Towards a new vertical coordinate to optimally resolve ocean mesoscale eddy dynamics in high-resolution models

Ocean Model Working Group

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Motivation: capturing mesoscale dynamics in high-resolution models

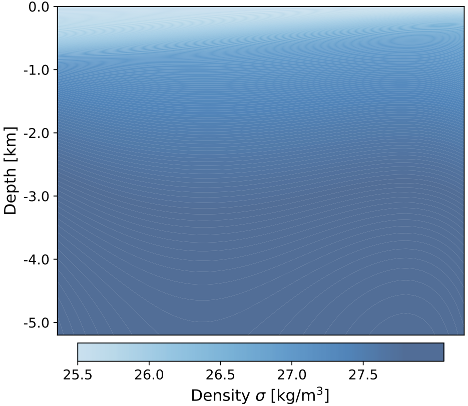
- To accurately capture the dynamics, the vertical must be sufficiently resolved. Evidence of insufficient resolution can be seen in:
  - 'Roll off' in QG horizontal turbulent spectra with lower vertical resolution
  - Sensitivities in MOM6 energetics depending on the choice of vertical grid
- As horizontal resolution of models increases, there is a corresponding demand on the vertical grid resolution
- Make the most effective use of vertical layers by placing them strategically
- Stewart et al., 2017 argue the importance of resolving the baroclinic modes which become relevant at finer meso- and submesoscales
  - Proposed geopotential grid with 50 well-positioned layers for the first baroclinic mode and additional 25 for each subsequent mode

Motivation: capturing mesoscale dynamics in high-resolution models

→ Investigate a promising vertical grid and compare representation of energetics in an idealized quasi-geostrophic (QG) regime. The grid is designed with an eye toward adapting it for use in primitive equation (PE) ocean models.

Proposed vertical coordinate

Defining an alpha coordinate: between geopotential and isopycnal



Buoyancy frequency quantifies stratification

$$N(z) = \left(\frac{-g\partial_z \rho}{\rho_{\text{ref}}} \right)^{1/2}$$

Define a family of coordinates based on N,

$$\xi = \int_0^z \left(\frac{N(z')}{N_{\text{ref}}} \right)^{\alpha} \mathrm{d}z'$$

$\alpha \geq 0$ controls strength of dependence on N

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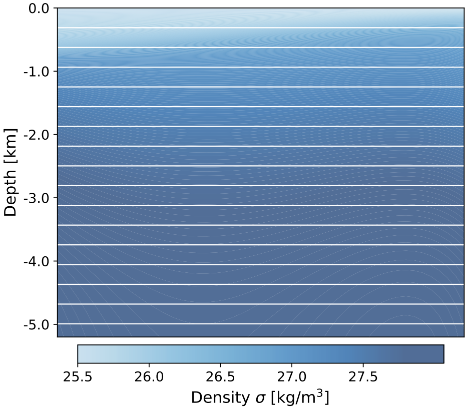

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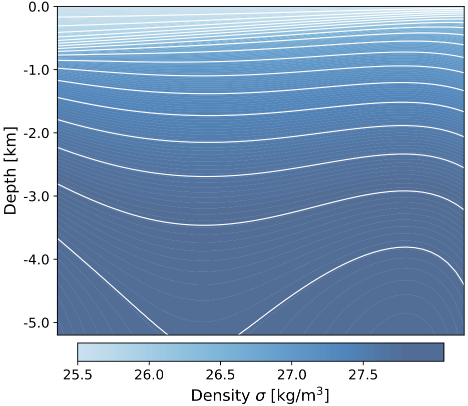

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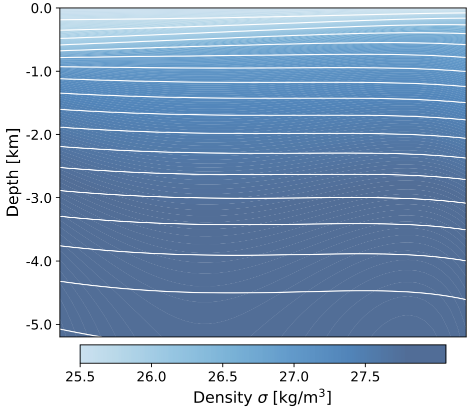
and $\alpha = 2$ is the isopycnal coordinate

Defining an alpha coordinate: between geopotential and isopycnal



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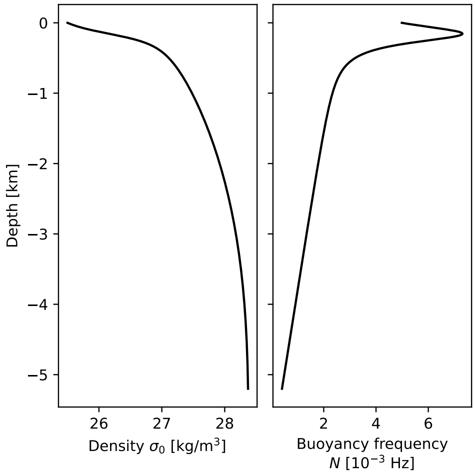
We propose $\alpha = 1$ "in between" the two be used

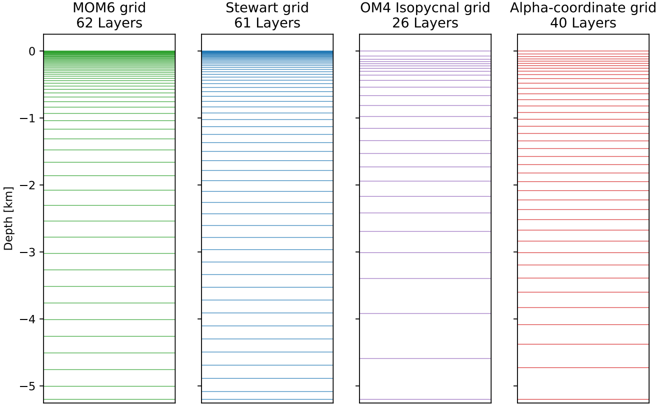
and $\alpha = 2$ is the isopycnal coordinate

Constructing the grid



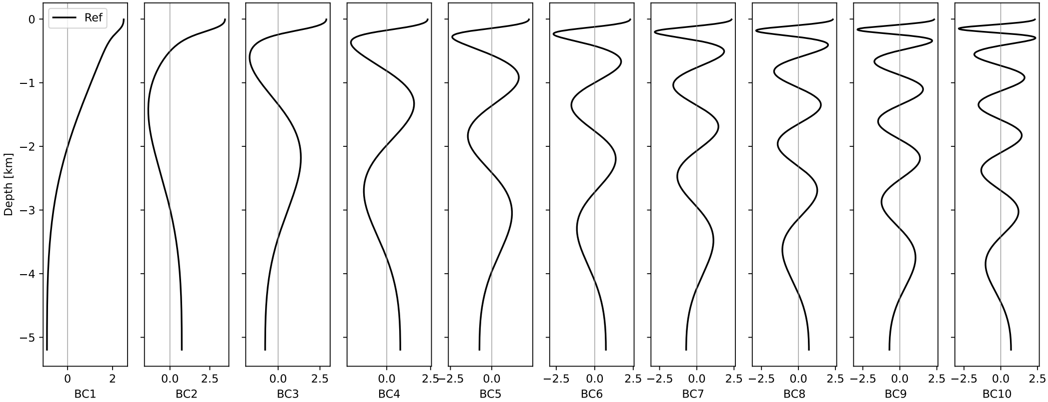
- Define a fixed idealized stratification that reflects realistic profiles
- For any grid, the layers must be specified in a coordinate
- $\alpha = 1$ we use equispaced points for the layers; the coordinate itself does the heavy lifting

Brief description of vertical grids

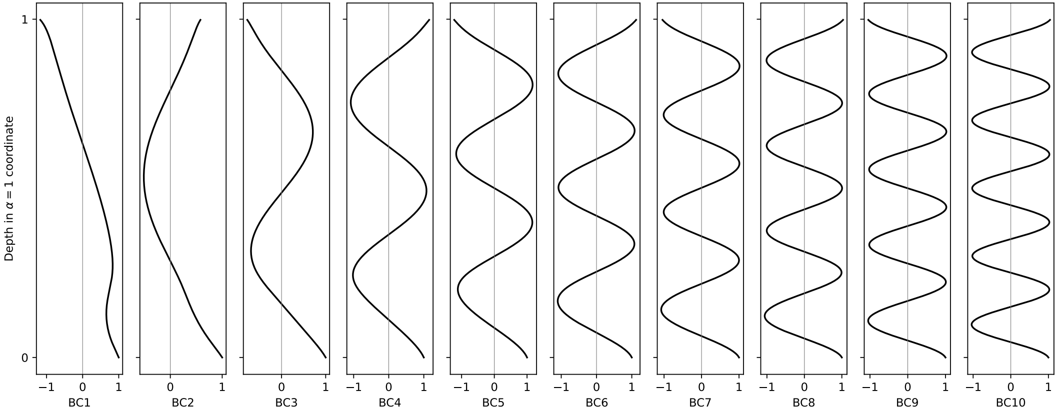


- $\alpha = 1$ (20,40,60)
- Stewart (65)
  (Stewart et al., 2017)
- MOM6 (65)
  (Marques et al., 2023)
- OM4 isopycnal / hybrid grid (75)
  (Adcroft et al., 2019)

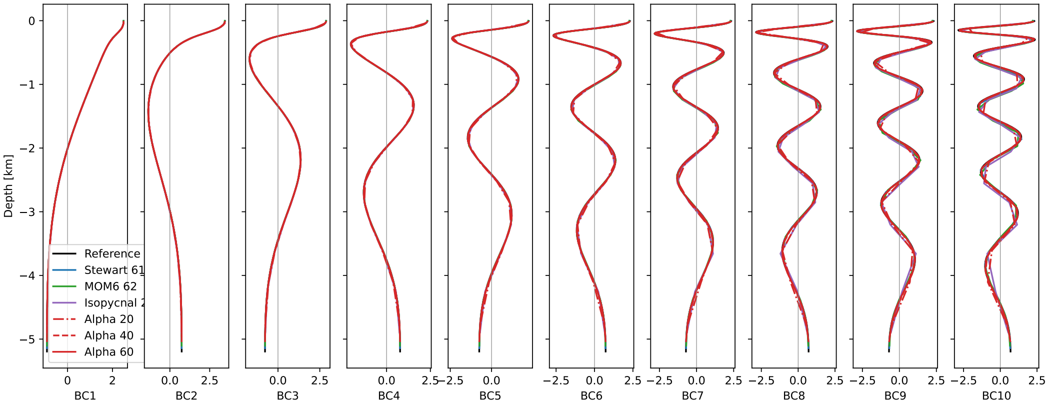
Baroclinic modes reflect increasing vertical complexity



Modes rapidly approach modulated cosines under $\alpha = 1$ coordinate



$\alpha = 1$ grid is well-suited to capture mode oscillations



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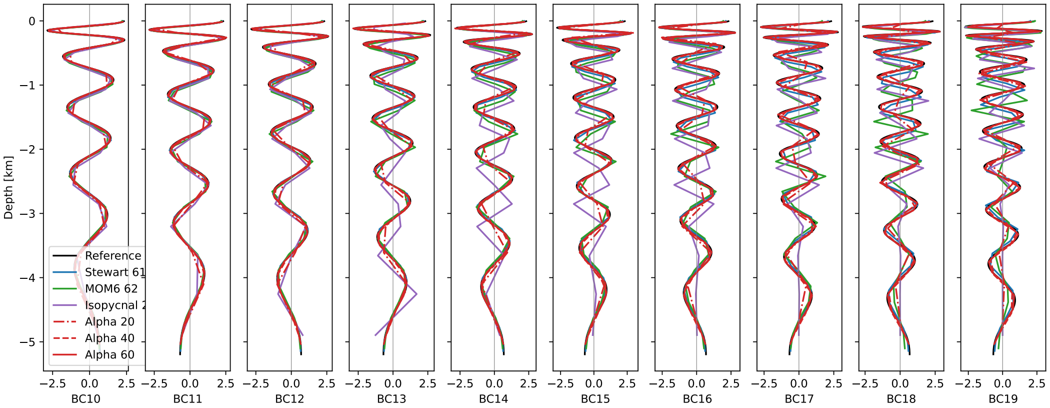
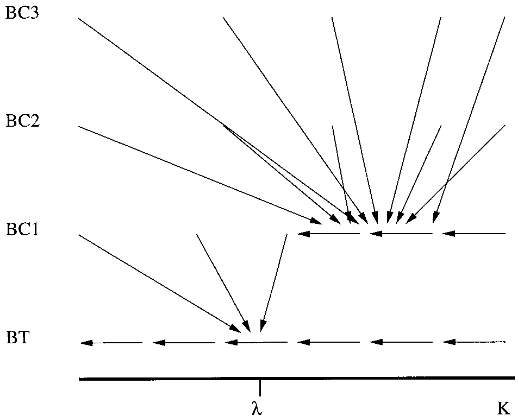


Energy cascade mediated by modal interaction coefficients



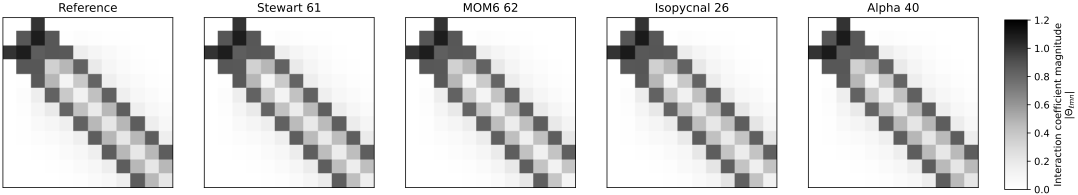
(Smith and Vallis, 2001)

The strength of interactions between vertical modes is controlled by a triple interaction coefficient,

$$\Theta_{\ell mn} = \int \phi_\ell \phi_m \phi_n dz$$
$$\rightarrow \quad \sum h_i \phi_{\ell,i} \phi_{m,i} \phi_{n,i}$$

The shape of the baroclinic modes, $\{\phi_m\}$, directly impacts the accuracy of the interactions and thus the energy pathways and cascade

QG interaction coefficients; consistent with accurate $\alpha = 1$ baroclinic modes



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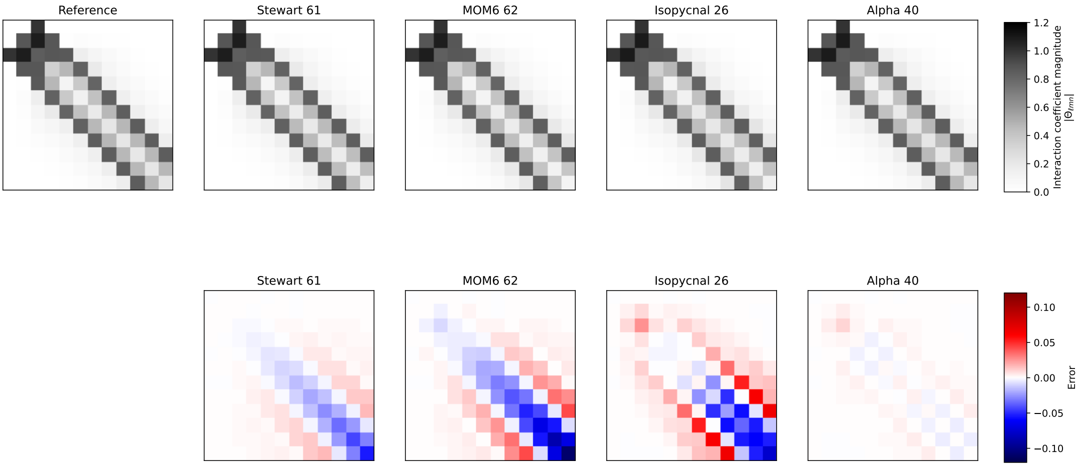
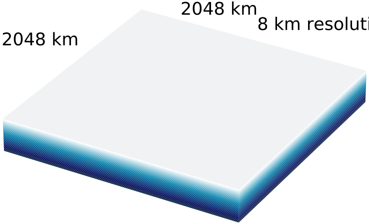


Dynamics: studying energetic behavior in nonlinear QG simulations

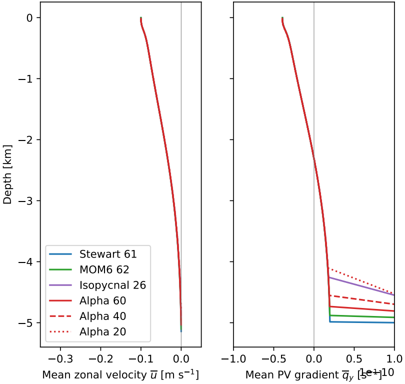
Simulation domain and setup

Fully nonlinear QG simulations

- Domain is periodic β-plane; flat, rigid top and bottom
- Fix background zonal velocity, $\overline{u}(z)$
- Baroclinic instabilities drive the turbulence in the modeled perturbation fields, $q', \psi'(x, y, z, t)$



Interior baroclinic instability excites a range of modes



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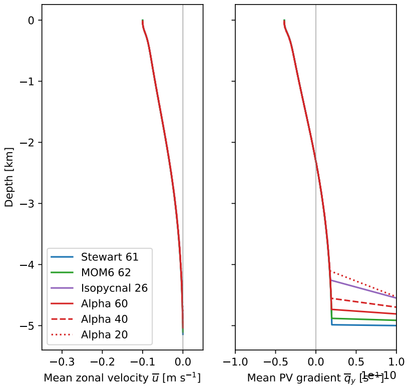

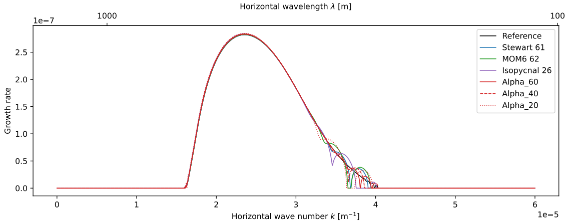

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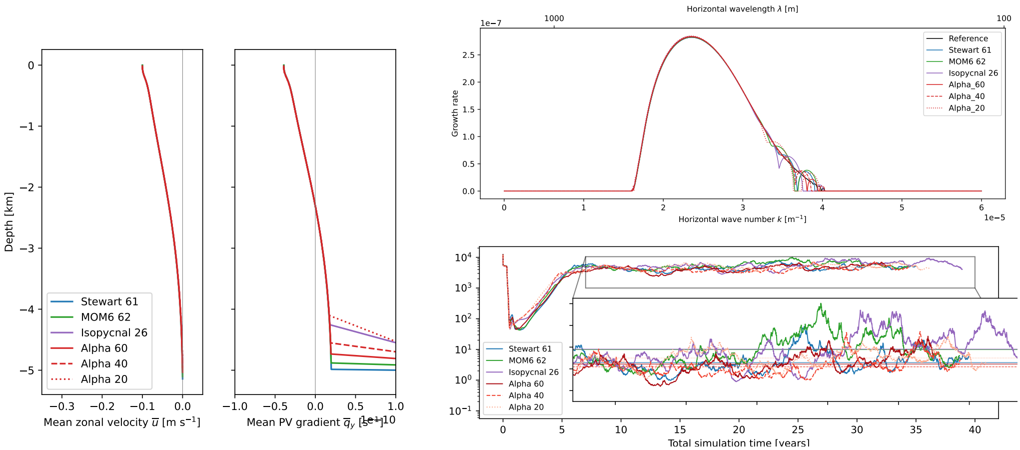
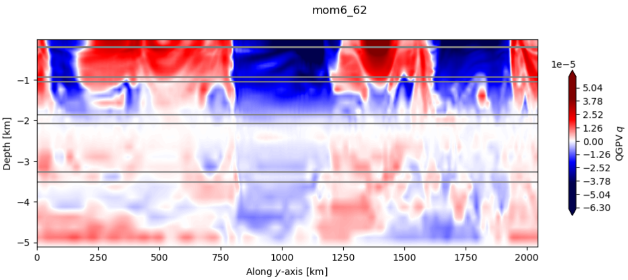
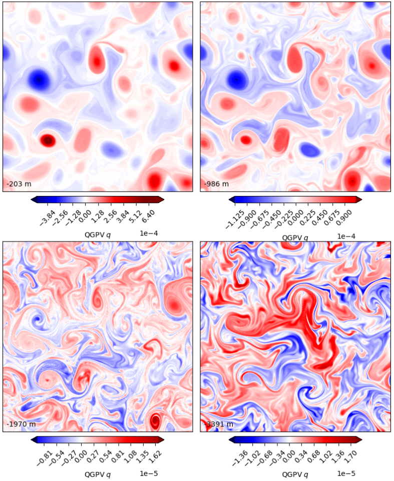


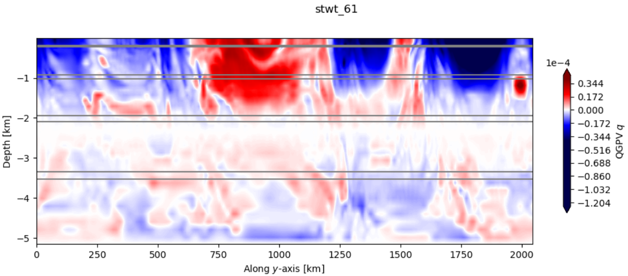
Look at turbulent statistics after spin up (every 25 days over 20 years)

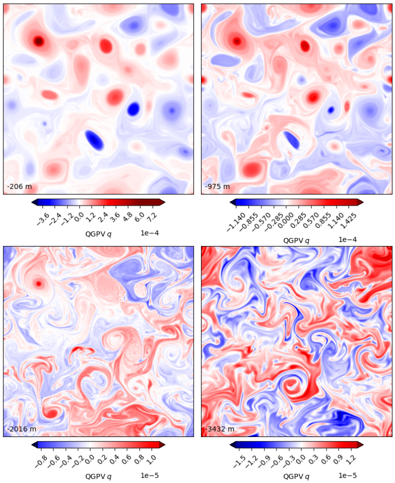
Cross section snapshots: MOM6 grid



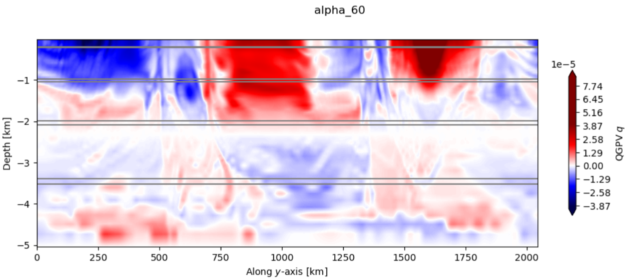


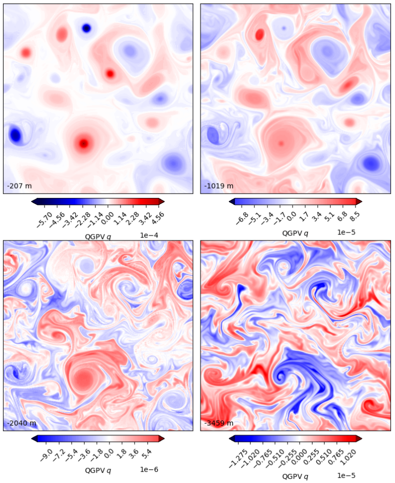
Cross section snapshots: Stewart grid



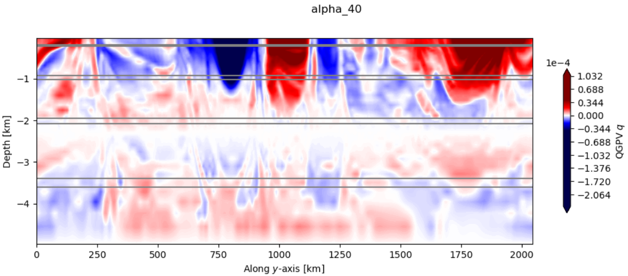


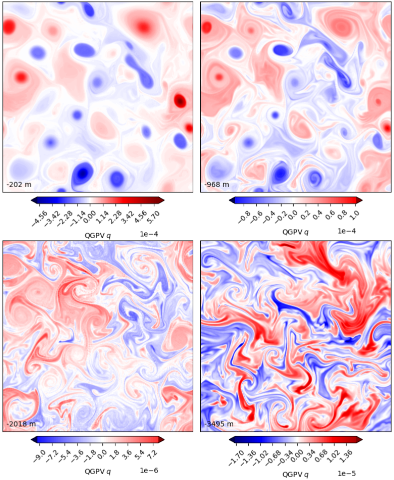
Cross section snapshots: $\alpha = 1$ grid with 60 layers



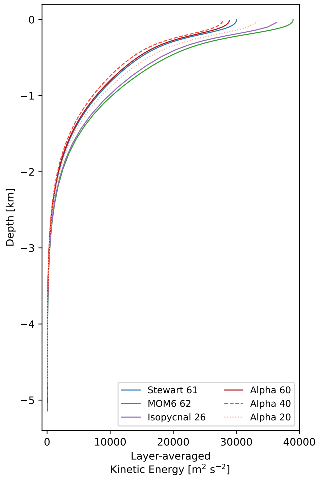


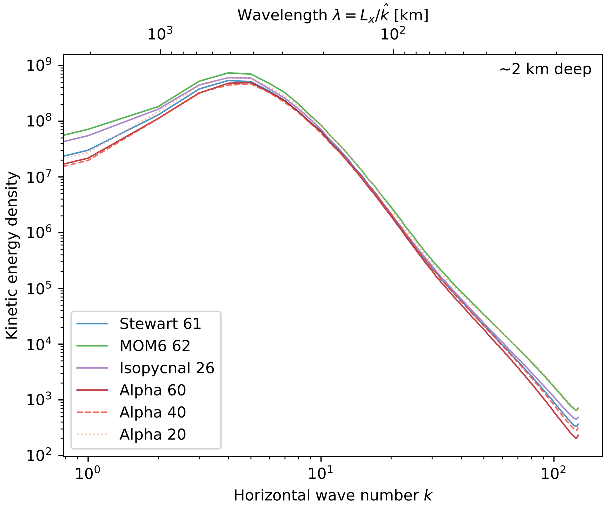
Cross section snapshots: $\alpha = 1$ grid with 40 layers





Diverging energetics across grids





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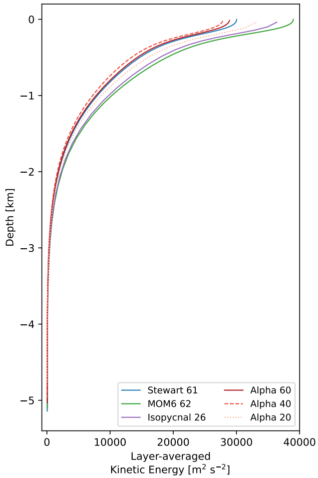

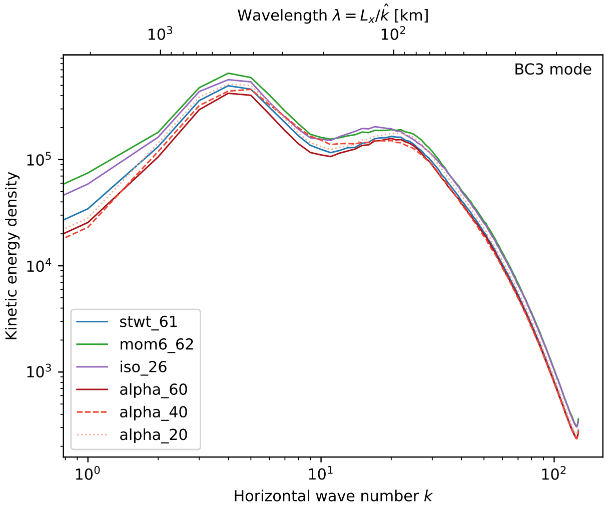

Heuristic behavior and theory suggest $\alpha = 1$ grid promising for mesoscale dynamics

- Equispaced $\alpha = 1$ grids provide a new, easily-computable means to efficiently resolve baroclinic modes
    - Straightforward definition of grid that can easily scale number of layers
    - Near optimal resolution of baroclinic modes out to the highest order
    - Adapts locally to stratification, which requires fewer layers globally to resolve modes than a geopotential grid
- Recreating energetic sensitivities in QG case study
- Convergence behavior can help us understand what the right answer might be within the variation displayed by grids
- Comparisons provide growing insight into the role of the vertical grid and resolution and the impact on the dynamics