A prototype deep-atmosphere version of the HOMME (SENH) dynamical core.

NCAR, 05-27-2025

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ITCZ among other features of equatorial climatology (e.g. the Madden Julian Oscillation) are biased in ESM simulations

Major modifications in E3SM Atmosphere Model (EAM)v2 to simulate at higher resolution

- Dynamical core in EAM: Higher Order Methods Modeling Environment (HOMME)
- New version solves non-hydrostatic equation set: suitable for high-res simulation (Taylor *et al*. (2020))
- This dynamical core is being reintroduced into CAM/CESM as part of the StormSPEED project
    - Enable scalable simulations at storm-resolving grid spacings.

Dynamical cores solve (approximated) variants of the Euler equations

(Typically in spherical coordinates in a rotating reference frame)

HOMME/EAMv2 non-hydrostatic equations:

u : zonal velocity
v : meridional velocity
w : vertical velocity
ρ : density
T : temperature
p : pressure

$$\frac{\mathrm{D}u}{\mathrm{D}t} - \frac{uv\tan(\varphi)}{a} + \frac{0}{a} = -\frac{1}{\rho a\cos\varphi}\frac{\partial p}{\partial \lambda} + 2\Omega v\sin(\varphi) - 0$$

$$\frac{\mathrm{D}v}{\mathrm{D}t} + \frac{u^2\tan(\varphi)}{a} + \frac{0}{a} = -\frac{1}{\rho a}\frac{\partial p}{\partial \varphi} - 2\Omega u\sin(\varphi)$$

$$\boxed{\frac{\mathrm{D}w}{\mathrm{D}t}} - \frac{0}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 0$$

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \frac{\rho}{r\cos\varphi}\left[\frac{\partial u}{\partial \lambda} + \frac{\partial(v\cos\varphi)}{\partial\varphi}\right] + \frac{\rho}{a^2}\frac{\partial\left(a^2 w\right)}{\partial z} = 0$$

$$c_v\frac{\mathrm{D}T}{\mathrm{D}t} + p\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{1}{\rho}\right) = J$$

$$p = \rho R_d T$$

$$\frac{\mathrm{D}(\ )}{\mathrm{D}t} = \frac{\partial(\ )}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial(\ )}{\partial\lambda} + \frac{v}{a}\frac{\partial(\ )}{\partial\phi} + w\frac{\partial(\ )}{\partial z}$$

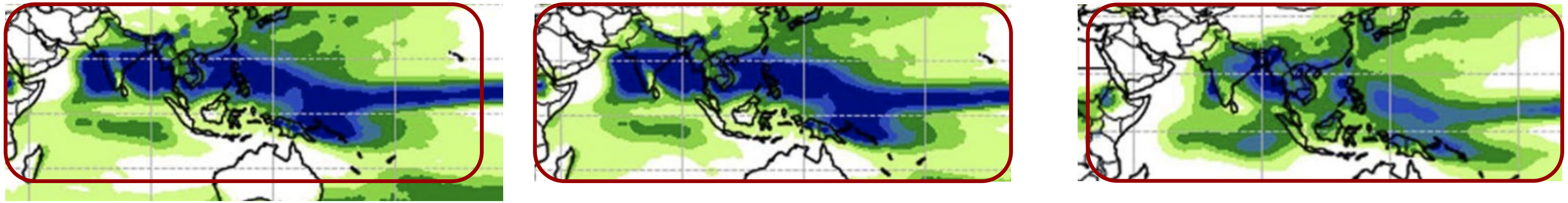
$$g = g_0 \cdot 1$$

Other approx.

=0 in hydrostatic models

Higher-resolution simulations do not resolve ITCZ bias

E3SM Hydrostatic HOMME | E3SM Non-hydrostatic HOMME | Observations



10-year averaged summer precipitation, units mm/day.
Nominal 28 km grid spacings
Reproduced from Liu *et al.* 2022

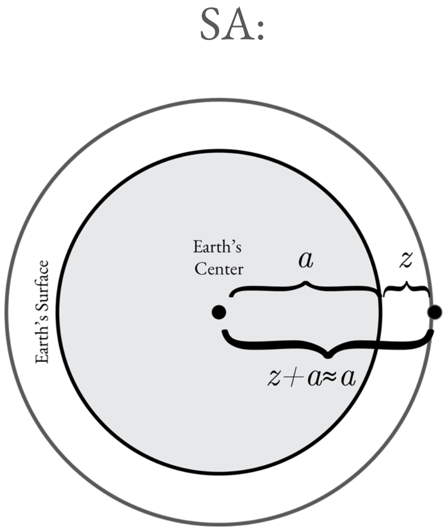
- Ma *et al*. (2019): most high-resolution CMIP6 models continue to exhibit major ITCZ biases.

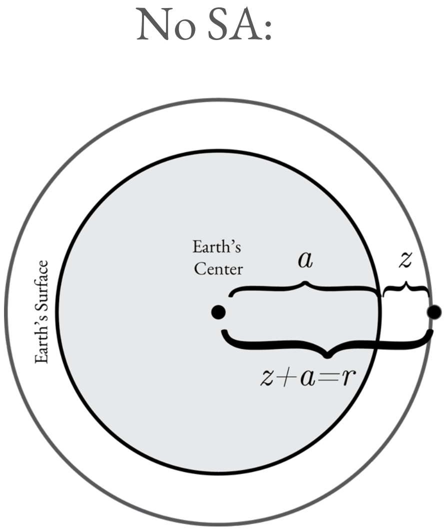
High-resolution non-hydrostatic simulations in E3SMv2 retain equatorial biases

Removing two additional problematic approximations from the dynamical core may improve CESM's (and E3SM's) equatorial climatology

Non-hydrostatic HOMME contains two additional approximations:

1) The Shallow-atmosphere (SA) approximation:





a: nominal radius of earth's surface

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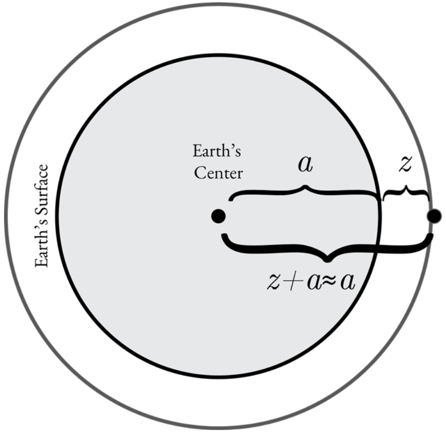
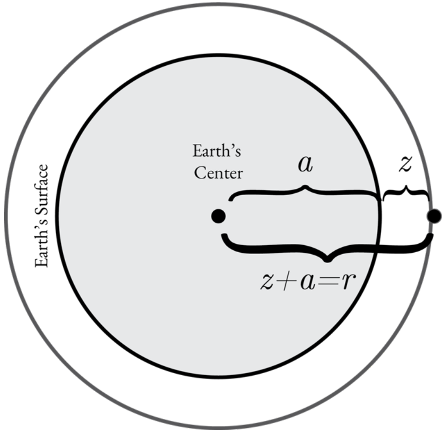
SA:



No SA:



$$g = g_0$$

$$g = g_0\left(\frac{a}{r}\right)^2$$

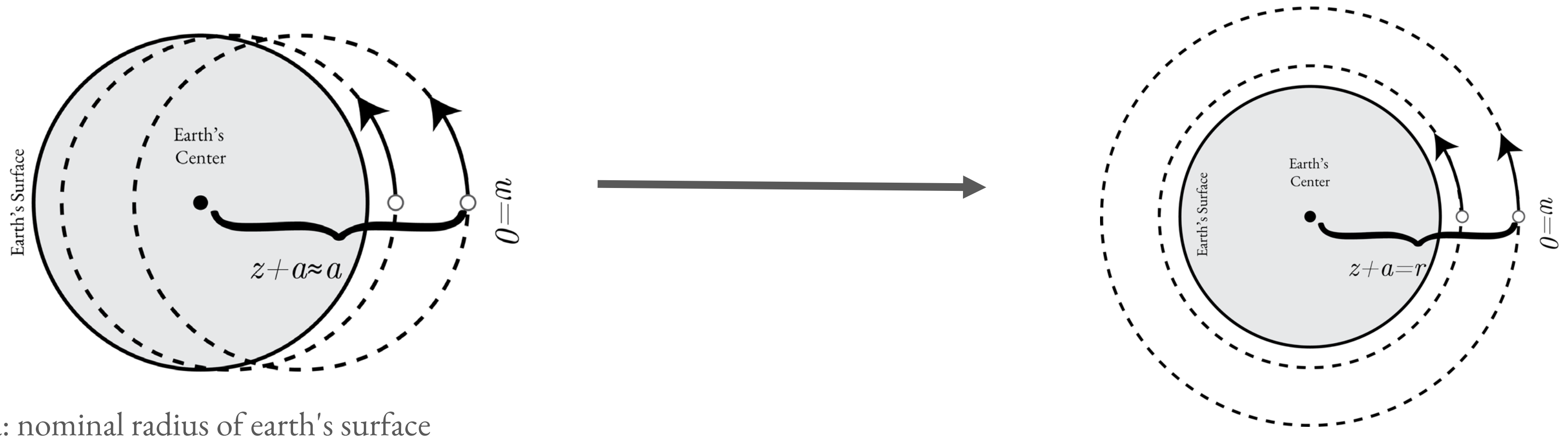
a: nominal radius of earth's surface

Non-hydrostatic HOMME contains two additional approximations:

1) The Traditional (T) approximation:

T: No T:

Discard "metric terms"



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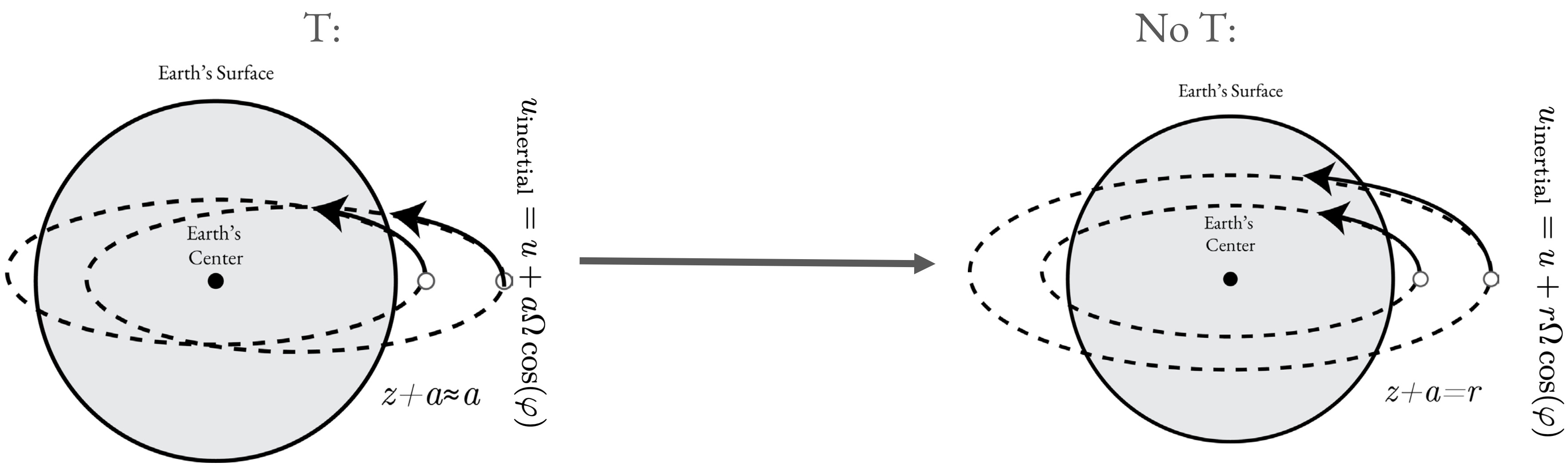

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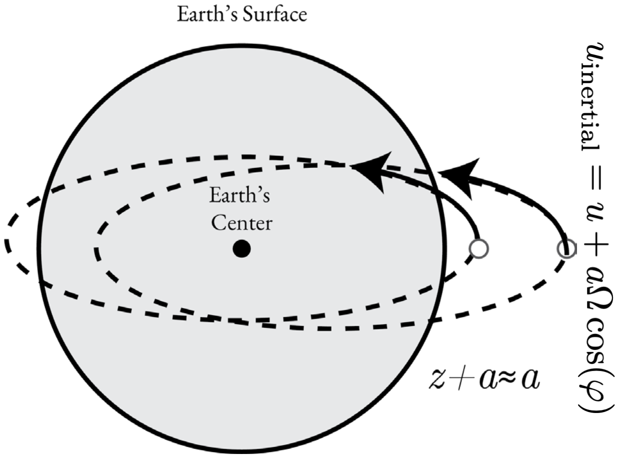

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- In rotating reference frame, this approximation discards terms of Coriolis force involving cosine of latitude
  - So-called Non-Traditional Coriolis Terms (NCTs)
- SA+T must be retained/discarded together to maintain conservation of momentum (White *et al., 2005)*

The Non-Hydrostatic Deep-Atmosphere equations:

$u:$ zonal velocity

$v:$ meridional velocity

$w:$ vertical velocity

$\rho:$ density

$T:$ temperature

$p:$ pressure

=0 in T approx

r = a in SA approx

=0 in hydrostatic models

$$\frac{\mathrm{D}u}{\mathrm{D}t} - \frac{uv\tan(\varphi)}{r} + \frac{uw}{r} = -\frac{1}{\rho r\cos\varphi}\frac{\partial p}{\partial\lambda} + 2\Omega v\sin(\varphi) - 2\Omega w\cos(\varphi)$$

$$\frac{\mathrm{D}v}{\mathrm{D}t} + \frac{u^2\tan(\varphi)}{r} + \frac{vw}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial\varphi} - 2\Omega u\sin(\varphi)$$

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$$g = g_0\left(\frac{a}{r}\right)^2$$

The SA + T approximations are justified for large-scale motion in the midlatitudes

In the presence of convective motion near the equator, are the NCTs still negligible?

Idealized studies indicate no

(e.g., Ong & Roundy (2019) and many other papers by Hing Ong)

Phase 1:
Preliminary results from Deep-Atmosphere HOMME/SENH

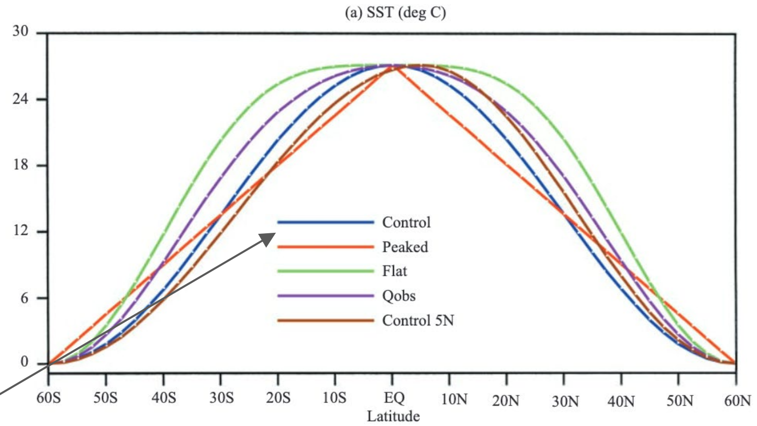
DA HOMME in the fewest changes

- All HOMME based on a Continuous-Galerkin Spectral Finite Element discretization of the cubed sphere (Taylor and Fournier, (2010))
  - Satisfies discrete versions of vector calc identities
  - Together with vertical coordinate: ensures mass, energy conservation
  - Implement DA HOMME without violating these conservation properties?
- Few steps to implement:
  - Modify *a* factors that appear in differential operators with factors of $\hat{r} = \dfrac{a + z}{a}$
  - Restore missing metric terms, NCTs from Traditional approximation
  - Modify time stepping that handles handle numerical stability restrictions in vertical.
  - Modify mass coordinate for deep atmosphere

CESM QPC6 Results:
(Produced via StormSPEED codebase)

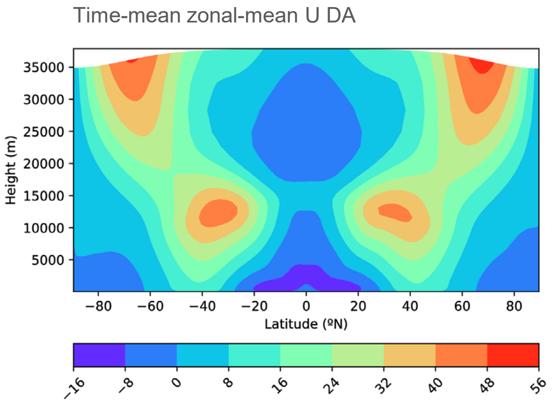
Aquaplanet simulations couple dycore to physics, idealized ocean:

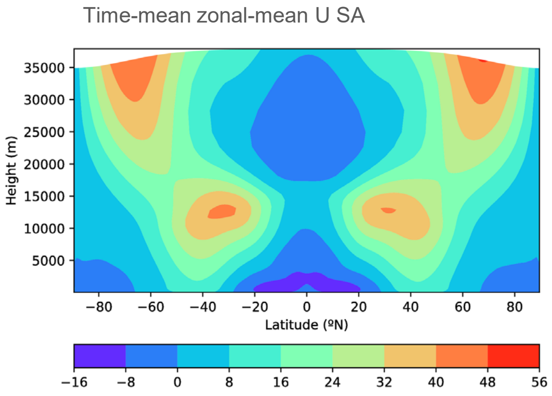
- Full-complexity atmospheric physics (in this case, CAM6)
  - E.g., complex microphysics, a deep convection parameterization, radiation, etc.
- Fluxes from lower boundary come from "data ocean"
  - Fully ocean-covered planet
  - No land mask, no topography
  - Idealized (e.g. zonally symmetric) Sea Surface Temperature pattern

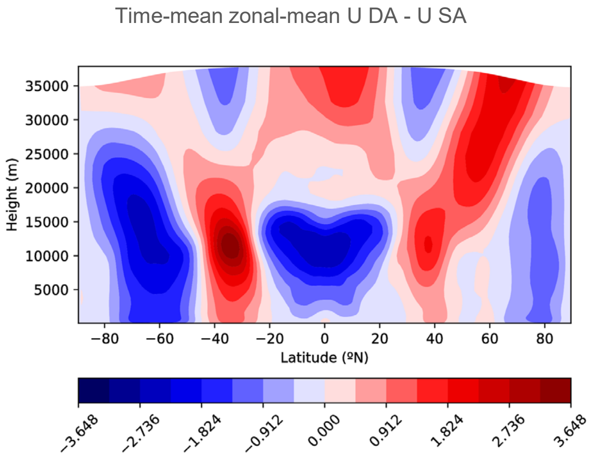


Reproduced from Neale and Hoskins (2001)

Approximately 10% difference in tropospheric zonal wind at the equator







2 m/s bias on top of 20 m/s wind

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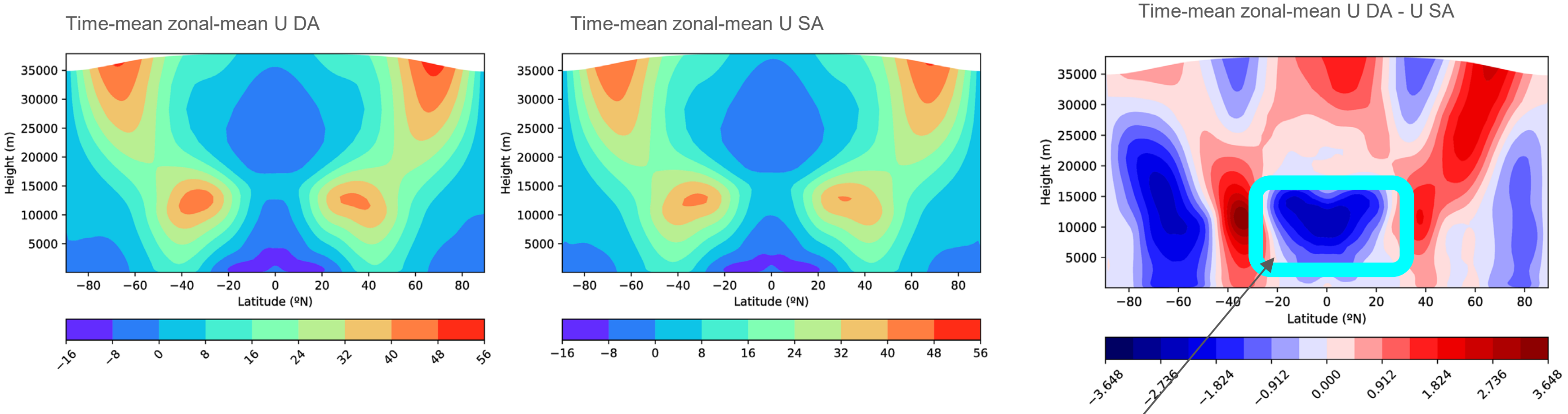

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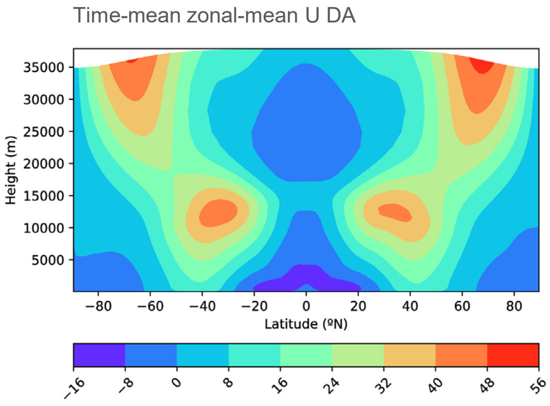
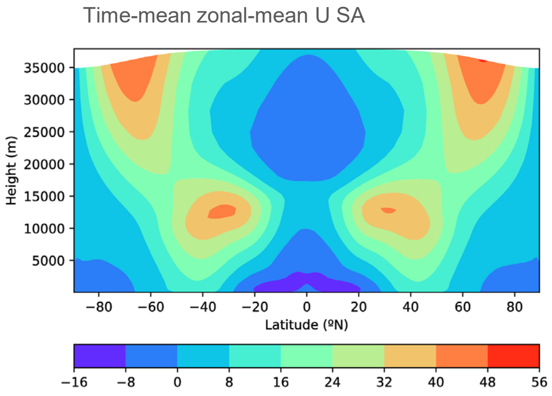
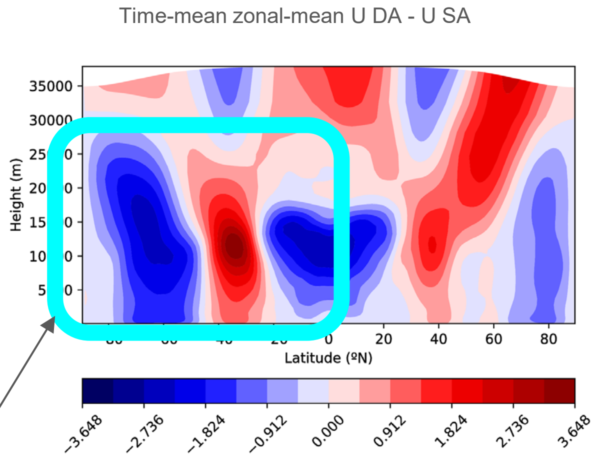
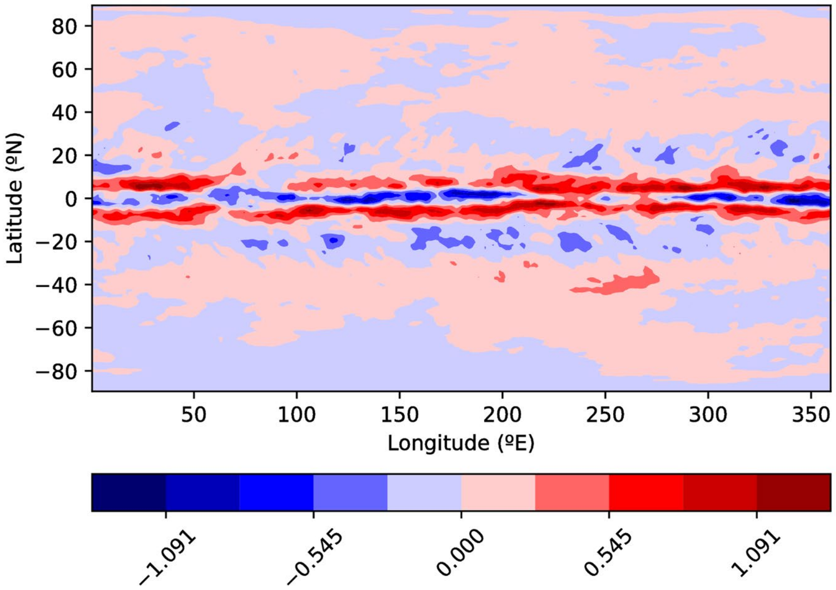


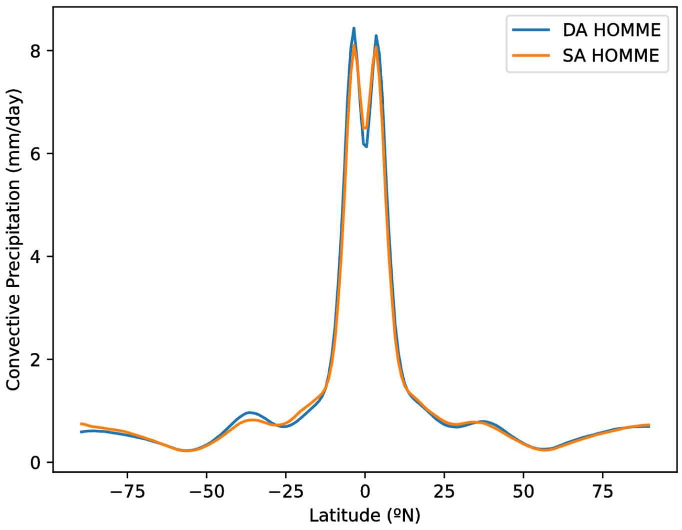




Changes are of equal magnitude outside of tropics, extending to the poles. Little study of what

In CESM QPC6 the deep atmosphere does not improve





Very preliminary: idealized tropical cyclone is influenced by TA approximation?

- Previous paper (Liang and Chan, 2005) indicate TA biases TC track, precipitation patterns.
- Reed & Jablonowski idealized tropical cyclone at ¼ º grid spacing shows different structure in DA HOMME (QPC6 physics, bc's)

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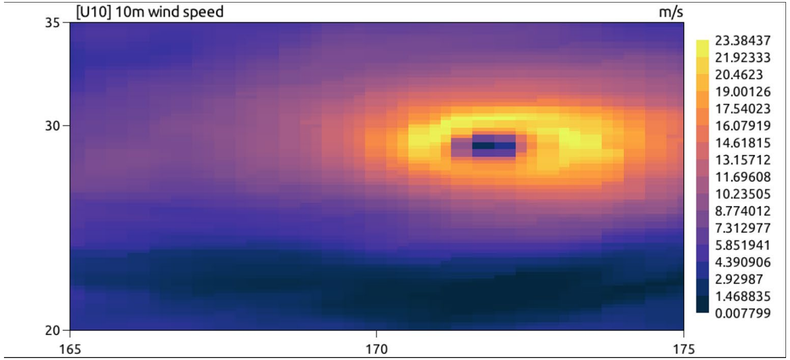
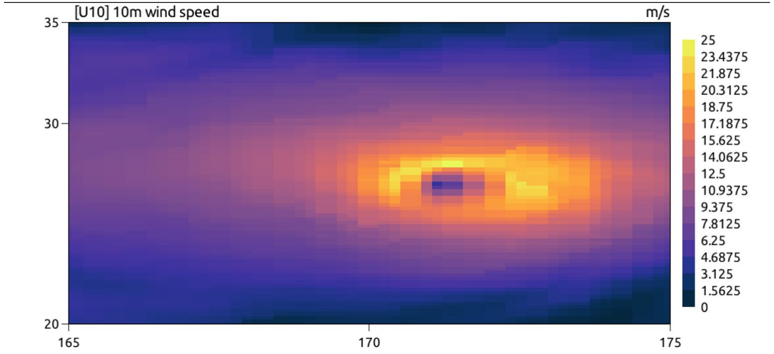
- Previous paper (Liang and Chan, 2005) indicate TA biases TC track, precipitation patterns.
- Reed & Jablonowski idealized tropical cyclone at ¼ º grid spacing shows different structure in DA HOMME (QPC6 physics, bc's)
- Position of pressure minimum after 10 days is measurably different!!

SA HOMME

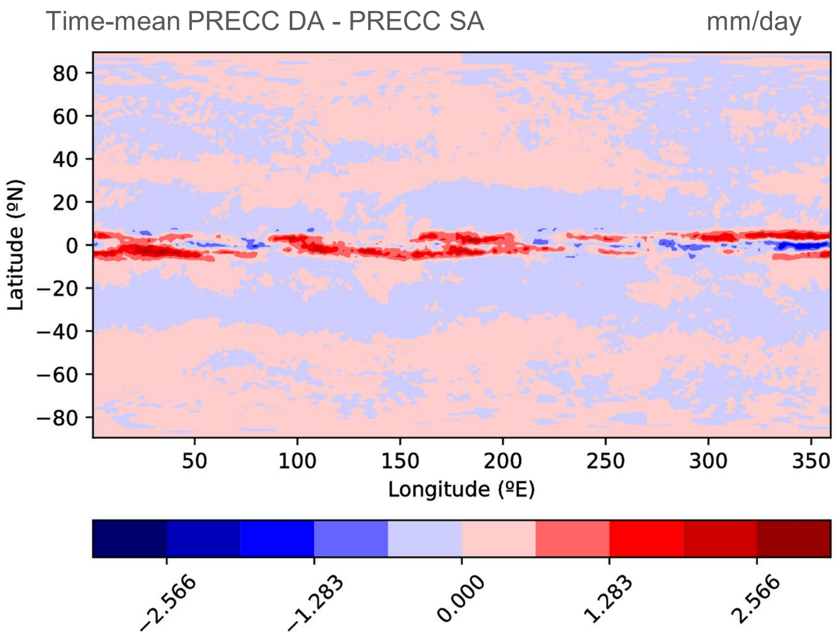


DA HOMME



E3SM (EAM v1) Results:

Approximately 10% difference in convective precip (PRECC)



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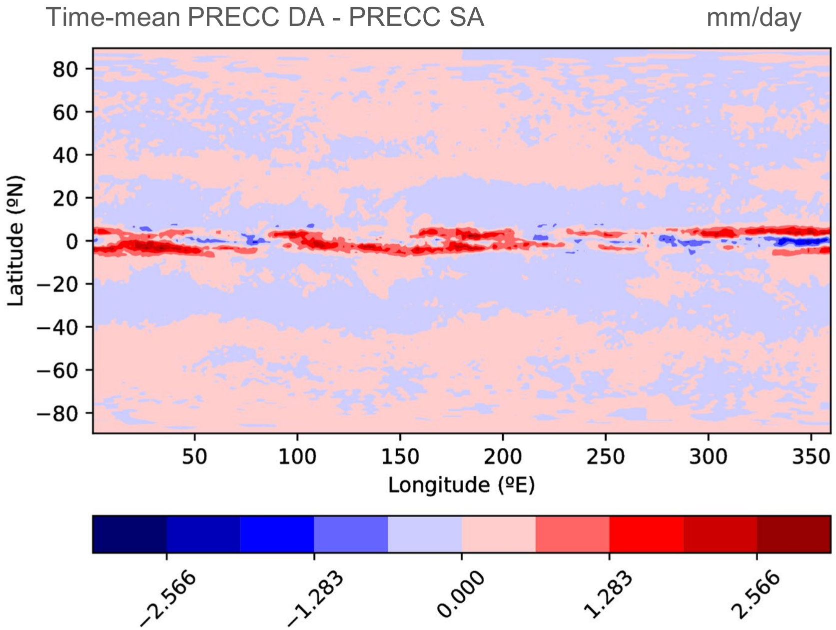
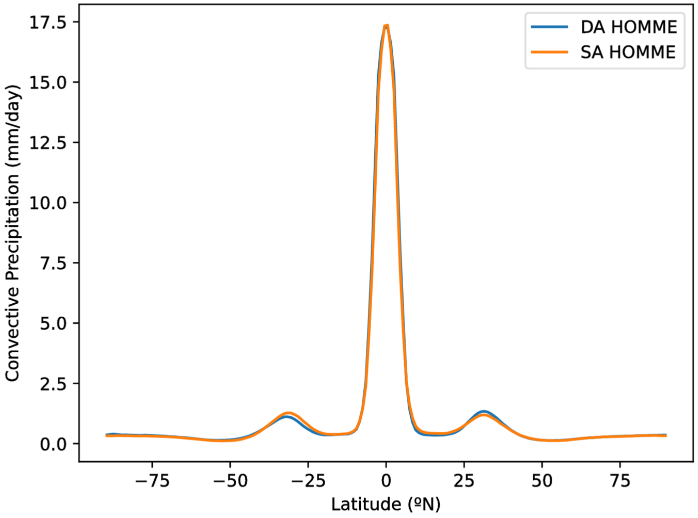
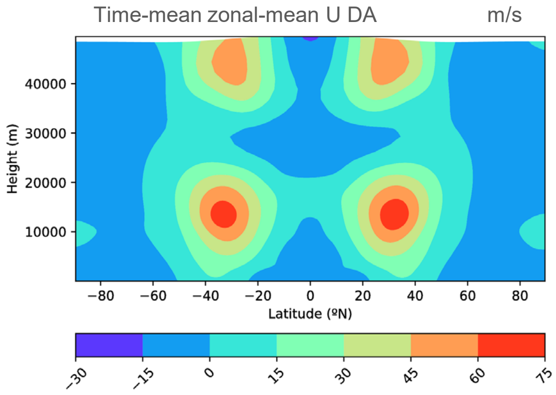
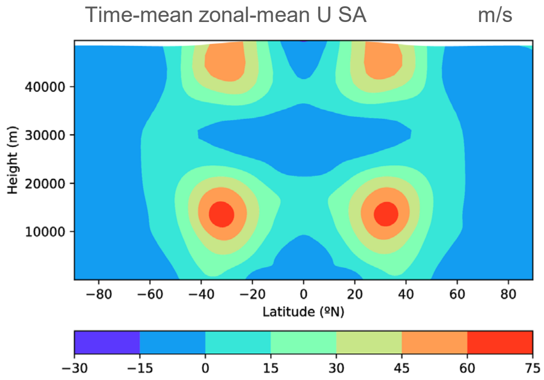


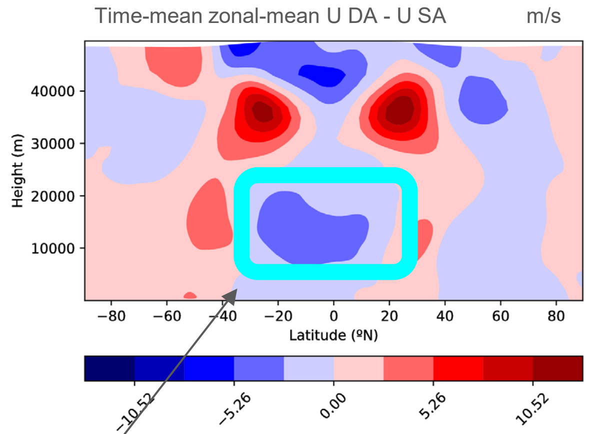
Disappears in zonal average?



Approximately 10% difference in tropospheric zonal wind

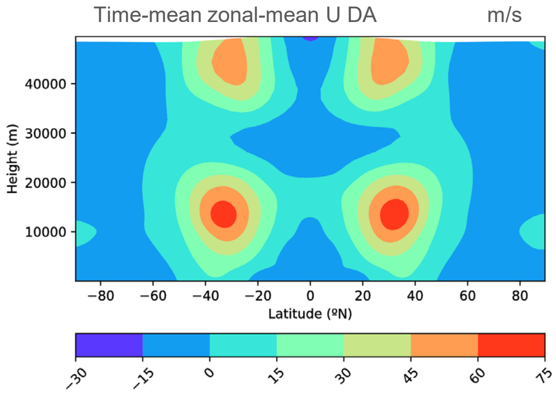


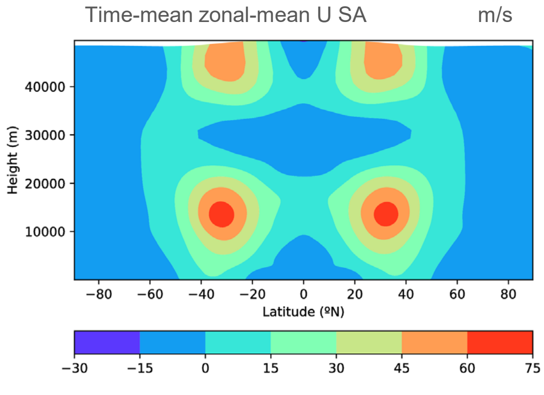


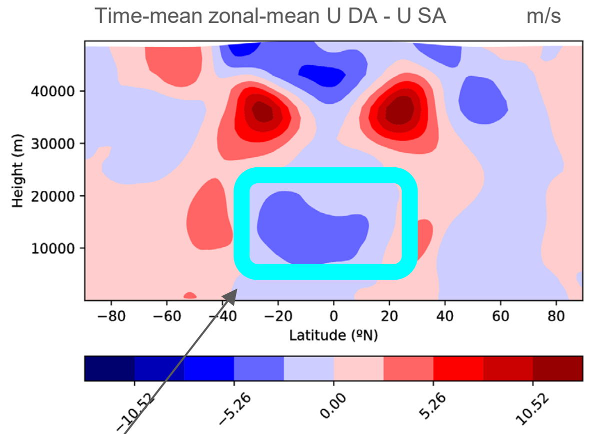


2 m/s bias on top of 20 m/s wind

Approximately 10% difference in tropospheric zonal wind



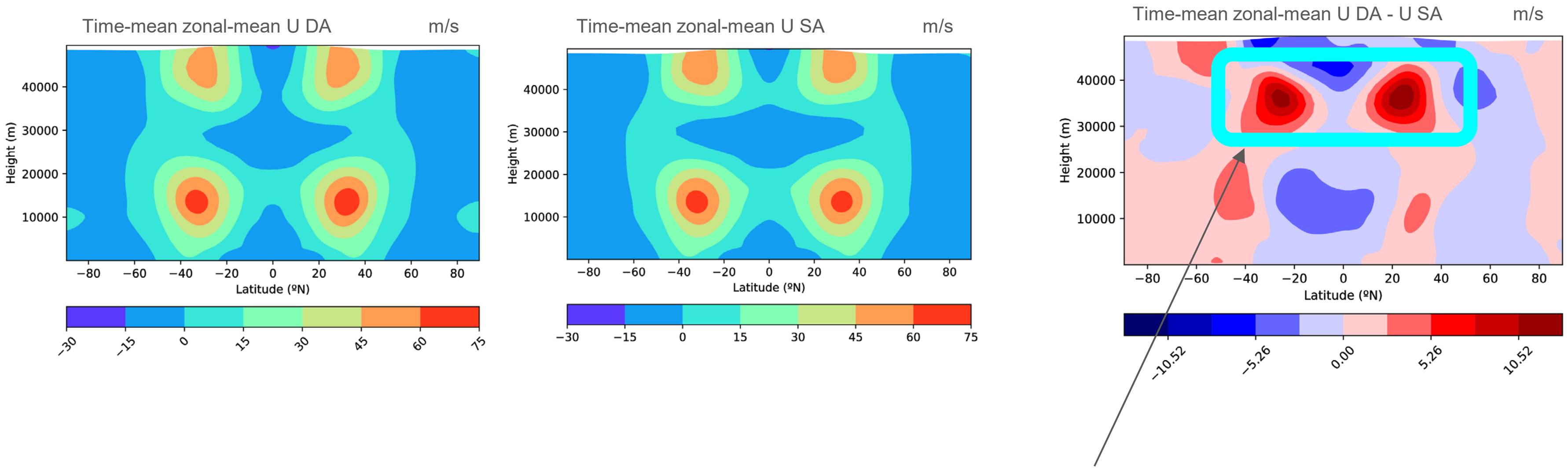




Magnitude resembles linearized result of Ong and Roundy (2019)!

2 m/s bias on top of 20 m/s wind

> 10% difference in stratospheric zonal wind



Stratospheric changes are not well studied. 10 m/s bias

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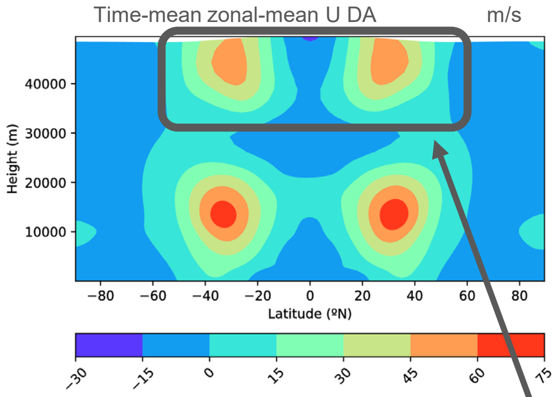

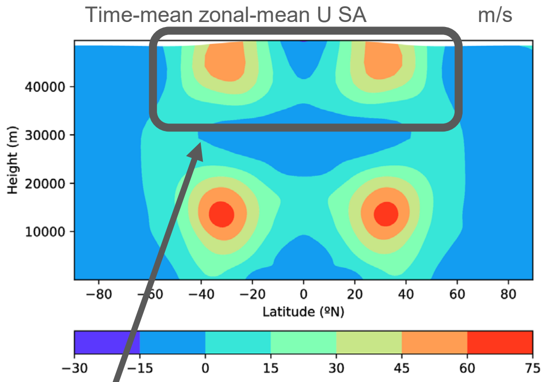

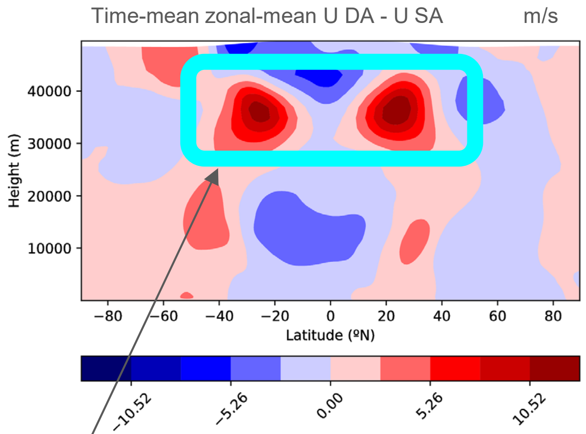

Strength stays the same, but position shifts.

Stratospheric changes are not well studied. 10 m/s bias

Summary:

- Designed a deep-atmosphere version of HOMME for E3SM,
  - could improve equatorial climatology, double ITCZ
- Porting this version of HOMME into the CAM StormSPEED codebase this summer!
- Beginning to systematically study deep-atmosphere equatorial climatology in E3SM/CESM
  - Systematic biases are observed not just in the equatorial troposphere
  - Stratospheric biases
  - Midlatitudinal biases.
- Phase 2:
  - E3SM/CESM simulations using Aquaplanets, Atmospheric Model Intercomparison Project (AMIP) simulations.
  - Global convection-permitting simulations at 3 km nominal grid spacing.

CAVEAT EMPTOR (bonus slides)

Prototype only implemented for most-used HOMME config:

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    - Floating vertical levels

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- Taylor *et al*. (2020) describe versions of Non-hydrostatic shallow HOMME for several vertical coords.
- Version used in E3SMv2/EAMv2 is terrain-following mass coordinate
  - Floating vertical levels
- HOMME with this choice of vertical coordinate:
  - Shallow-Atmosphere (SA) HOMME
- Modified SA HOMME to remove SA+T approximations:
  - Deep-Atmosphere (DA) HOMME

DA HOMME in the fewest changes

- All HOMME based on a Continuous-Galerkin Spectral Finite Element discretization of the cubed sphere (Taylor and Fournier, (2010))
    - Satisfies discrete versions of vector calc identities
    - Together with vertical coordinate: ensures mass, energy conservation
    - Implement DA HOMME without violating these conservation properties?

A mass coordinate suitable for the deep atmosphere

- Ignore details of terrain-following η

Redefined mass coordinate DA HOMME is difficult to initialize on small planets

- New DA HOMME η coordinate has non-linear relation with SA HOMME η coordinate
- This discrepancy introduces several complications for model initialization
  - We use numerical root-finding to initialize DA HOMME for small-planet simulations.
  - Problems are negligible on earth-sized planet.
  - (this took like 5 months to debug)

How did we validate DA HOMME is correct?

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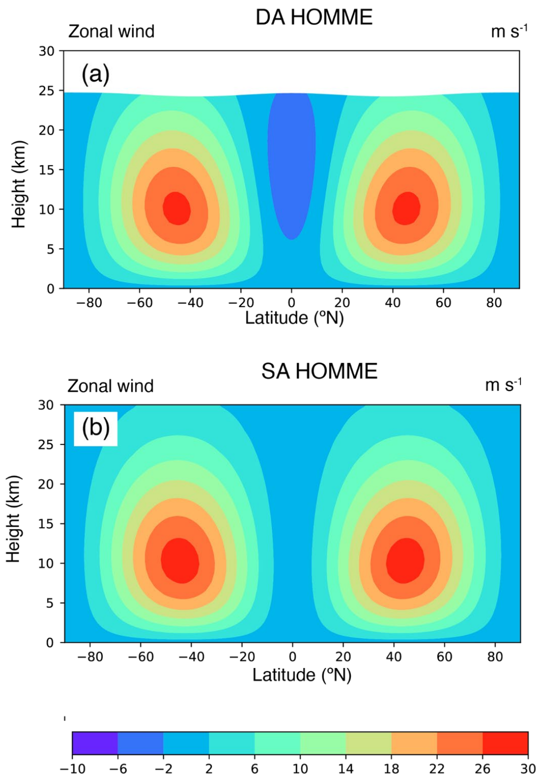
    - Test e.g. if equations are discretized correctly
  - Well-studied very idealized climate simulations with solns in literature
    - Tests if the model well-behaved in long simulations
- Small-earth simulation
  - Reduce radius of planet dynamical core is simulating: $a = \frac{a_{\mathrm{earth}}}{X}$
  - Increase rate of rotation to keep Coriolis magnitude comparable: $\Omega = X\Omega_{\mathrm{earth}}$
  - Larger difference between deep and shallow atmosphere dynamics!
  - Large-earth day → small-earth day

Verifying DA HOMME's ability to hold steady-state

- Ullrich *et al*. (2014) derives baroclinically unstable atmospheric steady-states for the deep and shallow atmospheres.

Verifying DA HOMME's ability to hold steady-state

- Ullrich *et al*. (2014) derives baroclinically unstable atmospheric steady-states for the deep and shallow atmospheres.
  - Deviation from initial conditions is error
  - Deviations amplify due to baroclinic instability.
    - Steady-state breaks down
  - Vertical velocity, w should remain identically zero, so max. magnitude of w is error proxy

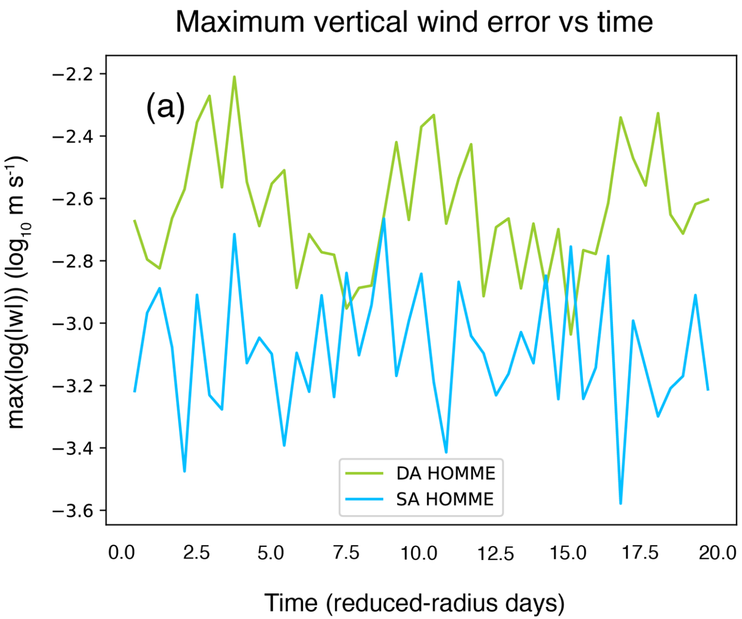


Verifying DA HOMME's ability to hold steady-state

- Simulations for 20 small-earth days, nominal radius $\frac{a_{\text{earth}}}{10}$
  - 30 vertical levels, 1º grid spacing.

Verifying DA HOMME's ability to hold steady-state

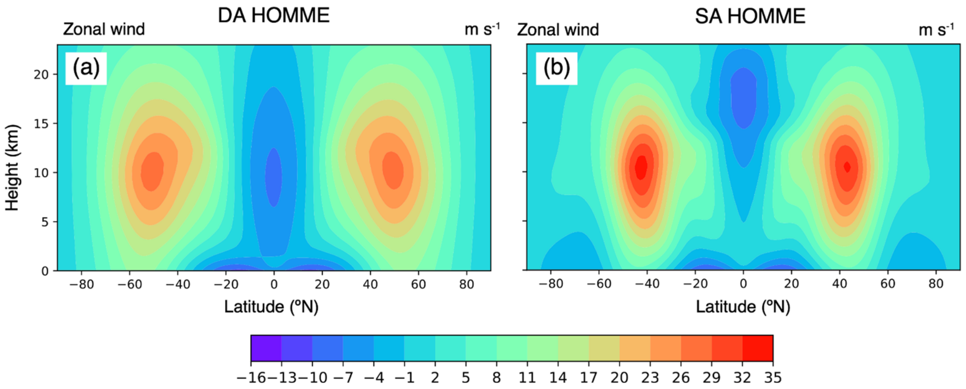
- Simulations for 20 small-earth days, nominal radius $\frac{a_{\mathrm{earth}}}{10}$
  - 30 vertical levels, 1º grid spacing.
- Error in DA HOMME is larger
  - Possibly due to discretization of Pressure Gradient Force
- Time to steady-state breakdown (20 small-earth days) is comparable for:
  - DA HOMME initialized with a deep-atmosphere steady-state
  - SA HOMME initialized with a shallow-atmosphere steady state



Maintaining a deep-atmosphere steady state indicates DA HOMME solves deep-atmosphere equations well (in E3SM)

A dynamical-core-only idealized climate-like test:

- Held-Suarez (1994) forcing:
  - Relaxation of temperature to a reference profile
  - Remove velocity at low levels
  - Induces general circulation
  - Climate equilibrates after a spin-up period
- Performed simulations:
  - for 2000 small-earth days on planet with radius
  - at 2º nominal grid spacing, 30 vertical levels.
- Results approximately match HS simulations in the literature (Yessad and Wedi, 2009)
  - Equatorial bias due to NCTs!



CESM:

- I have a prototype port of the fortran version of DA HOMME into the CAM StormSPEED repo that currently compiles
  - Have not run steady-state test yet (will do in the next day)
- Planning to run deep-vs-shallow QPC6 (default SST) in CESM this weekend.
- If all you care about is research configuration of CAM-SE with NCTs (disobeys conservation laws)
  - You can just add like 10 lines of code to CAM
- Question is: Can we use Wood and Staniforth 2003 trick for principled QHE-CAM-SE?
  - Maybe yes within dycore.
    - Modify prim_advance_mod.F90 and eos.F90. Not sure what the quasi-hydrostatic EOS looks like in CAM-SE, but likely not hard to derive.
    - Implications for spatial energy conservation. I know how it works for the HOMME Lorentz staggering. Not sure for CAM-SE.
  - We got lucky since HOMME vertical remap and ISLET transport scheme turned out not to need modification (just pass in same Δp with different interpretation). Might be different in CAM?
  - Changes to CAM physics are likely shared with the work I'll be doing on SENH.
- My degree roadmap doesn't include this work at the moment
  - That said, I care about making the SE/HOMME dycore better for the most users possible.
  - If I have the skills to do this, I'd like to make time for it.

Mild sleuthing (shoddy non-conservative method):

prim_advance_mod.F90

```fortran
        do j=1,np
          do i=1,np
            glnps1 = pgf_term(i,j,1)
            glnps2 = pgf_term(i,j,2)
            v1     = elem(ie)%state%v(i,j,1,k,n0)
            v2     = elem(ie)%state%v(i,j,2,k,n0)

            vtens1(i,j,k) =   &
                 + v2*(elem(ie)%fcor(i,j) + vort(i,j,k))        &
                 - vtemp(i,j,1) - glnps1

            vtens2(i,j,k) =   &
                 - v1*(elem(ie)%fcor(i,j) + vort(i,j,k))        &
                 - vtemp(i,j,2) - glnps2
            ttens(i,j,k)  =  - vgrad_T(i,j) + &
                 density_inv(i,j)*omega_full(i,j,k)*inv_cp_full(i,j,k,ie)
          end do
        end do
```

element_mod.F90

```fortran

     real (kind=r8)    :: fcor(np,np)                               ! Coreolis term

```

cube_mod.F90

```fortran
    rangle = rotate_grid * PI / 180._r8
    do j=1,np
       do i=1,np
             if ( rotate_grid /= 0) then
                lat = elem%spherep(i,j)%lat
                lon = elem%spherep(i,j)%lon
                elem%fcor(i,j)= 2*omega* &
                     (-cos(lon)*cos(lat)*sin(rangle) + sin(lat)*cos(rangle))
             else
                elem%fcor(i,j) = 2.0_r8*omega*SIN(elem%spherep(i,j)%lat)
             endif
       end do
    end do
```

Principled version:

- Differential operators in explicit time step routine need to be scaled by factors involving r (i.e. z + a).
  - Requires addit'l vertical scan as it is reconstructed by

$$\frac{G}{3a^2}(r^3 - r_{\mathrm{H}}^3) \equiv (Gr - Gr_{\mathrm{H}}) \left(\frac{r^2 + r r_{\mathrm{H}} + r_{\mathrm{H}}^2}{3a^2} \right) = \int_{\eta}^{\eta_{\mathrm{H}}} \frac{1}{\rho} \frac{\partial \Pi}{\partial \eta'} \, \mathrm{d}\eta',$$

  - (less bad than it looks, I've taken it directly from Wood 2003 and left their rather general notation)

- Energy conservation + diagnosis of Omega change.
  - Implicit change to EOS. Are invocations of EOS to compute ρ centralized in CAM-SE? If not, Uh-oh.

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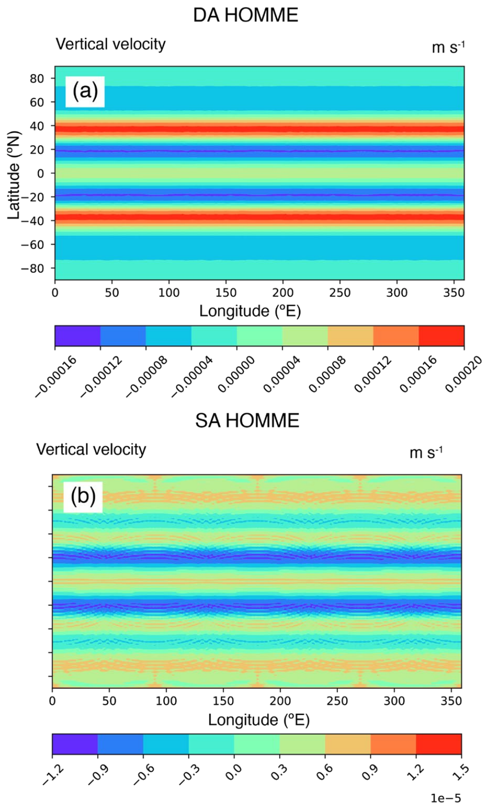
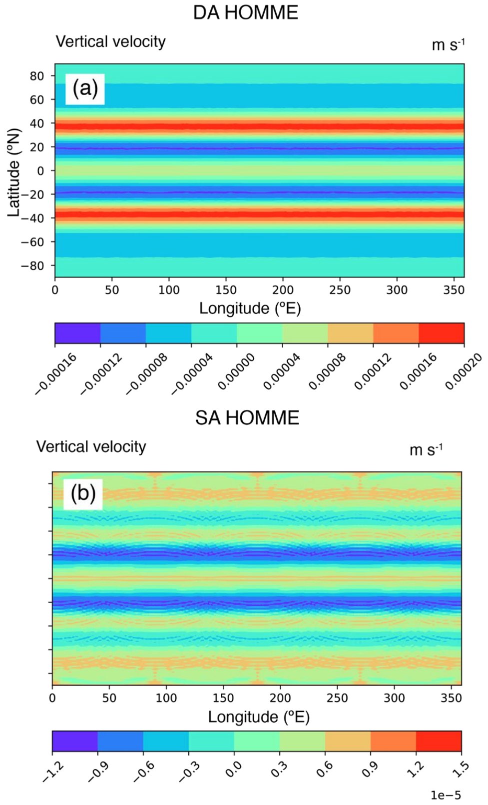
Why is error larger in DA HOMME?

- Error is larger in DA HOMME:
  - Zonally symmetric error in DA HOMME
  - Less zonally symmetric in SA HOMME



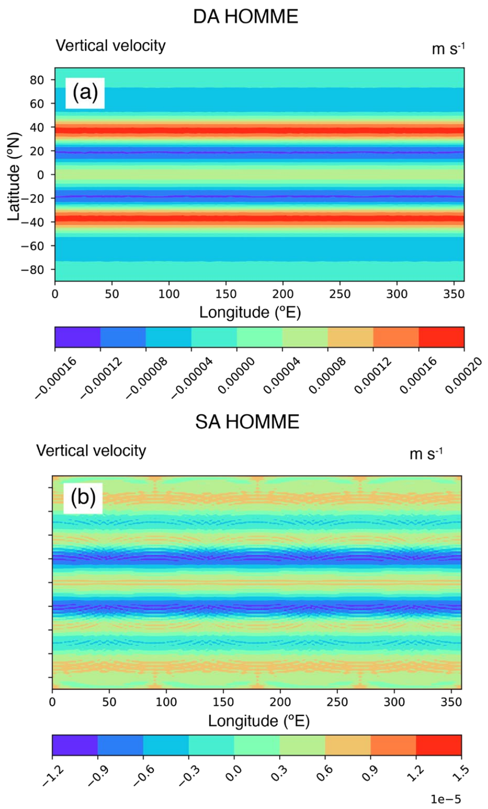
Why is error larger in DA HOMME?

- Error is larger in DA HOMME:
  - Zonally symmetric error in DA HOMME
  - Less zonally symmetric in SA HOMME
  - Root-finding means
    - Pressure gradient≠0 in DA HOMME
    - Pressure gradient=0 in SA HOMME

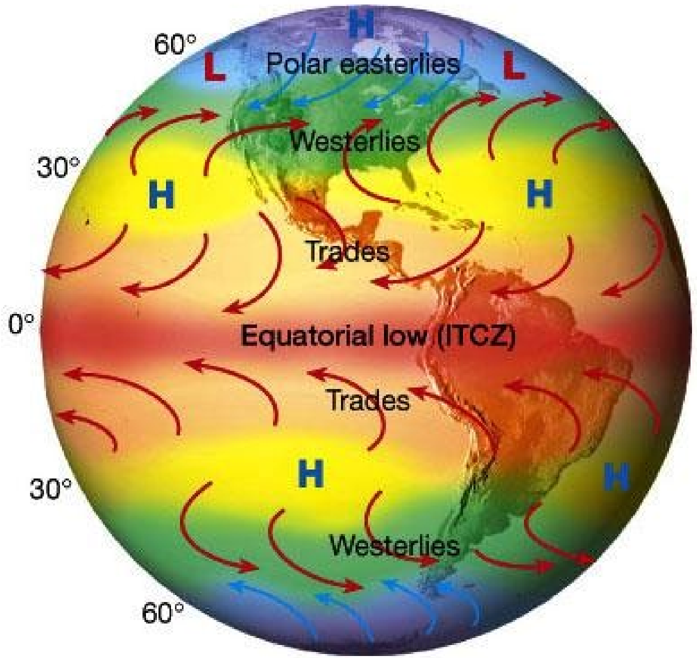


Why is error larger in DA HOMME?

- Error is larger in DA HOMME:
  - Zonally symmetric error in DA HOMME
  - Less zonally symmetric in SA HOMME
  - Root-finding means
    - Pressure gradient≠0 in DA HOMME
    - Pressure gradient=0 in SA HOMME
- Time to steady-state breakdown remains same.
  - Ongoing work on initialization should shed more light on this, but promising first results



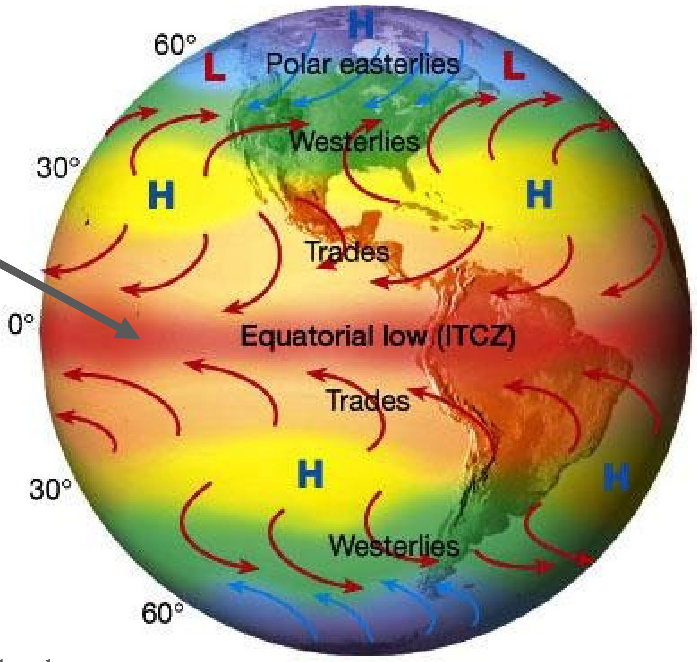
ESMs struggle to simulate the Intertropical Convergence Zone (ITCZ):



Reproduced from Lutgens and Tarbuck (2001)

ESMs struggle to simulate the Intertropical Convergence Zone (ITCZ):

Converging trade winds induce upward motion, induces precipitation



Reproduced from Lutgens and Tarbuck (2001)

HOMME with hydrostatic approximation cannot resolve convection

Hydrostatic

$$\frac{\partial p}{\partial z} = -\rho g$$

Appropriate for synoptic motions, models with O(100 km) resolution

Vertical velocity not a prognostic variable of the model

Convection is parameterized.

Non-hydrostatic

$$\frac{\mathrm{D}w}{\mathrm{D}t} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g$$

Appropriate for high-resolution models with O(<30 km) resolution

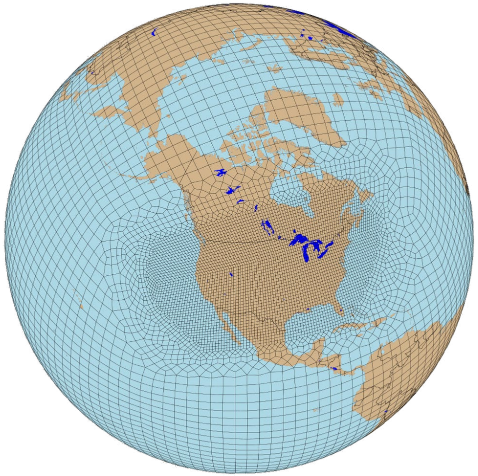
Vertical velocity *w* restored as prognostic variable of the model

Convection (partly) simulated by the dynamical core

$$\frac{\mathrm{D}(\ )}{\mathrm{D}t} = \frac{\partial(\ )}{\partial t} + \frac{u}{r\cos\phi}\frac{\partial(\ )}{\partial\lambda} + \frac{v}{r}\frac{\partial(\ )}{\partial\phi} + w\frac{\partial(\ )}{\partial r}$$

E3SM + HOMME can be run with grid refinement:

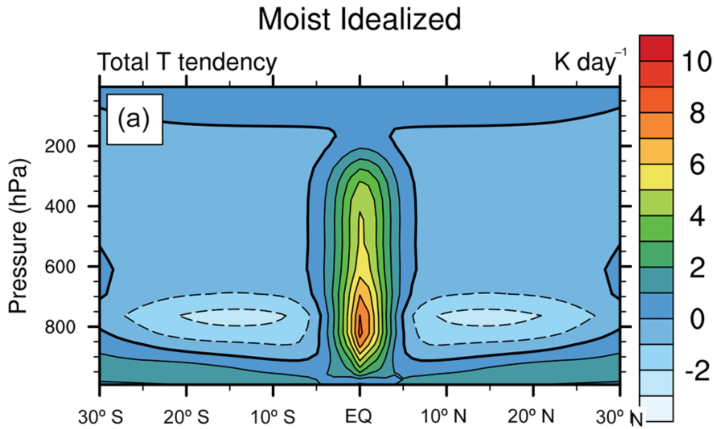
- Recent efforts to scientifically validate E3SM on refined grids (Tang *et al*. 2023)
- For our purposes we will design (or select from literature) a grid with approximately zonally-symmetric refinement about the equator.
    - Previously successfully generated a similar grid for the MPAS dynamical core with 30 km minimum grid spacing
    - Likely to target 7 km minimum nominal grid spacing or so for E3SM.



E.g. 2D refined grids can be generated and used for a full 3D atmosphere out of the box. Reproduced from Tang *et al.* (2018)

Moist Held-Suarez Test: idealized ITCZ without coupling?

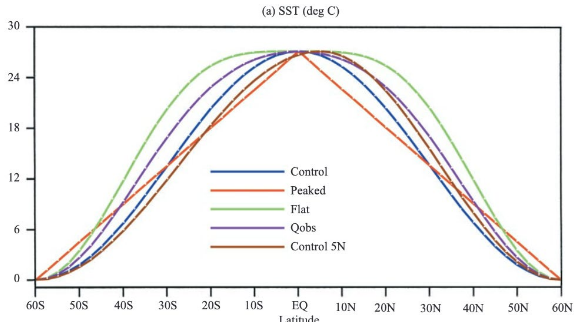
- Thatcher and Jablonowski (2016) developed analogue of the Held-Suarez test case
  - Idealized large-scale condensation, boundary mixing, and surface heat fluxes are added direct to the dynamical core
  - Mimics climatologies of experiments with more complex physics (particularly aqua-planet)
- Large scale condensation may not mimic heating profile of convection near the equator
- However, it does contain an ITCZ and could show differences in climatological "precipitation"



Time-mean zonal-mean plot of thermal forcing for a Moist Held-Suarez test in the CAM5 dynamical core. Reproduced from Thatcher and Jablonowski (2016).

Aqua-planet simulations allow years-long simulation of ITCZ

- Aqua-planet simulations simplify lower atmospheric boundary*
  - Typically a flat ocean covering the entire planet with prescribed SSTs.
  - Run with full-complexity physics (potentially deep convection)
  - Run for multiple years
- Offer potential to study wave dynamics, propagating convective systems
- Principal findings will relate to impacts of deep-atmosphere on precipitation climatology
  - Cheaper to run for multiple years than e.g. AMIP



Prescribed surface temperature profiles for the subset of aqua-planets with axisymmetric SSTs described in Neale and Hoskins (2000), reproduced from said paper.

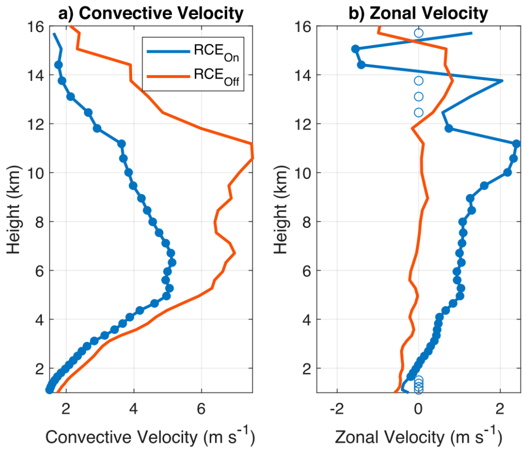
* the "aqua-planet" umbrella covers many configurations, see e.g. Neale and Hoskins (2000), Williamson *et al.* (2012), Blackburn *et al*. (2013) or others.

AMIP findings have best chance to apply to earth's climate

- Activate atmosphere and land models
- Couple to prescribed boundary conditions for ocean, sea-ice, greenhouse emissions, aerosols
- Can be compared to archived E3SMv1, E3SMv2 shallow AMIP runs if same initial conditions are run
- High realism of this model makes findings about MJO, ITCZ most likely to apply to CMIP-stype simulations.
- Running on custom refined grids may present issues with boundary data!

The problem with the TA: ITCZ

- Igel and Biello (2020) found TA results in drastic biases in convection-permitting simulations of cumulus convection, Radiative-convective Equilibrium experiments (RCE).
- TA may induce biases at scales ranging from synoptic scale to individual cumulus columns.



A comparison of RCE experiments with NCTs present (RCE_{ON}) and absent (RCE_{OFF}) for profiles of (a) mean vertical velocity where w > 1m/s and (b) zonal velocity in convection. Reproduced from Igel and Biello (2020).

The problem with the TA: normal modes, wave propagation

- Linearized studies dynamical studies find pronounced wave propagation errors at the equator due to TA. See e.g. Thuburn *et al*. (2002); Kasahara (2003a/b, 2004); Durran and Bretherton (2004).
- Errors from TA become worse as horizontal resolution increases!
- Figure right: frequency of normal modes can vary by a small amount in the presence of the SA+TA approximations
  - Unlikely to be significant

Frequency of normal modes with zonal wavenumber 1 in a shallow/deep rotating atmosphere.

Mode type	Meridional mode	Vertical mode	Frequency, shallow atmosphere, constant g	Frequency, deep atmosphere, constant g	Frequency, deep atmosphere, variable g
Acoustic	0	0 (external)	5.45667×10^{-5} -1.32896×10^{-4}	5.44156×10^{-5} -1.32748×10^{-4}	5.44145×10^{-5} -1.32747×10^{-4}
Acoustic	2	0 (external)	2.87183×10^{-4} -2.92754×10^{-4}	2.86538×10^{-4} -2.92117×10^{-4}	2.86533×10^{-4} -2.92112×10^{-4}
Acoustic	0	2	3.27377×10^{-2} -3.27377×10^{-2}	3.27234×10^{-2} -3.27235×10^{-2}	3.25373×10^{-2} -3.25374×10^{-2}
Gravity	0 (Kelvin)	2	3.14113×10^{-5}	3.12593×10^{-5}	3.10370×10^{-5}
Gravity	2	2	1.87932×10^{-4} -1.95262×10^{-4}	1.87105×10^{-4} -1.94349×10^{-4}	1.86170×10^{-4} -1.93459×10^{-4}
Rossby	0	0 (external)	-1.45975×10^{-5}	-1.45721×10^{-5}	-1.45719×10^{-5}
Rossby	2	0 (external)	-3.06824×10^{-6}	-3.06671×10^{-6}	-3.06671×10^{-6}
Rossby	0	2	-9.58848×10^{-6}	-9.52404×10^{-6}	-9.46493×10^{-6}

All modes are symmetric about the equator with zonal wave number $m = 1$. Where two values are shown these are for an eastward- and westward-propagating pair of modes.

Reproduced from Thuburn *et al.* 2002

HOMME's funky EOS:

Shallow mass coordinate:

$$\pi(\eta) = \int_z^\infty g_0 \rho \, \mathrm{d}z \left(= \int_{\eta(z)}^0 g_0 \rho \frac{\partial z}{\partial \eta} \, \mathrm{d}\eta \right) \longrightarrow \frac{\partial \pi}{\partial \eta} = -g_0 \rho \frac{\partial z}{\partial \eta}$$

Deep mass coordinate:

$$\pi(\eta) = \int_z^\infty g_0 \hat{r}^2 \rho \, \mathrm{d}z \left(= \int_{\eta(z)}^0 g_0 \hat{r}^2 \rho \frac{\partial z}{\partial \eta} \, \mathrm{d}\eta \right) \longrightarrow \frac{\partial \pi}{\partial \eta} = -g_0 \rho \hat{r}^2 \frac{\partial z}{\partial \eta}$$

HOMME's funky EOS:

Shallow mass coordinate:

$$\frac{\partial \pi}{\partial \eta} = g_0 \rho \frac{\partial z}{\partial \eta} \longrightarrow \quad p = \left(\frac{\frac{\partial \pi}{\partial z}}{-g_0 \frac{\partial z}{\partial \eta}} \right) R_d T_v \implies g_0 \frac{\partial z}{\partial \eta} = -\frac{R_d T_v}{p} \frac{\partial \pi}{\partial \eta}$$

Deep mass coordinate:

$$\frac{\partial \pi}{\partial \eta} = g_0 \rho \hat{r}^2 \frac{\partial z}{\partial \eta} \longrightarrow \quad p = \left(\frac{\frac{\partial \pi}{\partial z}}{-g_0 \hat{r}^2 \frac{\partial z}{\partial \eta}} \right) R_d T_v \implies g_0 \hat{r}^2 \frac{\partial z}{\partial \eta} = -\frac{R_d T_v}{p} \frac{\partial \pi}{\partial \eta}$$