An Adaptive Grid (AG) vertical coordinate for ocean models

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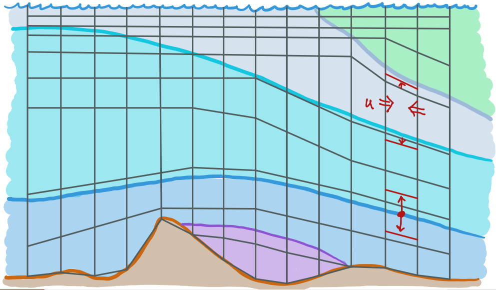
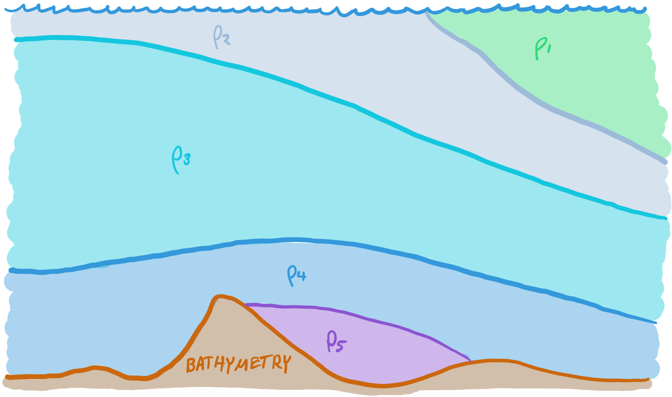


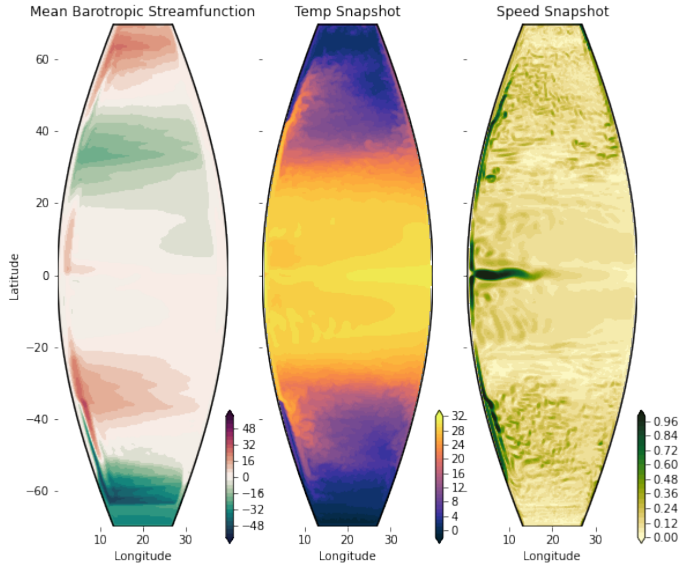
Vertical Coordinates

- The vertical direction in an ocean model is **special**!
- Aspect ratio – 1:2,000 – like a sheet of paper
- Gravity $\rightarrow$ Hydrostatic pressure
- Stratification
- Transport in vertical is $\sim 10^5\times$ smaller than in the horizontal
- Not parallelisable
- Sloping boundaries



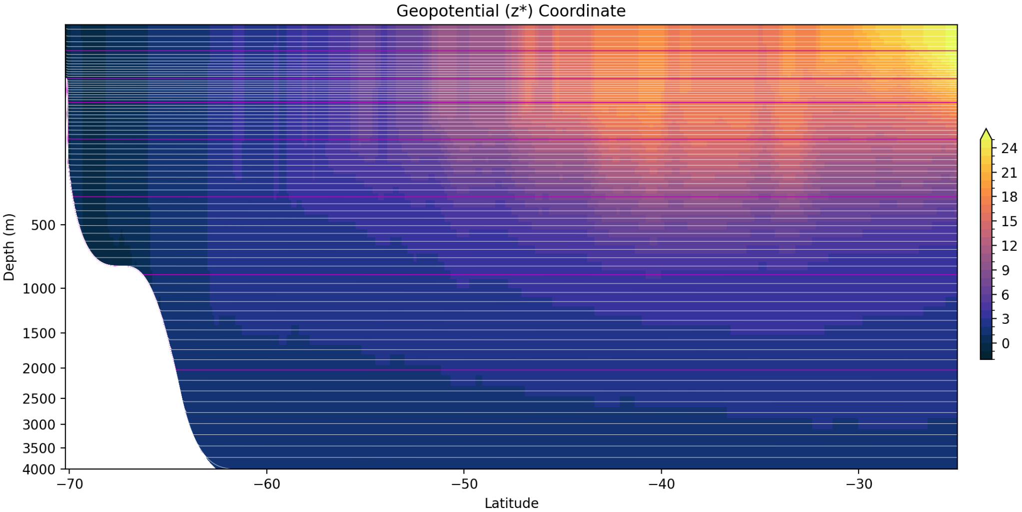
The treatment of the vertical direction is not just a question of resolution, but involves careful thought about how coordinates are specified.

Comparing Vertical Coordinates in a Sector Ocean Model.

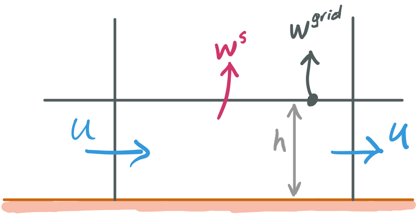


- MOM6 at 0.25° resolution
- 75 vertical levels
- 40° wide, pole-to-pole
- Reentrant Southern Ocean
- Steady wind forcing
- Surface temperature restoring
- No salinity, simple EoS
- Sloping sidewalls
- Antarctic continental shelf

Geopotential (quasi-Eulerian) Vertical Coordinates (z^*)



MOM6 uses Arbitrary Lagrangian Eulerian (ALE) Coordinates



Example: Lowest grid cell in a 1-D model!
Divide w into cross-surface & grid components:

$$w = w^{\mathrm{s}} - w^{\mathrm{grid}}$$

Evolve layer thickness $h \to h^{\mathrm{new}}$ over timestep Δt

Quasi-Eulerian (z^*)

$$w^{\mathrm{grid}} \approx 0$$

$$w^{\mathrm{s}} = -w^{\mathrm{grid}} - \partial_x(hu)$$

$$h^{\mathrm{new}} = h + w^{\mathrm{grid}}\Delta t$$

ALE with Vertical Lagrangian Remap

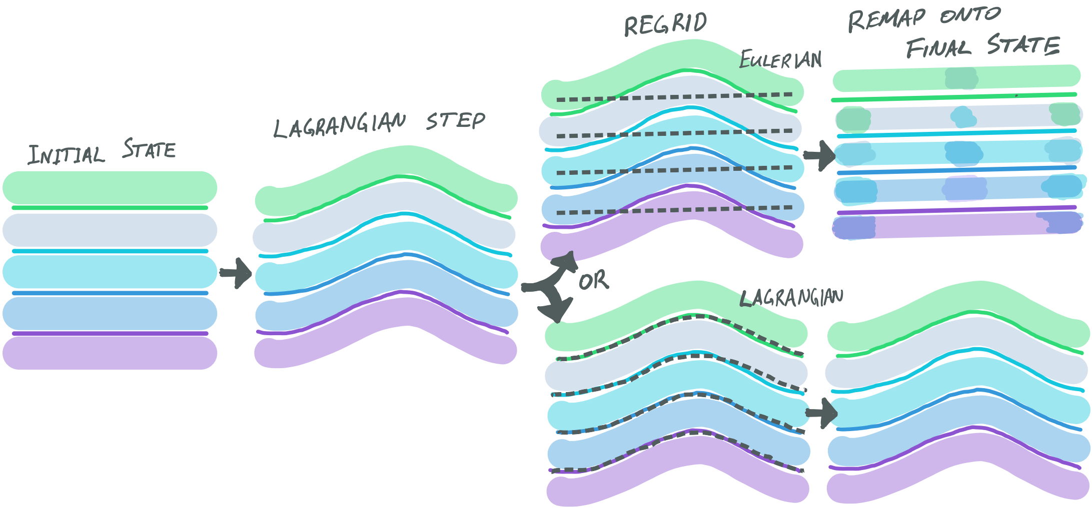
$$w^{\mathrm{grid}} = -\partial_x(hu)$$

$$h^{\dagger} = h + w^{\mathrm{grid}}\Delta t$$

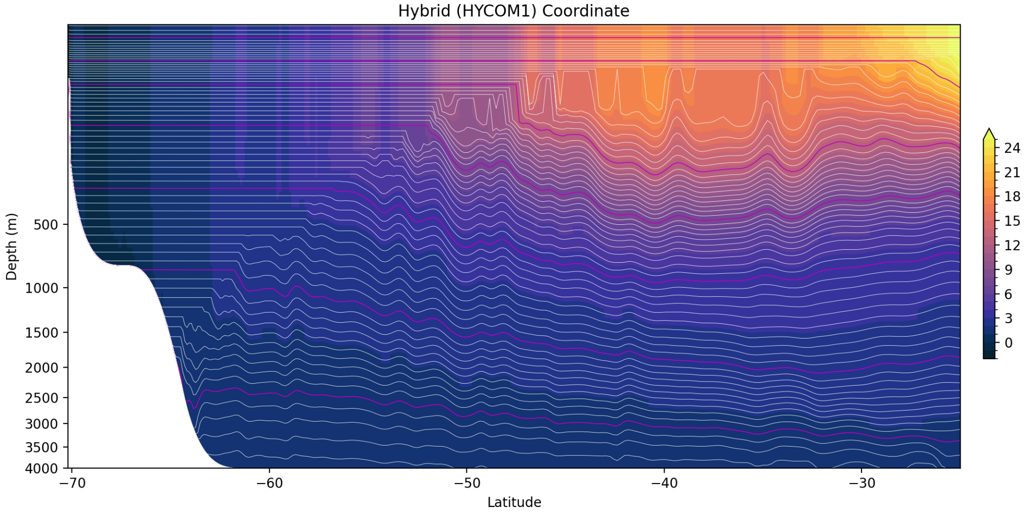
$$h^{\mathrm{new}} = h^{\mathrm{target}}$$

$$w^{s} = -(h^{\mathrm{target}} - h^{\dagger})/\Delta t$$

Arbitrary Lagrangian Eulerian with Vertical Lagrangian Remap



Hybrid z^*-Isopycnal Coordinates



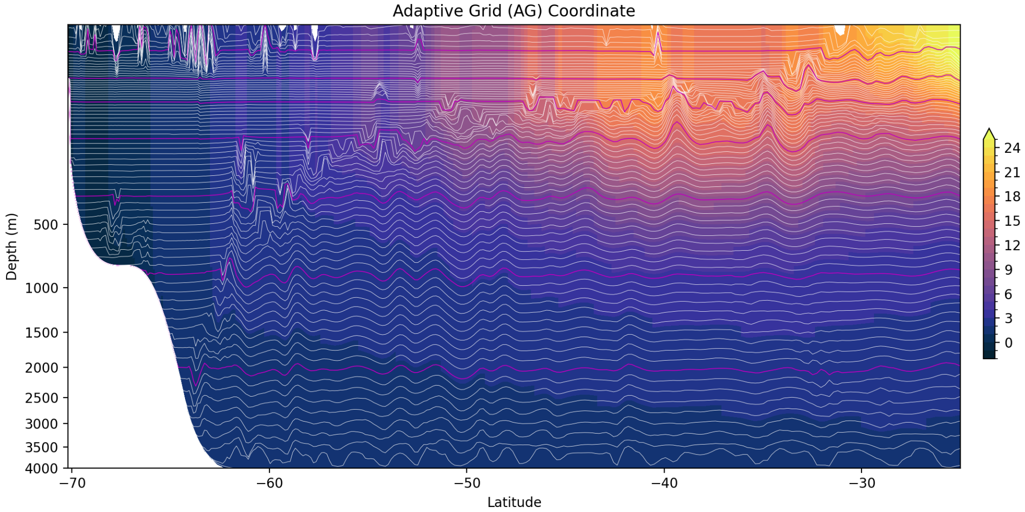
Adaptive Grid (AG) Coordinates

$$\partial_t z_k = -\nabla_H \cdot \Bigg(\underbrace{\omega_\sigma \frac{\kappa \nabla_H \sigma}{\sqrt{(\partial_z \sigma)^2 + (\nabla_H \sigma)^2}}}_{\text{density adaptivity}} + \underbrace{\omega_z \kappa \nabla_H z_k}_{\substack{\text{lateral}\\ \text{smoothing}}} \Bigg) + \underbrace{\tau_r^{-1} (z_k^* - z_k)}_{\text{vertical restoring}} + \underbrace{F_{\text{con}}}_{\substack{\text{convective}\\ \text{adjustment}}}$$

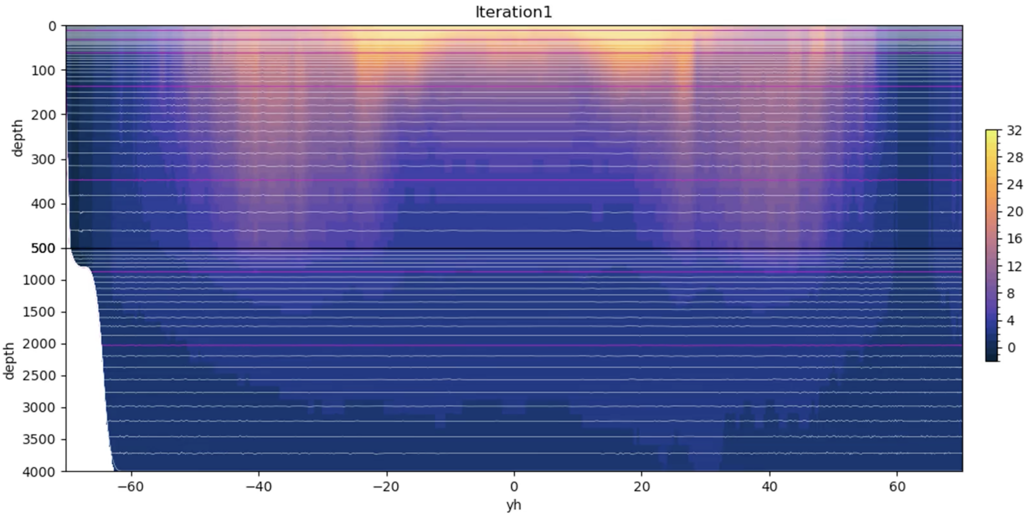
- **Density Adaptivity:** Orients the coordinate interface to reduce along-layer gradients. Operates in stratified regions to create a coordinate that locally approximates isopycnal surfaces.
- **Lateral Smoothing:** When density surfaces are too steep, such as in the surface mixed layer, this term switches on. It acts to smooth the interface height, and produces geopotential-like interfaces.
- **Vertical Restoring:** Weak restoring to avoid slow drift of interface.
- **Convective Adjustment:** Needed for downslope gravity currents . . .

(Gibson, 2019; Gibson et al., In Prep)

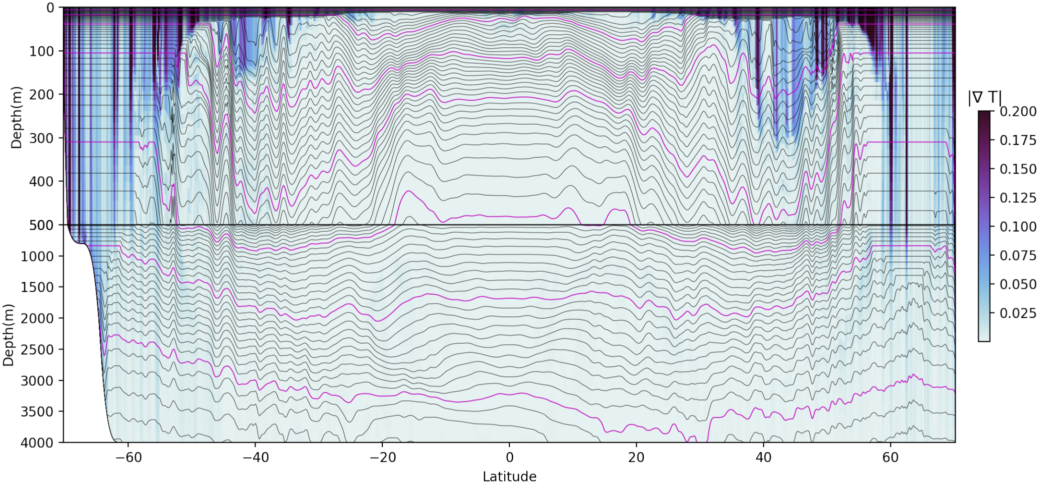
Adaptive Grid (AG) Coordinates



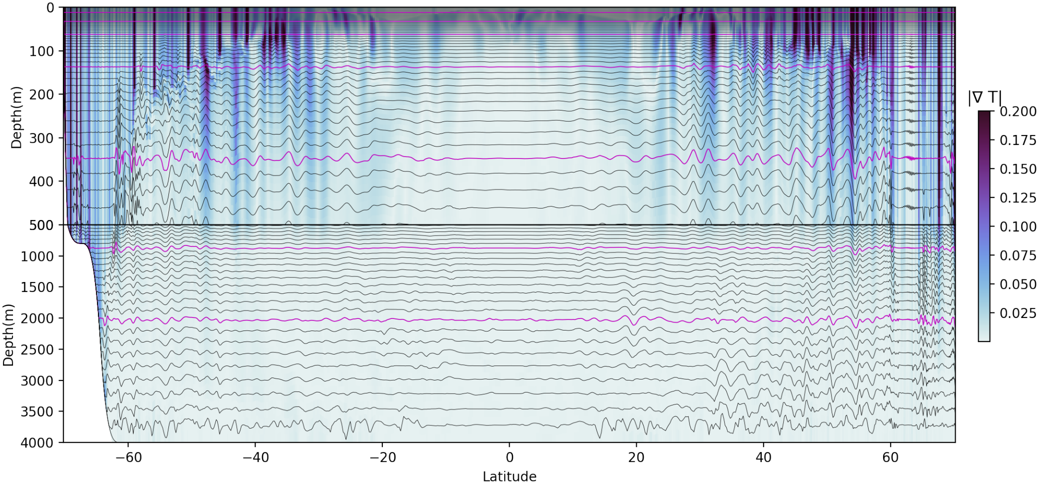
Adaptive Grid (AG) Coordinates



Hybrid z^*-Isopycnal Coordinates – Lateral Temp Gradients



Adaptive Coordinates – Lateral Temp Gradients

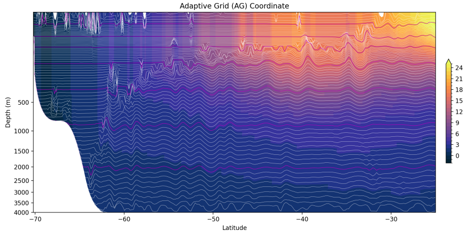


Next Steps ...

Add a more scale-selective (biharmonic) background smoothing:

$$\partial_t z_k = -\nabla_H \cdot \Bigg(\underbrace{\frac{\omega_\sigma \kappa \nabla_H \sigma}{\sqrt{(\partial_z \sigma)^2 + (\nabla_H \sigma)^2}}}_{\text{density adaptivity}} + \underbrace{\omega_z \kappa \nabla_H z_k}_{\substack{\text{lateral}\\ \text{smoothing}}} - \underbrace{\kappa_4 \nabla_H^3 z_k}_{\substack{\text{biharmonic}\\ \text{smoothing}}} \Bigg) + \underbrace{\tau_r^{-1} (z_k^* - z_k)}_{\text{vertical restoring}} + \underbrace{F_{\mathsf{con}}}_{\substack{\text{convective}\\ \text{adjustment}}}$$

Currently being implemented, but we're open to ideas!



Summary

- AG Coordinate runs and is stable
- Works well in stratified and unstratified regions
- Transition region remains a challenge
- Testing higher order smoothing ...