Leith+E backscatter in CESM-MOM6

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Ocean models at eddy-permitting resolutions have some mesoscale eddies, but they are not fully realistic. They tend to be too weak.

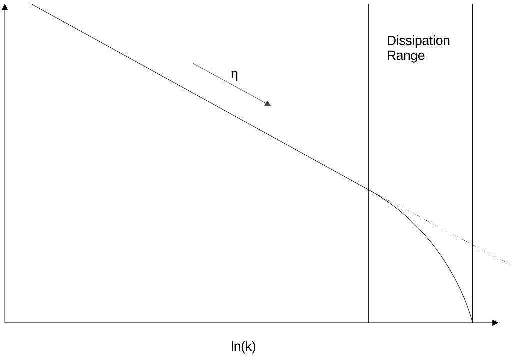
Real mesoscale eddies are mainly driven by baroclinic instability and damped by a zoo of processes. What about in ocean models?

In models they are driven by baroclinic instability and damped by viscosity.

Both can be wrong, but we will focus on damping by viscosity.

- If viscosity is too strong then eddies will be too weak.
- If viscosity is too weak the velocity field will be grainy which will lead to spurious mixing.

The Smagorinsky viscosity is based on the idea of truncating a forward cascade of kinetic energy.



Smagorinsky

Viscous/Kolmogorov dissipation scale for energy:

$$\ell = \left(\frac{\nu_2^3}{\epsilon}\right)^{1/4}$$

Balance energy cascade with dissipation by harmonic viscosity:

$$\epsilon = \nu_2 |\nabla \boldsymbol{u}|^2$$

Adjust ν_2 so that $\ell \sim \Delta x$:

$$\nu_2 \propto (\Delta x)^2 |\nabla \boldsymbol{u}|.$$

2D Leith, Biharmonic

Viscous dissipation scale for enstrophy:

$$\ell = \left(\frac{\nu_4^3}{\eta} \right)^{1/12}$$

Balance enstrophy cascade with dissipation by biharmonic viscosity:

$$\eta = \nu_4 |\nabla^2 \omega|^2$$

Adjust ν_4 so that $\ell \sim \Delta x$:

$$\nu_4 \propto (\Delta x)^6 |\nabla^2 \omega|.$$

These Leith closures (and related QG-Leith closures) have been shown by Fox-Kemper, Pearson, Bachman, and collaborators to have a salutary effect on eddying simulations.

Leith closures are based on reasoning about the *enstrophy* cascade. The turbulence theory also predicts negligible *energy* transfer within the same range of scales.

They still tend to over-dissipate energy though, esp. QG-Leith.

Here we can combine KE backscatter and (hyper)viscosity to construct schemes that respect both the energetic and enstrophetic properties of ocean macroturbulence.

We will use a closure of the form

$$\partial_t \boldsymbol{u} = \ldots + \nabla \cdot (\nu_2 \nabla \bar{\boldsymbol{u}}) - \nabla \cdot (\nu_4 \nabla^3 \boldsymbol{u})$$

where $\bar{\cdot}$ denotes a low-pass spatial filter that *zeroes out grid-scale features.*

Now use two constraints to set $\nu_2 < 0$ and $\nu_4 > 0$:

- The enstrophy dissipation scale is proportional to the grid scale. Use the biharmonic dimensional formula (on a prev. slide) and
$$\eta = \nu_2 |\nabla \bar{\omega}|^2 + \nu_4 |\nabla^2 \omega|^2.$$
- The two terms together dissipate a fraction c_K of the rate associated with the biharmonic term

$$\nu_2 \overline{\omega}^2 = -\nu_4 c_K |\nabla \omega|^2.$$

Together these imply

$$
\begin{aligned}
\nu_4 &= \left(\frac{\Upsilon \Delta x}{\pi}\right)^6 \left[(\nabla^2\omega)^2 - m|\nabla\overline{\omega}|^2\right]^{1/2}, \\
\nu_2 &= -m\nu_4 \\
m &= c_K \frac{|\nabla\omega|^2}{\overline{\omega}^2}.
\end{aligned}
$$

There are several problems with this, including (i) solution does't always exist, and (ii) it doesn't guarantee that scales near the grid scale are damped.

Updates to the raw formula:

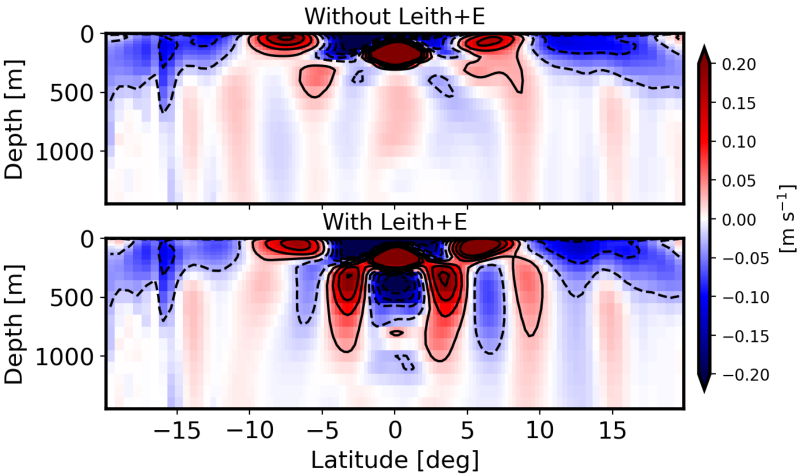
- If pure biharmonic Leith doesn't rise above a minimum then there's probably not a cascade there to truncated, so set $\nu_2 = m = 0$ and $\nu_4 = \min$.
- If $\nu_2 = -m\nu_4$ is too big, reduce m to its max allowed value.
- Apply spatial smoothing to both m and ν_4, the latter via $\sqrt{\overline{\nu_4^2}}$.

I implemented this in MOM6 in summer 2023. H. Yassin evaluated it and MEKE backscatter in CESM-MOM6 at $1/4^\circ$.

A year ago we thought that using Leith *without backscatter* ($c_K = 0$) in the 2/3 degree model might help with the EUC.

This backscatter only turns on if the biharmonic Leith coefficient rises above a background ($10^{12} m^4/s$). Because of the dependence on grid scale $(\Delta x)^6$, this mostly happens in the tropics.

Gustavo Marques ran a simulation that accidentally left the backscatter turned on, with surprising results.

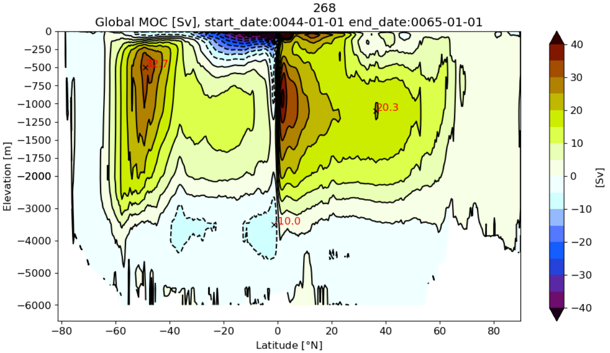


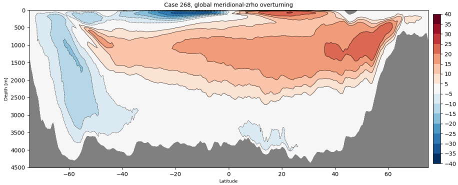
Backscatter generates realistic zonal jets: stacked at the equator, with Tsuchiya jets on the flanks.

These jets ventilate the eastern equatorial Pacific at $\sim$400 m depth, significantly reducing a bias in the Oxygen Minimum Zone extent.

One downside of this backscatter is that it leads to a large MOC cell upwelling at the equator.

We have reduced this by tapering the backscatter smoothly to zero from 1 km to 1.5 km deep, but we have not been able to eliminate it.


268
Global MOC [Sv], start_date:0044-01-01 end_date:0065-01-01
32.7
20.3
10.0
Elevation [m]
Latitude [°N]
[Sv]


Case 268, global meridional-zrho overturning
Depth [m]
Latitude

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