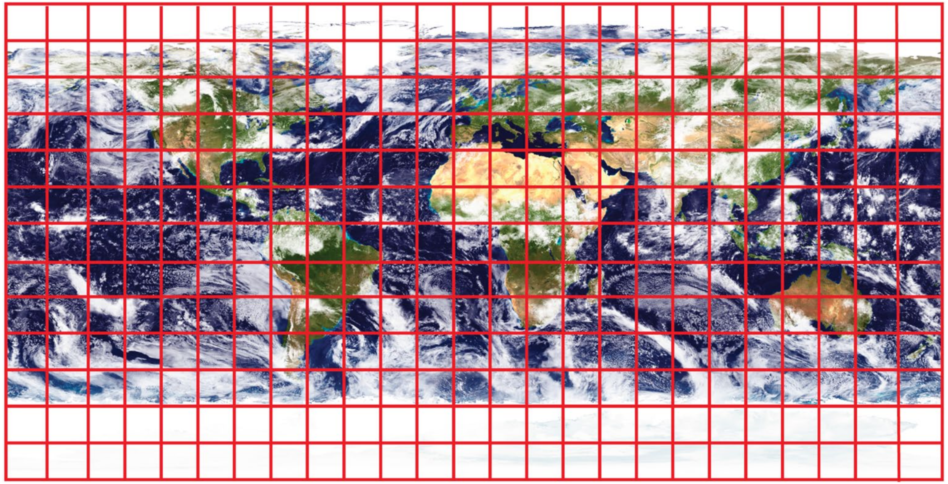
Source: NASA Earth Observatory

Horizontal computational space



Red lines: regular latitude-longitude grid

Dynamical core



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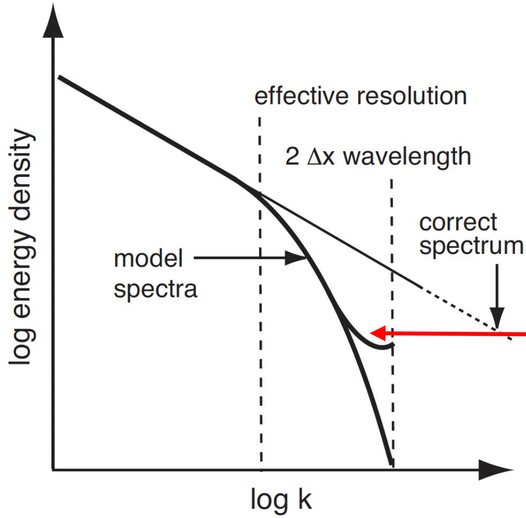
- Grid cell size (dx) defines the resolution of the computational grid (≠ effective resolution!)
- The ***dynamical core*** solves the governing fluid and thermodynamic equations on resolved scales
- In modeling jargon, the dynamical core is referred to as the ***dycore*** or the ***dynamics***
- The near-grid-scale solutions (e.g., 2*dx waves) are not accurately represented and are removed by implicit and/or explicit numerical filters

Effective Resolution: smallest scale (highest wavenumber k = k_{eff}) that a model can accurately represent

- k_{eff} can be assessed analytically for linearized equations (Von Neumann analysis)
- In a full model, once can assess k_{eff} using the kinetic energy of, e.g., the horizontal wind (see below).

Effective resolution is typically 4-10 times the grid spacing, depending on numerical method



Fall-off in energy at the grid-scale due to scale-selective numerical filters in dycore

Figure from Skamarock (2011): Schematic depicting the possible behavior of spectral tails from model forecasts

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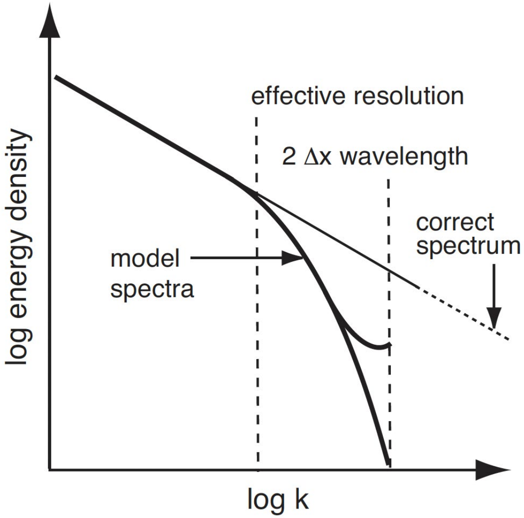

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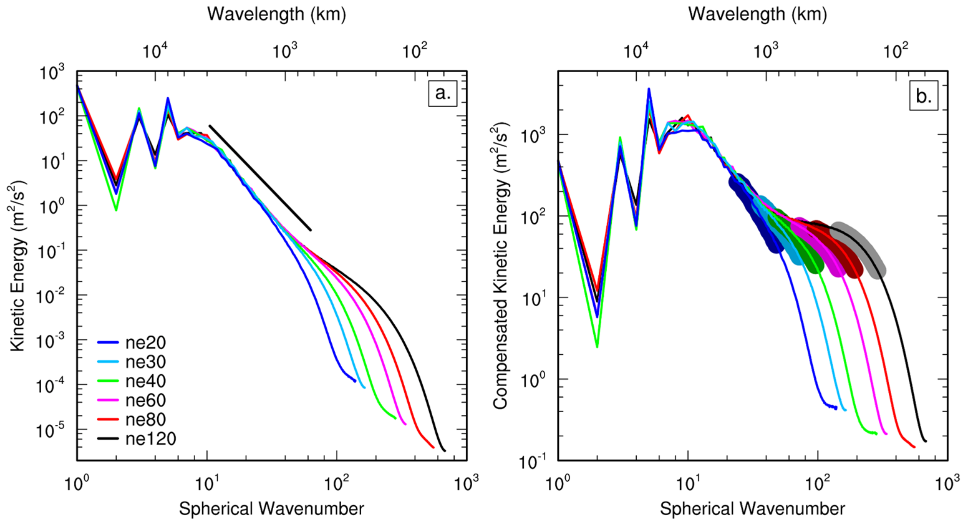
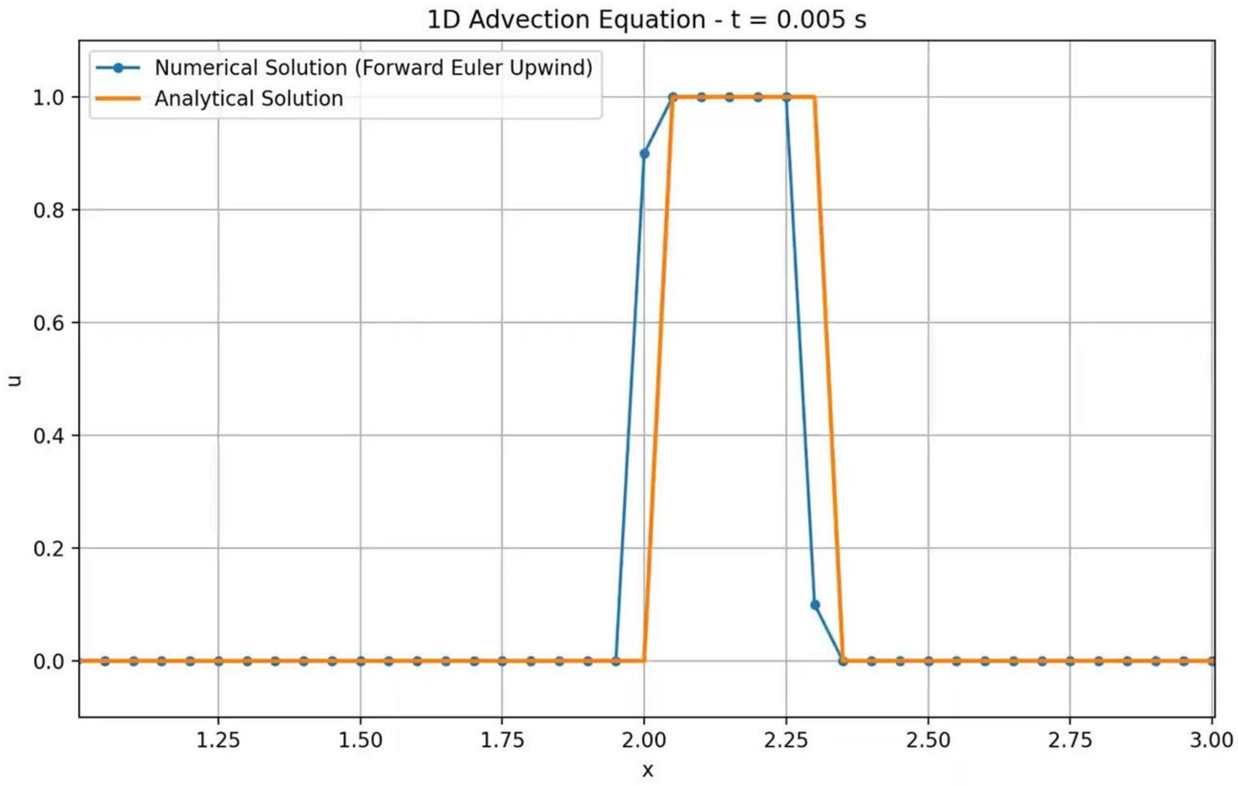


Figure from Herrington and Reed. 2020: Kinetic energy spectrum (left) and compensated kinetic energy spectrum (right) at the 200-hPa level in the simulations

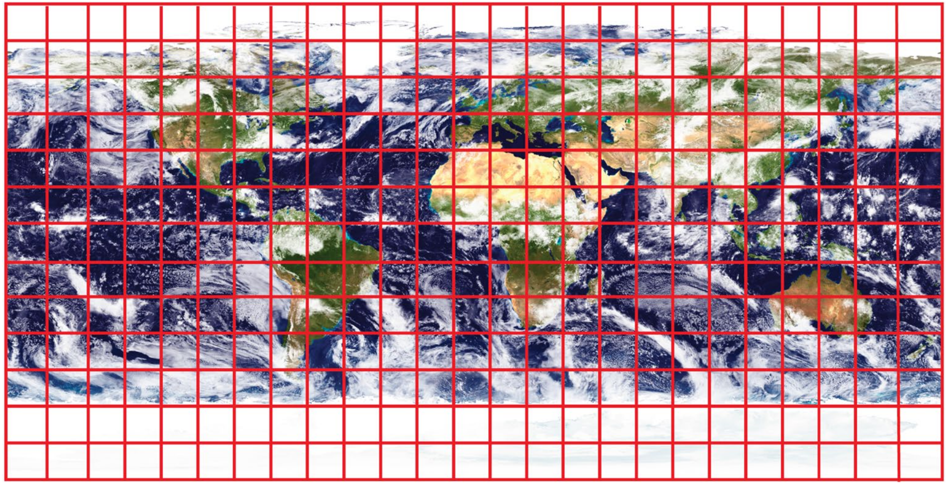
Numerical errors in the dynamical core



Physics parameterizations

See “Atmosphere Modeling II: Parameterizations” talk this PM

Kanishk Gohil & Ben Stephens



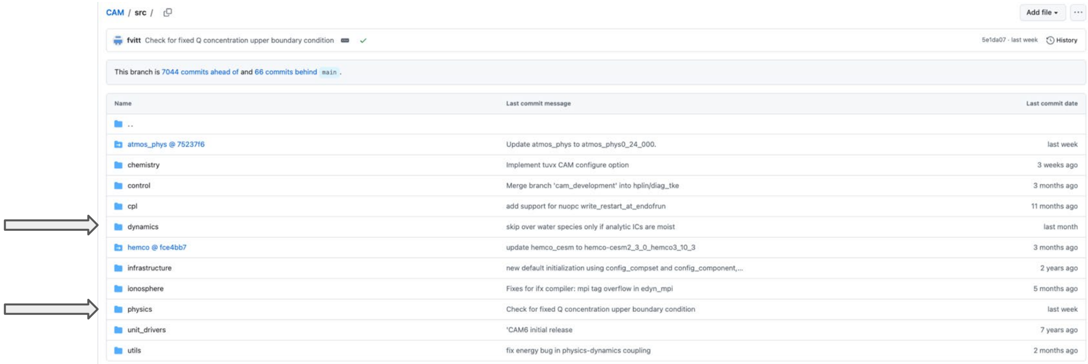
Red lines: regular latitude-longitude grid

- Many important processes occur at the sub-grid-scale that must be parameterized
- ***Parameterizations*** compute the grid-cell average tendencies due to sub-grid processes in terms of the resolved atmospheric state
- In modeling jargon, parameterizations are referred to as the ***physics***
- Sometimes called ***column physics***, which emphasizes that physics only operate in a column

Model code

Parameterization suite

Moist processes : deep convection, shallow convection, large-scale condensation
Radiation and Clouds : cloud microphysics, precipitation processes, radiation
Turbulent mixing : planetary boundary layer parameterization, vertical diffusion, gravity wave drag



‘Resolved’ dynamics

‘Roughly speaking, the dynamical core solves the governing fluid and thermodynamic equations on resolved scales,while the parameterizations represent sub-grid-scale processes and other processes not included in the dynamical core such as radiative transfer.’ - Thuburn (2008)

Multi-scale nature of atmospheric dynamics (from Thuburn 2011)

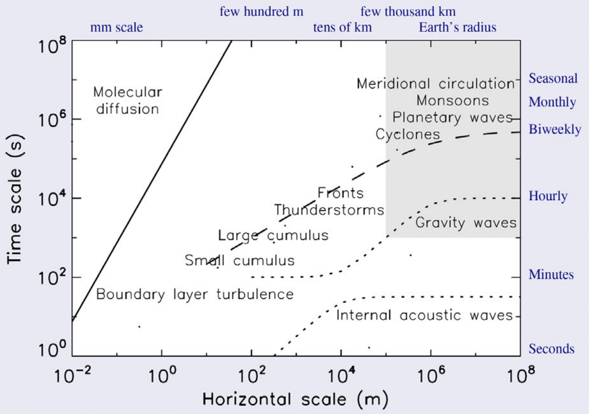


Figure from Thuburn (2011): Schematic depicting the time scales and horizontal scales of a range of atmospheric phenomena.

- O(10^4 km): large-scale circulations (Hadley circulation)
- O(10^4 km): undulations in the jet stream and pressure patterns associated with the largest scale Rossby waves (called *planetary waves*)
- O(10^3 km): cyclones and anticyclones
- O(10 km): the transition zones between relatively warm and cool air masses can collapse in scale to form fronts with widths of a few tens of km
- O(10^3 km - 100 m): convection can be organized on a huge range of different scales (tropical intraseasonal oscillations; supercell complexes and squall lines; individual small cumulus clouds formed from turbulent boundary layer eddies)

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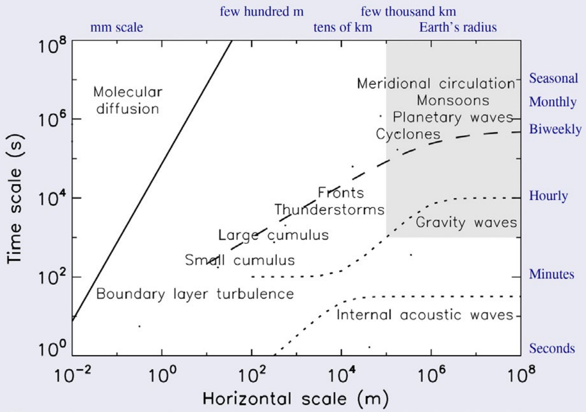

- ❑ All phenomena along the dashed line are important for weather and climate, and so need to be represented in numerical models
- ❑ Important phenomena occur at all scales – there is no significant spectral gap! Moreover there are strong interactions between phenomena at different scales, and these interaction need to be represented
- ❑ The lack any spectral gap makes the modeling of weather and climate very challenging

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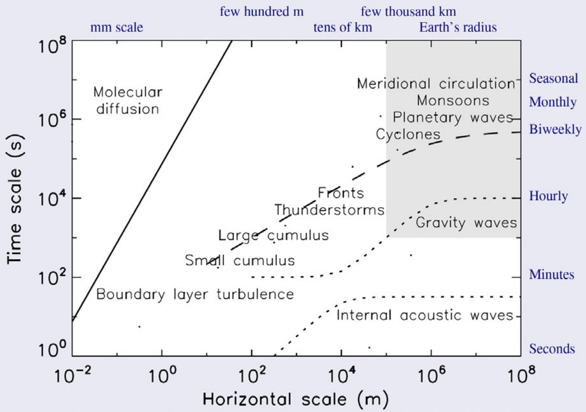

- ❑ Two dotted curves correspond to dispersion relations for internal inertio-gravity waves and internal acoustic waves (fast processes)
- ❑ these lines lie significantly below the energetically dominant processes on the dashed line
  - ❑ they are energetically weak compared to the dominant processes along the dashed curve
  - ❑ we do relatively little damage if we distort their propagation
  - ❑ the fact that these waves are fast puts constraints on the size of dt (at least for explicit and semi-implicit time-stepping schemes)!

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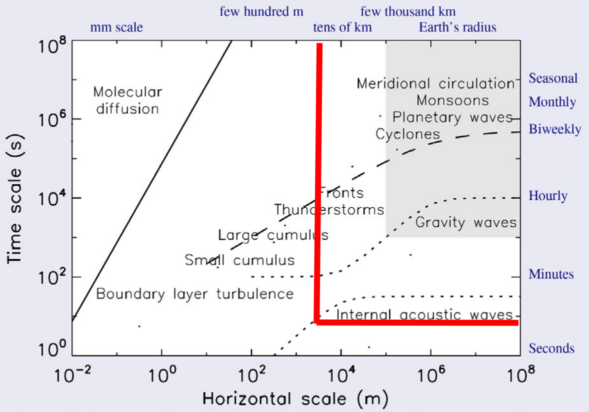

Horizontal Resolution

- the shaded region shows the resolved space/time scales in typical current day climate models (approximately 1˚– 2˚ resolution)
- we develop CAM at 1˚ resolution, but it has been ran down to ~3 km using the MPAS non-hydrostatic dycore
- as the resolution is increased some `large-scale' parameterizations may no longer be necessary (e.g., deep convection) and we might need to redesign some parameterizations that were developed for horizontal resolutions of hundreds of km's

- O(10^4 km): large-scale circulations (Hadley circulation)
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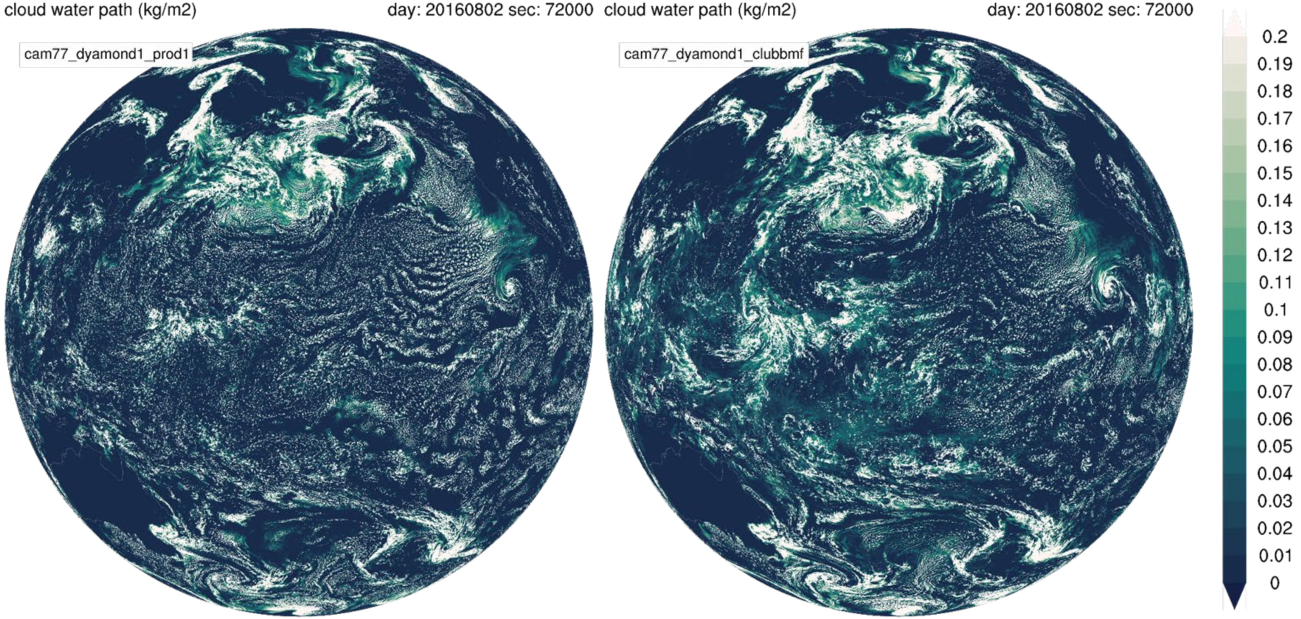
Do we still need to parameterize cumulus convection at ~3 km grid spacing?

CAM-MPAS simulations (3.75 km grid spacing, 58 vertical levels)

- With the ‘deep convection’ scheme off (left), the model struggles to simulate convective transitions (shallow cumulus → congestus → cumulonimbus), and instead produces ‘sprinkles’ of deep convection in most places
- Adding parameterized convection (right) produces a more realistic spectrum of cloud types



Designing an atmospheric model – Scope

What is the scope?

- ❑ NWP? Climate?
- ❑ Meteorological processes:
  - ❑ Planetary waves?
  - ❑ Extratropical storms?
  - ❑ Squall lines?
  - ❑ Tropical cyclones?
  - ❑ Supercells?
- ❑ Chemistry? Stratosphere? Space Weather?

These choices constrain the scales that need to be resolved by the dynamical core, which may require different **equation sets** (e.g., non-hydrostatic, deep atmosphere) and prognostic variables

Common large-scale model assumptions

The following approximations are made to the compressible Euler equations:

- **spherical geoid**: geopotential Φ is only a function of radial distance from the center of the Earth r: $\Phi = \Phi(r)$ (for planet Earth the true gravitational acceleration is much stronger than the centrifugal force).
  $\Rightarrow$ Effective gravity acts only in radial direction
- **quasi-hydrostatic approximation** (also simply referred to as *hydrostatic approximation*): Involves ignoring the acceleration term in the vertical component of the momentum equations so that it reads:
$$\rho g = -\frac{\partial p}{\partial z}, \tag{1}$$
  where g gravity, ρ density and p pressure. Good approximation down to horizontal scales greater than approximately $10km$.
- **shallow atmosphere**: a collection of approximations. Coriolis terms involving the horizontal components of Ω are neglected (Ω is angular velocity), factors $1/r$ are replaced with $1/a$ where a is the mean radius of the Earth and certain other metric terms are neglected so that the system retains conservation laws for energy and angular momentum.

Common vertical coordinate: terrain following

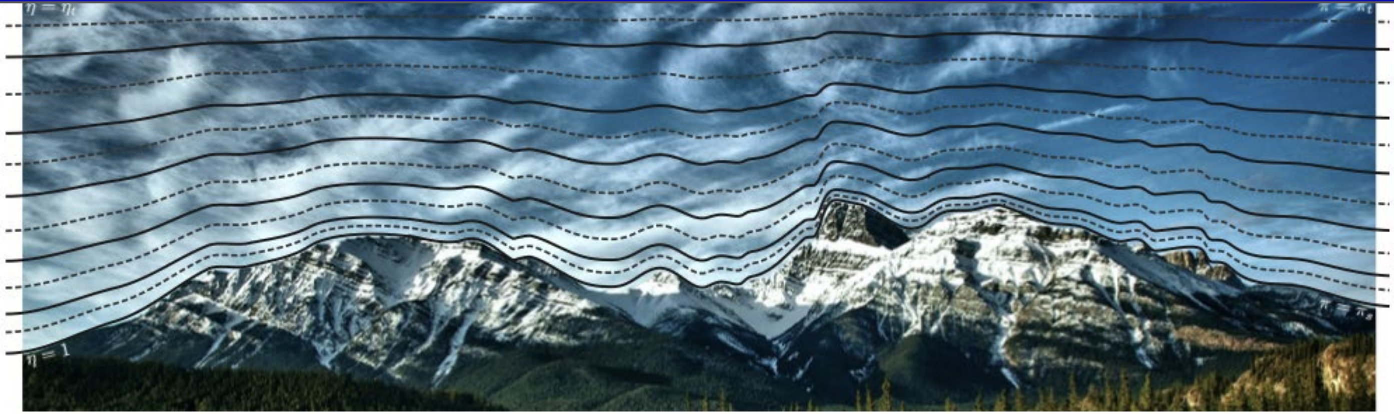


Figure courtesy of David Hall (CU Boulder).

Sigma layers at the bottom (following terrain) with isobaric (pressure) layers aloft.

Pressure at model level interfaces

$$p_{k+1/2} = A_{k+1/2}\, p_0 + B_{k+1/2}\, p_s,$$

where p_s is surface pressure, p_0 is the model top pressure, and $A_{k+1/2}(\in [0:1])$ and $B_{k+1/2}(\in [1:0])$ hybrid coefficients (in model code: *hyai* and *hybi*). Similarly for model level mid-points.

Note: vertical index is 1 at model top and *klev* at surface.

Vertically Lagrangian floating coordinate (Lin 2004)

- FV, FV3, SE use a Lagrangian ('floating') vertical coordinate such that

$$\frac{d\xi}{dt} = 0,$$

i.e., vertical surfaces are material surfaces (no flow across them)

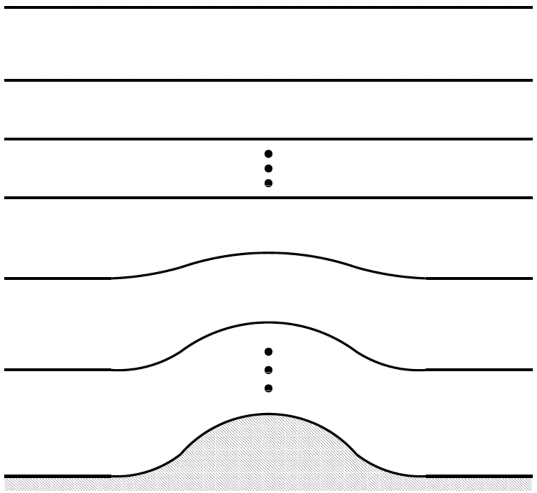


Figure shows 'usual' hybrid $\sigma - p$ vertical coordinate $\eta(p_s, p)$ (where p_s is surface pressure):

- $\eta(p_s, p)$ is a monotonic function of p.
- $\eta(p_s, p_s) = 1$
- $\eta(p_s, 0) = 0$
- $\eta(p_s, p_{top}) = \eta_{top}$.

Boundary conditions are:

- $\frac{d\eta(p_s, p_s)}{dt} = 0$
- $\frac{d\eta(p_s, p_{top})}{dt} = \omega(p_{top}) = 0$

(ω is vertical velocity in pressure coordinates)

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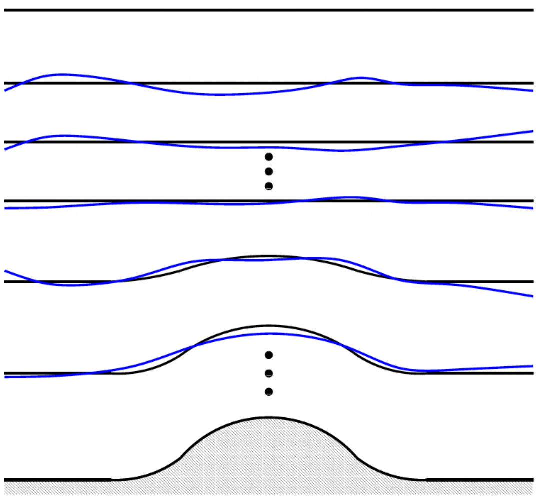


Figure:

- set $\xi = \eta$ at time t_{start} (black lines).
- for $t > t_{start}$ the vertical levels deform as they move with the flow (blue lines).
- to avoid excessive deformation of the vertical levels (non-uniform vertical resolution) the prognostic variables defined in the Lagrangian layers ξ are periodically remapped (= conservative interpolation) back to the Eulerian reference coordinates η (more on this later).

Why use floating Lagrangian vertical coordinates? Vertical advection terms disappear (3D model becomes 'stacked shallow-water models'; only 2D numerical methods are needed)

Dynamical core equations (in spectral-element dynamical core)

2.5. Equations of Motion

The $\eta^{(d)}$-coordinate adiabatic and frictionless atmospheric primitive equations assuming floating Lagrangian vertical coordinates (Lin, 2004; Starr, 1945) can be written in vector invariant form as

$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta + f)\,\hat{\vec{k}} \times \mathbf{v} + \nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2 + \Phi\right) + \frac{1}{\rho}\nabla_{\eta^{(d)}} p = 0, \tag{49}$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla_{\eta^{(d)}} T - \frac{1}{c_p \rho}\omega = 0, \tag{50}$$

Dry mass coordinate formulation of continuity equation

$$\frac{\partial}{\partial t}\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)}\right) + \nabla_{\eta^{(d)}}\cdot\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)} \mathbf{v}\right) = 0, \quad \ell \in \mathcal{L}_{\text{all}}, \tag{51}$$

where Φ is the geopotential height ($\Phi = g\,z$), $\hat{\vec{k}}$ is the unit vector normal to the surface of the sphere, $\mathbf{v} = (u, v)$ is the velocity vector with u being the zonal velocity component and v the meridional velocity component, $\zeta = \hat{\vec{k}} \cdot \nabla\times\mathbf{v}$ is vorticity, f Coriolis parameter, and $\omega = \mathrm{d}p/\mathrm{d}T$ is the (moist) pressure vertical velocity with $\mathrm{d}/\mathrm{d}T = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\eta_d}$ being the material/total derivative along $\eta^{(d)}$.

https://agupubs.onlinelibrary.wiley.com/doi/epdf/10.1029/2017MS001257

Dynamical core considerations: thermodynamics

Moist air is considered a mixture of dry air and various forms of water:

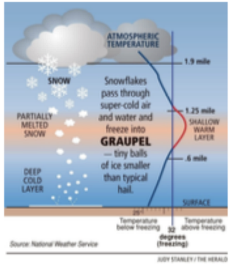
Water in the atmosphere

1. **Water vapor (gaseous phase of water): weight of water vapor in the atmosphere corresponds to approximately ~2.4hPa**
2. **Liquid water (clouds): condensation of water vapor form droplets**

3. **Frozen water / ice (clouds): ice crystals**

Cirrus clouds (high clouds)

High resolution models also transport graubel, hail, snow, ...



Common thermodynamic assumptions:

- Atmosphere ideal gas
- Condensates take up no volume
- Constant latent heats

The set of all components of moist air are referred to as:

$$\mathcal{L}_{all} \equiv \{d, wv, cl, ci, rn, sw, gr\} .$$

Note that dry air and water vapor are gases and $'cl'$, $'ci'$, $'rn'$, $'sw'$ and $'gr'$ are condensates (very different thermodynamic properties!)

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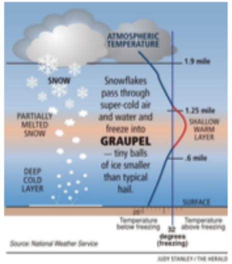

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Also note:

- we don't have separate momentum equations for components of air - so they move with the same "large-scale" velocity field
- We don't have individual thermodynamic equations for each water especies so we assume they all have the same temperature
- Moist thermodynamics is perhaps where there is room for improvement in global models!

See, e.g., Lauritzen et al. (2022)

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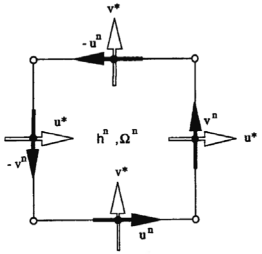
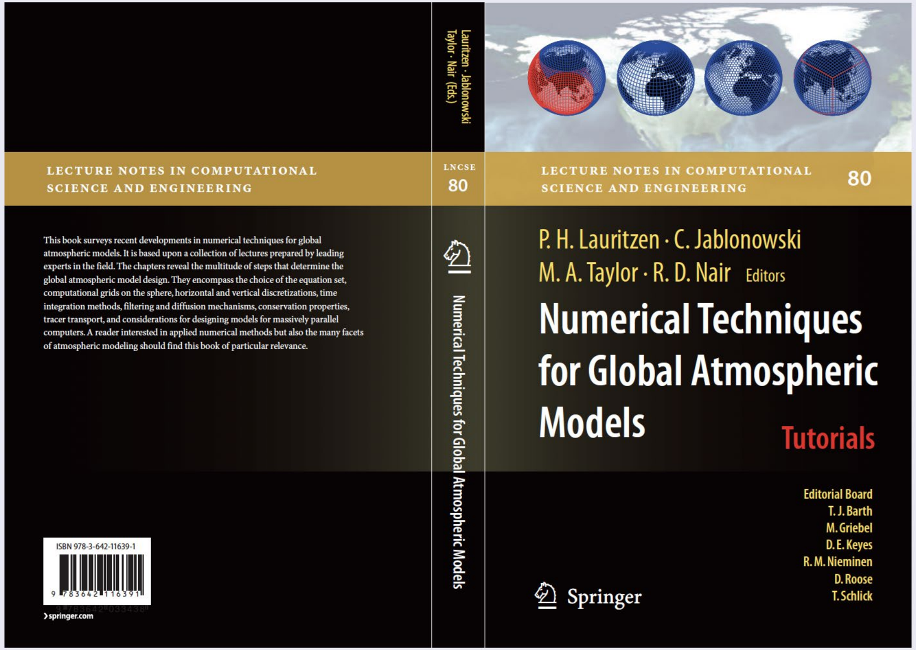
Other dynamical core design considerations:

- Grid type, discretization method, order of accuracy, arrangement of variables
- Mass and energy conservation
- Tracer transport properties – shape preservation, preserves linear correlations, scalability



Interested in numerical methods for global models?



- ❑ Book based on the lectures given at the 2008 NCAR ASP (Advance Study Program) Summer Colloquium
- ❑ 16 Chapters; authors include J.Thuburn, J.Tribbia, D.Durran, T.Ringler, W.Skamarock, R.Rood, J.Dennis, Editors, ….Foreword by D. Randall

Interested in numerical methods for global models?

DCMIP-2025
The Dynamical Core Model Intercomparison Project and Summer School

National Center for Atmospheric Research (NCAR), Mesa Lab, Boulder, CO, USA

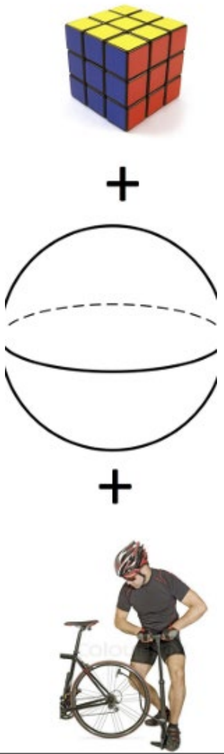
June/2-6/2025

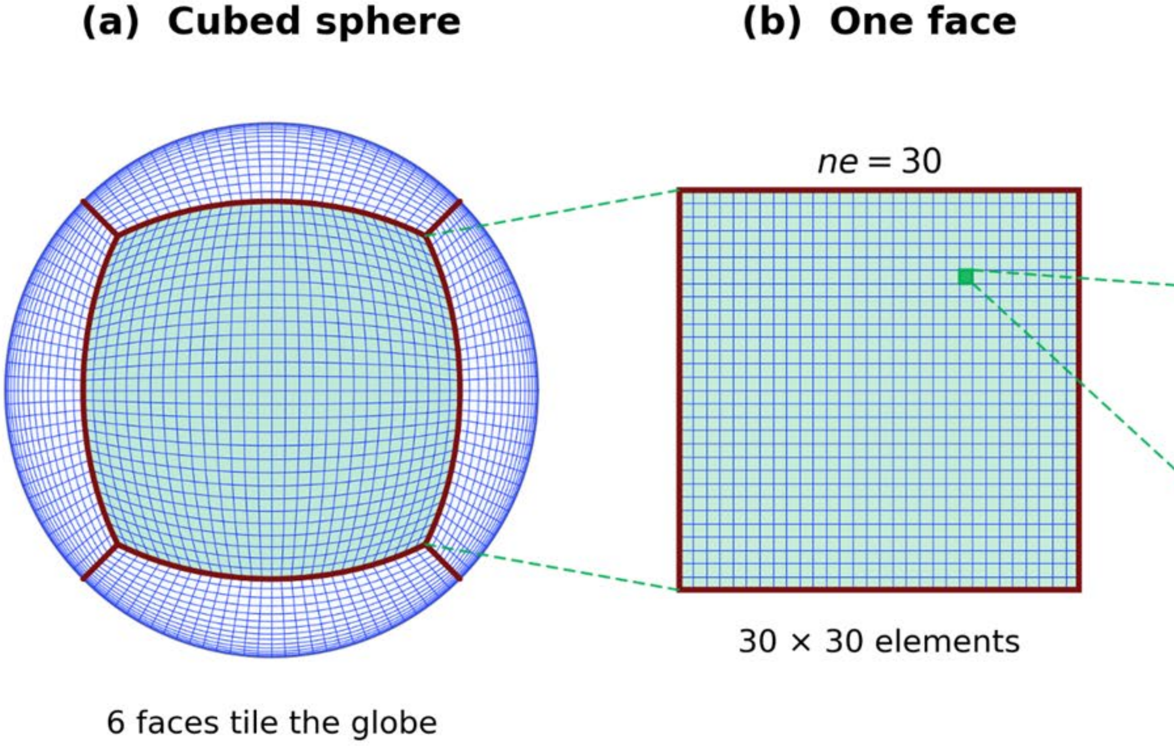
The DCMIP-2025 Vision

The DCMIP-2025 summer school surveys the design decisions in atmospheric General Circulation Models (GCMs) with a focus on their dynamical cores and takes a unique look at Machine Learning emulators for GCMs. DCMIP is built upon lectures, hands-on dynamical core & ML modeling projects, shared cyberinfrastructure, and computational tools that utilize the Community Earth System Model (CESM) from NCAR.

All talks available in dcmip.org and recordings on YouTube!

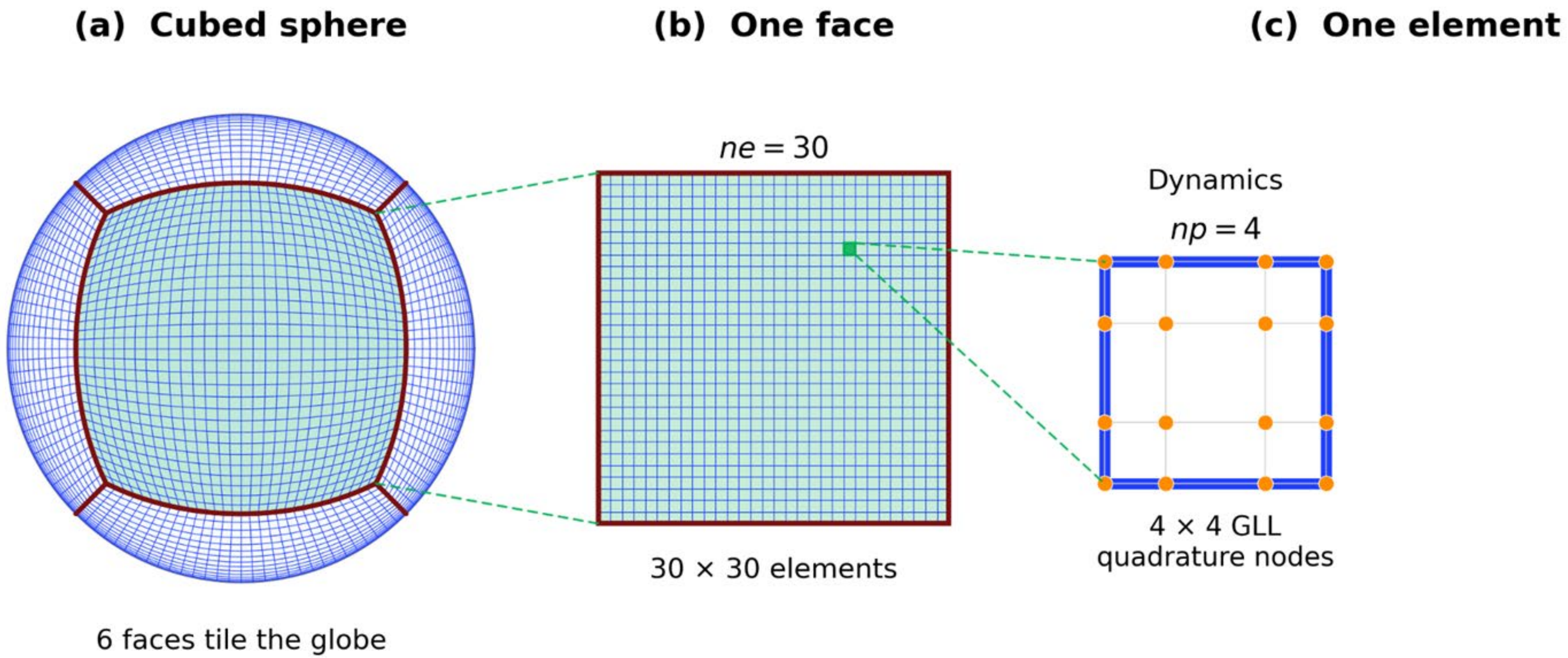
CESM3 standard resolution dynamical core SE-CSLAM dynamical core: grids





SE-CSLAM = Spectral-Element with Conservative Semi-LAgrangian Multi-tracer transport scheme

SE-CSLAM dynamical core: grids

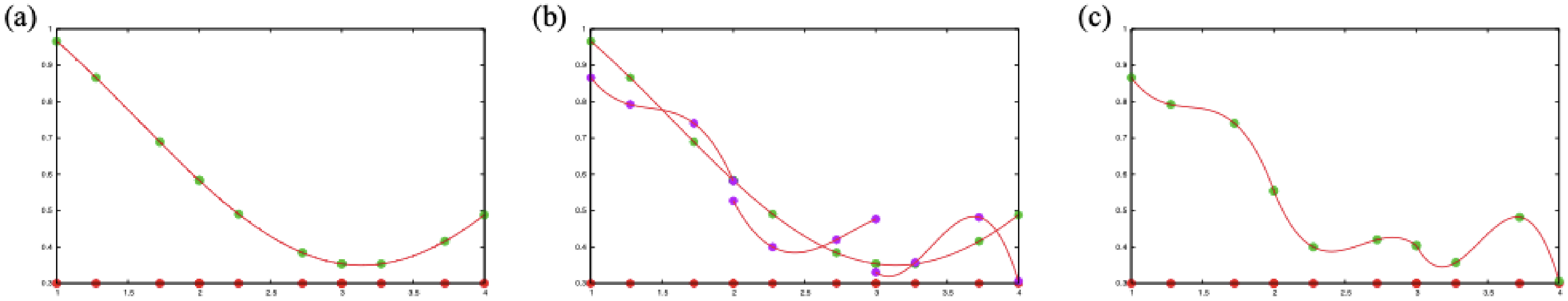


GLL: Gauss–Lobatto–Legendre

SE-CSLAM = Spectral-Element with Conservative Semi-LAgrangian Multi-tracer transport scheme

SE-CSLAM dynamical core: 1D schematic of spectral-element method

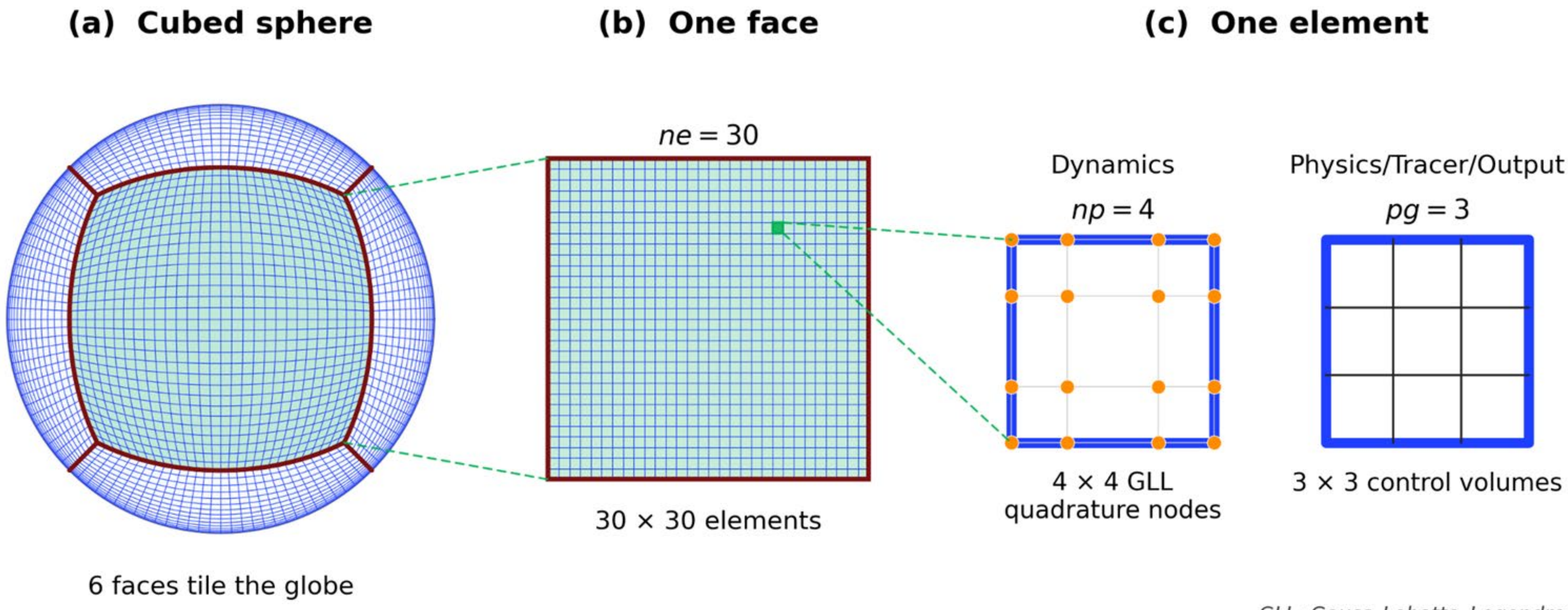
CAM-SE uses a continuous Galerkin finite element method (Taylor et al., 1997) referred to as **Spectral Elements (SE)**:



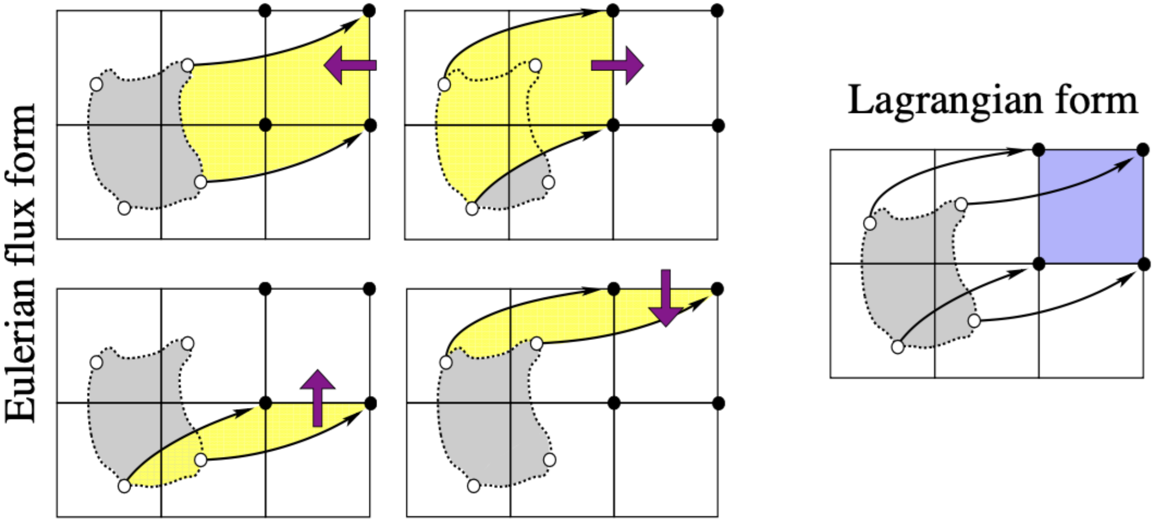
How do solutions in each element 'communicate' with each other?

- The solution is projected onto the space of globally continuous (C^0) piecewise polynomials
- $\rightarrow$ point values are forced to be C^0 continuous along element boundaries by averaging.
- Note: this is the only operation in which information 'propagates' between elements
- MPI data-communication: only information on the boundary of elements!
- For more details see explanation/discussion in Herrington et al. (2018).

SE-CSLAM dynamical core: grids



Finite-volume discretization of continuity equation: Tracking mass



The yellow areas are 'swept' through the control volume faces during one time-step. The grey area is the corresponding Lagrangian area (area moving with the flow with no flow through its boundaries that ends up at the Eulerian control volume after one time-step). Black arrows show parcel trajectories.

Note **equivalence** between Eulerian flux-form and Lagrangian form!

(Lauritzen et al., 2011b)

Separate physics, transport and dynamics grid

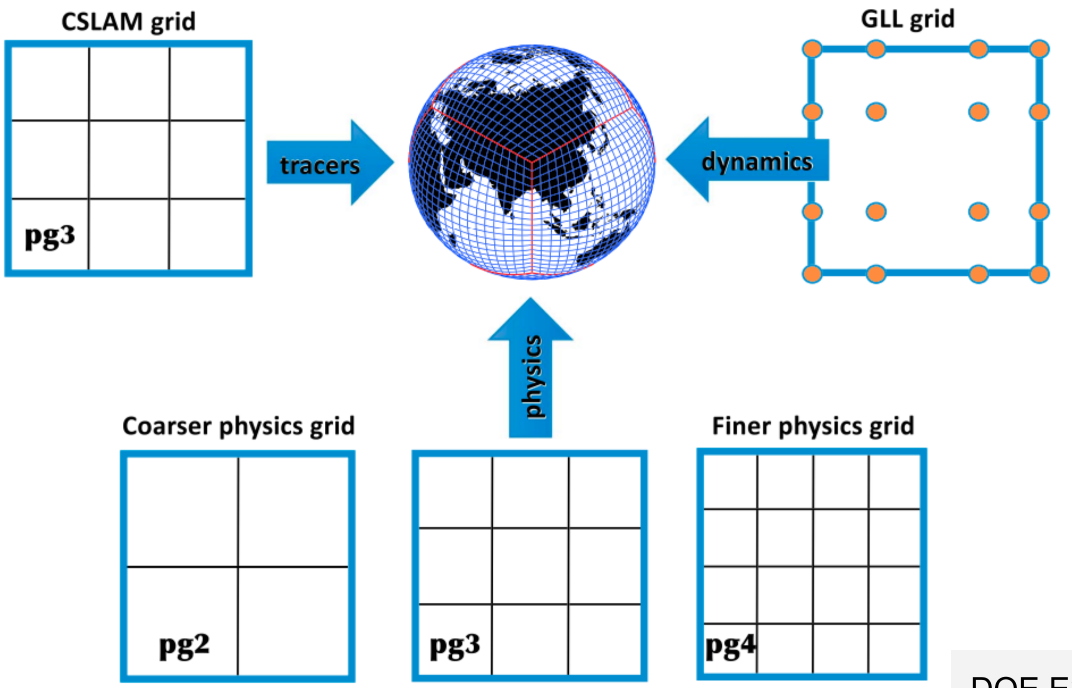


Figure 1. An overview of the different grids in CAM-SE-CSLAM.

For CESM3 we use pg3 grid for CAM physics!

Separating grids is not trivial - mapping between grids must be done carefully!
https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2019ms001684

Transport scheme:
Conservative Semi-LAgrangian Multi-tracer scheme
(consistent coupling with spectral-elements dycore described here https://journals.ametsoc.org/view/journals/mwre/145/3/mwr-d-16-0258.1.xml)

Note: Dry-mass vertical coordinate makes CSLAM-SE dycore coupling more consistent!

DOE E3SM is using similar approach (but transport scheme faster and supports variable resolution grids)

UK Met Office is exploring separation of grids as well

Vertical grid resolution/spacing in CAM7

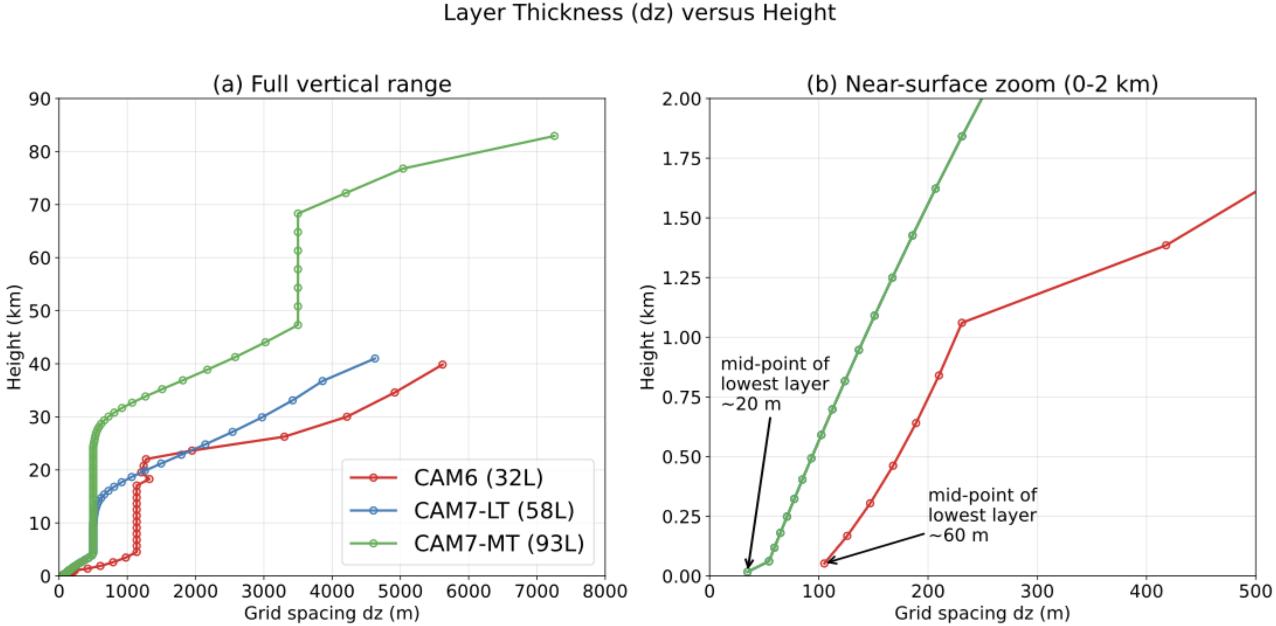


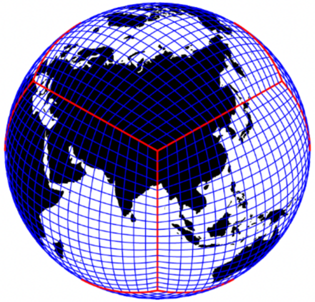
Figure 2. Grid spacing (dz) as a function of height for different CAM model configurations. (a) Full vertical range and (b) zoom over the lowest ~2 km (near-surface region). Default CAM7-MT has a model top near ~80 km compared to ~40 km in CAM6, with grid spacing in the stratosphere comparable to WACCM6 (not shown) and significantly increased resolution in the boundary layer relative to CAM6.

CAM’s dycores

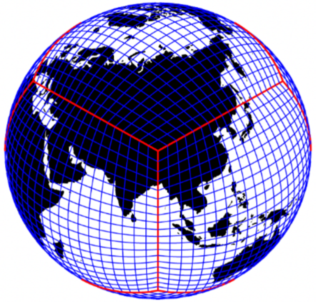
Eulerian Spectral Transform (EUL)

Finite-Volume (FV)

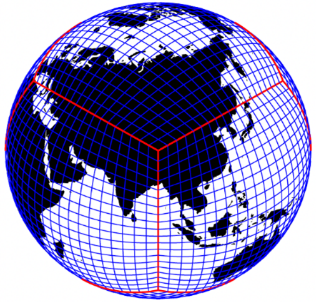
Finite-Volume Cubed-Sphere (FV3)



Spectral-Element CSLAM (SE-CSLAM)



Spectral-Element Non-Hydrostatic (SE-NH)



Model for Prediction Across Scales (MPAS)

The EUL dycore has been decommissioned

CAM’s dycores

Eulerian Spectral Transform **(EUL)**

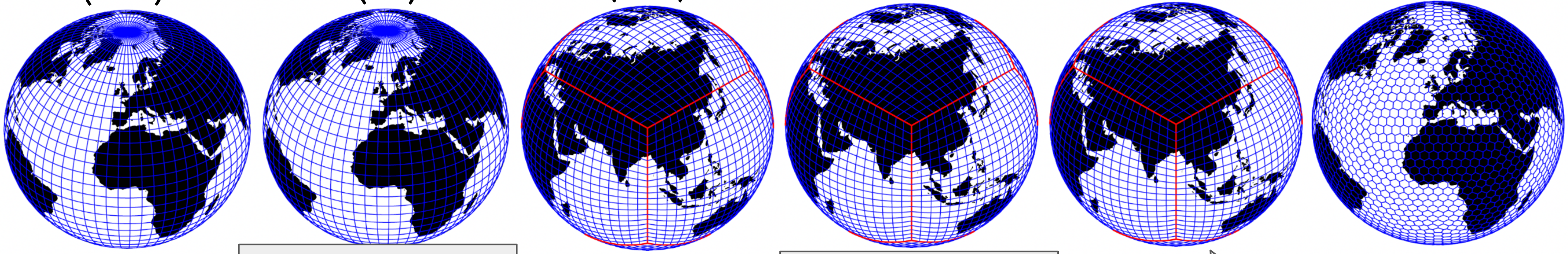
Finite-Volume **(FV)**

Finite-Volume Cubed-Sphere **(FV3)**

Spectral-Element CSLAM **(SE-CSLAM)**

Spectral-Element Non-Hydrostatic **(SE-NH)**

Model for Prediction Across Scales **(MPAS)**



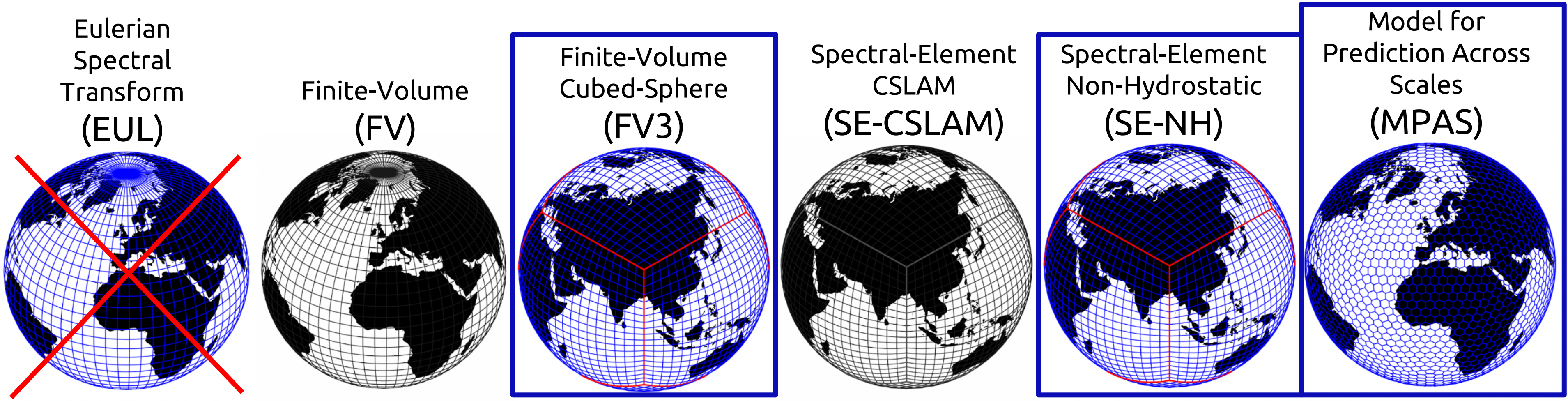
Previous default dycore for 1 degree applications

Limited support!

New Default dycore for 1 degree applications

On a fork of CESM
https://github.com/NCAR/StormSPEED
https://sites.google.com/umich.edu/nsf-stormspeed

CAM’s non-hydrostatic dycores

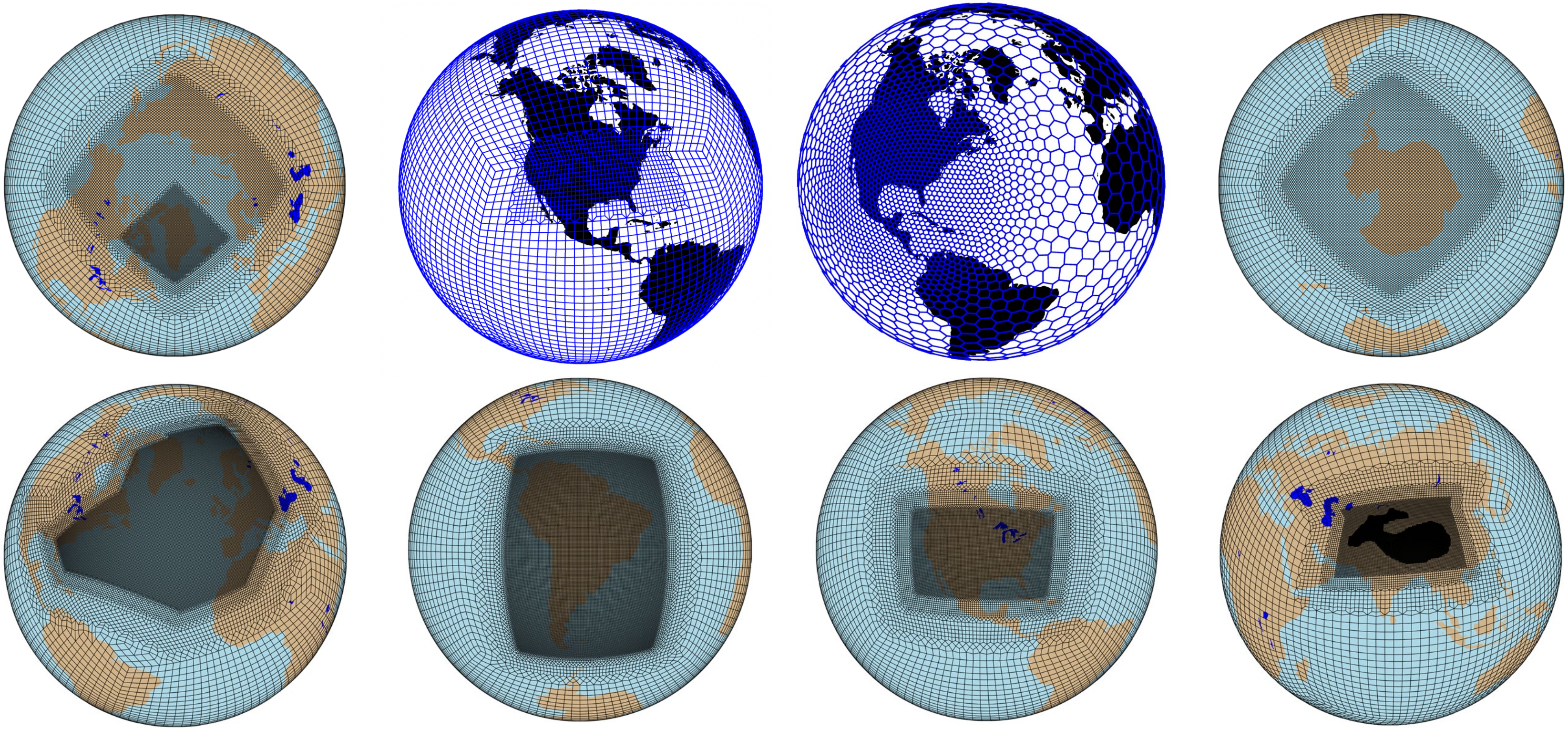


- ❑ CAM has three dycores based on non-hydrostatic equation sets
- ❑ Scale analysis indicates non-hydro req’d for length scales O(10 km) and less (k→m)
  (e.g., Wedi and Smolarkiewicz, 2009)

Mesh refinement

See Adam Herrington's talk on Friday!

SE, MPAS & FV3 support mesh refinement



CAM model tops

The vertical extent is from the surface to:

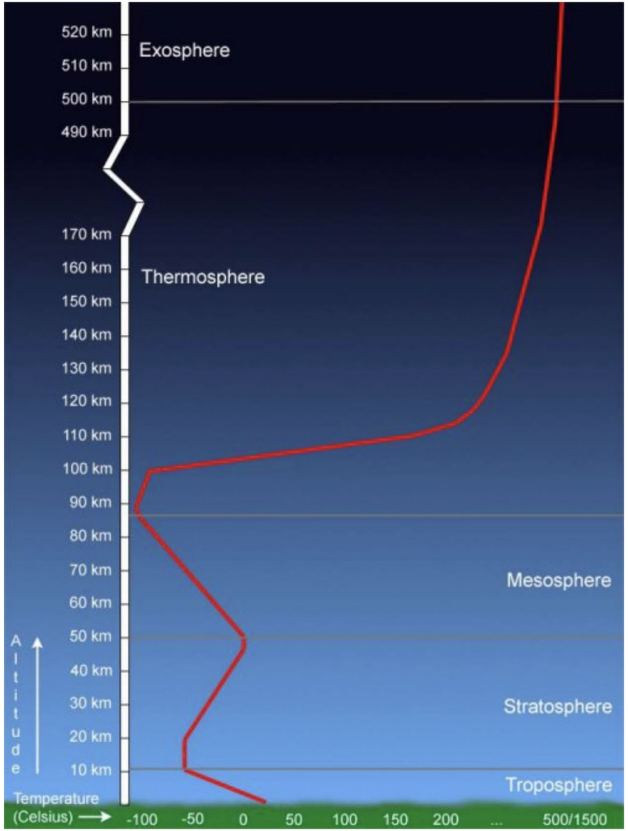
- approximately 42 km / 2 hPa
- approximately 140 km /10^{-6} hPa for WACCM6 (Whole Atmosphere Community Climate Model)
- approximately 600 km / 10^{-9} hPa for WACCM-X

Note:

For CAM7 we introduced a mid-top version with 93 vertical levels and model top at 80 km, in addition to a low-top version with 58 levels and 40 km top

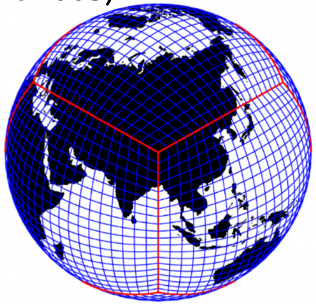
Viscous sponge layer

In practice, GCMs need additional damping in the top few levels of the model to prevent wave reflections off the model top

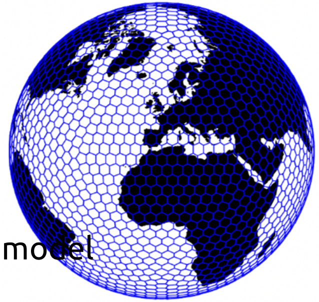


CAM’s non-hydrostatic dycores

- SE-NH
  - Dynamical core based on HOMME (High-Order Method Modeling Environment; Thomas and Loft 2005)
  - Cubed-sphere tiled with quadrilateral elements; fourth order basis set within each element
  - Reformulated with non-hydrostatic equation set (Taylor et al. 2019)
  - Efficient semi-Lagrangian tracer transport (Bradley et al. 2022)
  - Physics evaluated on separate finite-volume grid (Hannah et al. 2021)
  - Mass conservation to machine precision and good total energy conservation properties



- FV3
  - Cubed-sphere version of FV
  - Flux-form semi-Lagrangian advection on staggered ‘CD’ grid (Lin and Rood 1997)
  - https://www.gfdl.noaa.gov/fv3/fv3-documentation-and-references/



- MPAS
  - Centroidal Voronoi tessellation of the sphere (hexagons)
  - Finite-volume discretization on staggered C-grid
  - Fully compressible non-hydrostatic discretization similar to Weather Research Forecast (WRF) model (Skamarock and Klemp 2009)
  - Height-based terrain-following vertical coordinate (Klemp 2011)
  - Coupled to CAM physics’ pressure-based vertical coordinate (Huang et al. 2022)

CAM-SE: (Lauritzen et al., 2018) Total energy conservation (Lauritzen and Williamson, 2019)

The total energy equation can be written on the form (Kasahara, 1974)

$$\frac{\partial}{\partial t}\left[\rho\left(\frac{\partial z}{\partial \eta^{(d)}}\right)(K + c_v T + gz)\right] + \nabla_{\eta^{(d)}} \cdot \left[\rho \vec{v}\left(\frac{\partial z}{\partial \eta^{(d)}}\right)(K + gz + c_p T)\right] = -\frac{\partial}{\partial \eta^{(d)}}\left(p\frac{\partial z}{\partial t}\right), \tag{9}$$

where c_p is heat capacity at constant pressure, z height, and $K = \frac{1}{2}\vec{v}\cdot\vec{v}$.

In z-based vertical coordinate (for a moment assume $\eta^{(d)} \equiv z$), then integrating energy equation in the vertical and using that z is constant at the model top (z_{top}) and surface (z_s) we get

$$\frac{\partial}{\partial t}\int_{z=z_s}^{z=z_{top}} (K + c_v T + gz)\,\rho dz + \nabla_z \cdot \int_{z=z_s}^{z=z_{top}} \vec{v}\,(K + gz + c_p T)\,\rho dz = 0. \tag{10}$$

Note: clear separation of kinetic (K), potential (gz) and internal ($c_v T$) energy. Integrating in the horizontal over the entire sphere the flux term drops out, and it is clear that the total energy is conserved for the frictionless and adiabatic system of equations.

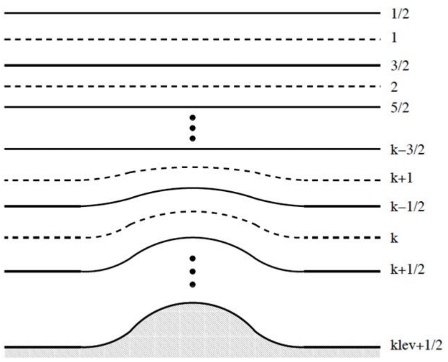
In a hybrid-sigma vertical coordinate and assuming that pressure model top is constant we get

$$\frac{1}{g}\frac{\partial}{\partial t}\int_{\eta=0}^{\eta=1}\left(\frac{\partial \mathcal{P}^{(d)}}{\partial \eta^{(d)}}\right)\sum_{\ell}\left[m^{(\ell)}\left(K + c_p^{(\ell)} T + \Phi_s\right)\right] d\eta^{(d)} = 0 \tag{11}$$

where $\sum_\ell$ is sum over dry air, water vapor, cloud liquid, cloud ice, rain and snow ($\ell =$ 'd', 'wv', 'cl', 'ci', 'rn', 'sw').

Vertical coordinate: hybrid sigma ($\sigma = p/p_s$)-pressure (p) coordinate

- FV, FV3, SE use a hybrid sigma-pressure vertical coordinate (η)



Sigma layers at the bottom (following terrain) with isobaric (pressure) layers aloft.

- Pressure at model level interfaces:

$$p_{k+½} = A_{k+½}\, p_0 + B_{k+½}\, p_s$$

- where p_s is the surface pressure, p_0 is a reference pressure, and $A_{k+½}$ and $B_{k+½}$ are hybrid coefficients (in model code: hyai and hybi). Similarly for model level mid-points.

Note. The vertical index is 1 at the model top, and *klev* at the surface

Vertically Lagrangian floating coordinate (Lin 2004)

- FV, FV3, SE use a Lagrangian ('floating') vertical coordinate such that

$$\frac{d\xi}{dt} = 0,$$

i.e., vertical surfaces are material surfaces (no flow across them)

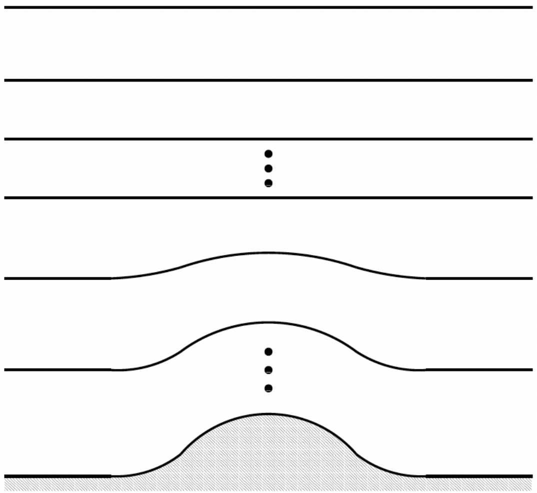


Figure shows 'usual' hybrid $\sigma - p$ vertical coordinate $\eta(p_s, p)$ (where p_s is surface pressure):

- $\eta(p_s, p)$ is a monotonic function of p.
- $\eta(p_s, p_s) = 1$
- $\eta(p_s, 0) = 0$
- $\eta(p_s, p_{top}) = \eta_{top}$.

Boundary conditions are:

- $\frac{d\eta(p_s, p_s)}{dt} = 0$
- $\frac{d\eta(p_s, p_{top})}{dt} = \omega(p_{top}) = 0$

(ω is vertical velocity in pressure coordinates)

Vertically Lagrangian floating coordinate (Lin 2004)

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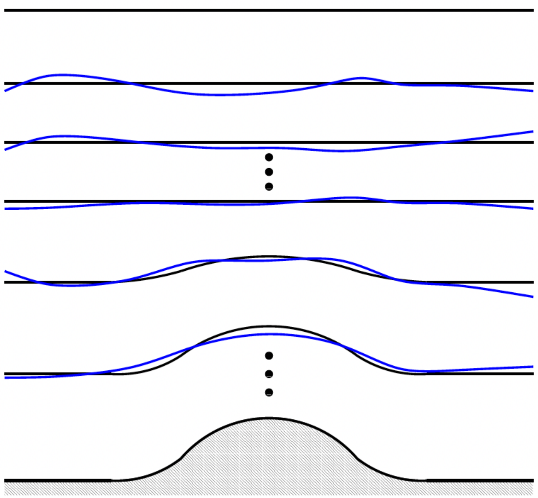


Figure:

- set $\xi = \eta$ at time t_{start} (black lines).
- for $t > t_{start}$ the vertical levels deform as they move with the flow (blue lines).
- to avoid excessive deformation of the vertical levels (non-uniform vertical resolution) the prognostic variables defined in the Lagrangian layers ξ are periodically remapped (= conservative interpolation) back to the Eulerian reference coordinates η (more on this later).

Why use floating Lagrangian vertical coordinates? Vertical advection terms disappear (3D model becomes 'stacked shallow-water models'; only 2D numerical methods are needed)

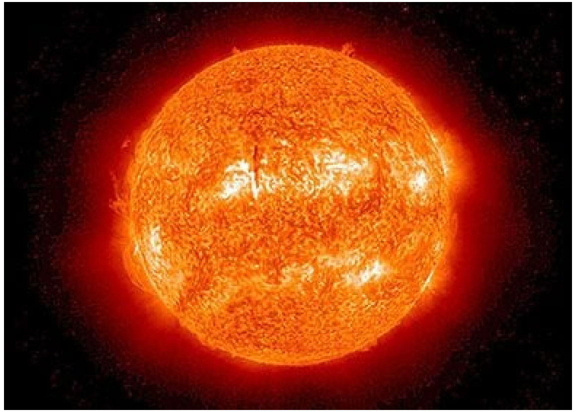
What's in a physics suite?

See "Atmosphere Modeling II: Parameterizations" talk this PM

Christina McCluskey, Meg Fowler and Ben Stephens

A physics suite contains several processes:

- ❑ Radiation
- ❑ PBL scheme (turbulent fluxes, surface drag)
- ❑ PBL form drag due to sub-grid orography
- ❑ Gravity-wave drag (convection, orography)
- ❑ Cumulus convection (separate schemes for shallow and deep convection)
- ❑ *Stratiform macro- and micro-physics
- ❑ Aerosol physics
- ❑ Chemistry



The specific choice of processes represented, and what scheme is used to represent a process depends on the scope of the model

*Performs the saturation adjustment, which is instantaneous and therefore the fastest process in the model. Historically, the state after the saturation adjustment is considered to be the most realistic atmospheric state in the time-loop.