CESM Ocean Model Winter Working Group Meeting, 2023

Scale-dependent vertical structure of eddy kinetic energy in an idealized isopycnal ocean model

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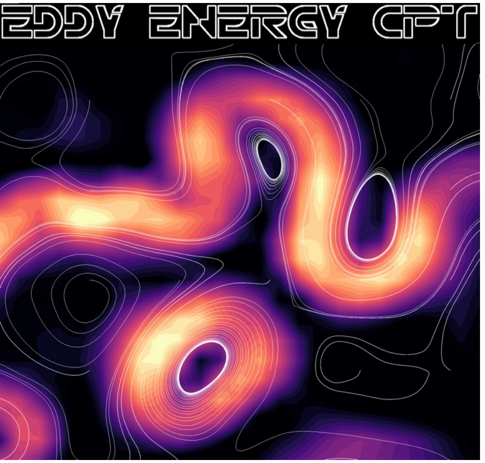
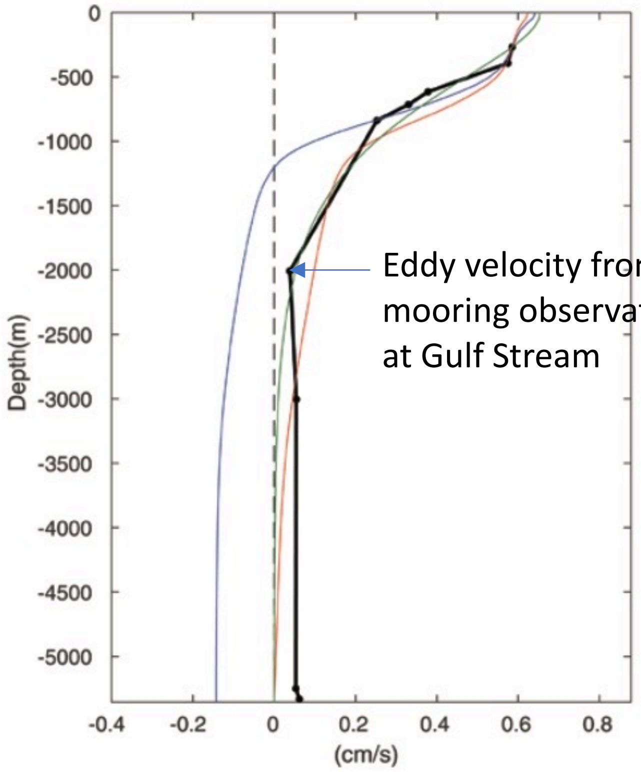
Thank Elizabeth Yankovsky, Jacob Steinberg, Shafer Smith, Malte Jansen, Baylor Fox-Kemper and the other members of the Ocean Transport and Eddy Energy Climate Process Team for helpful discussions



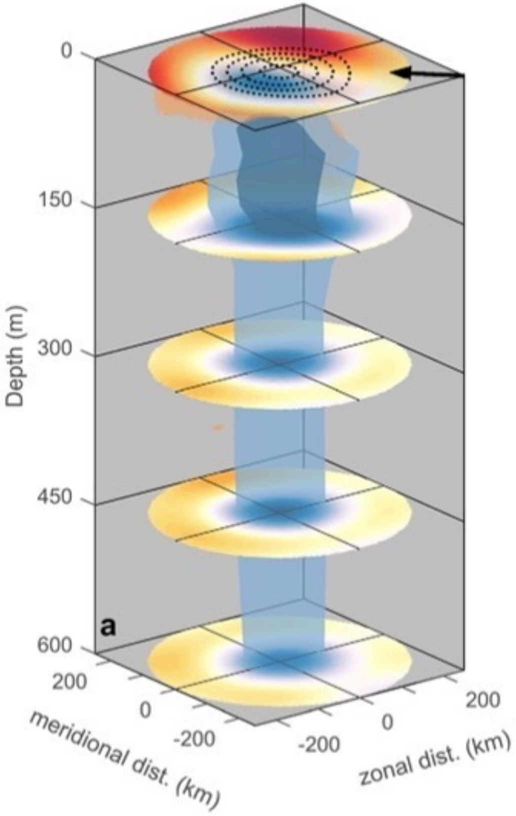
Vertical Structure of Mesoscale Eddies

- Mesoscale eddies have significant vertical structures of velocity and density anomalies
- These vertical structures need to be accounted for in eddy parameterization



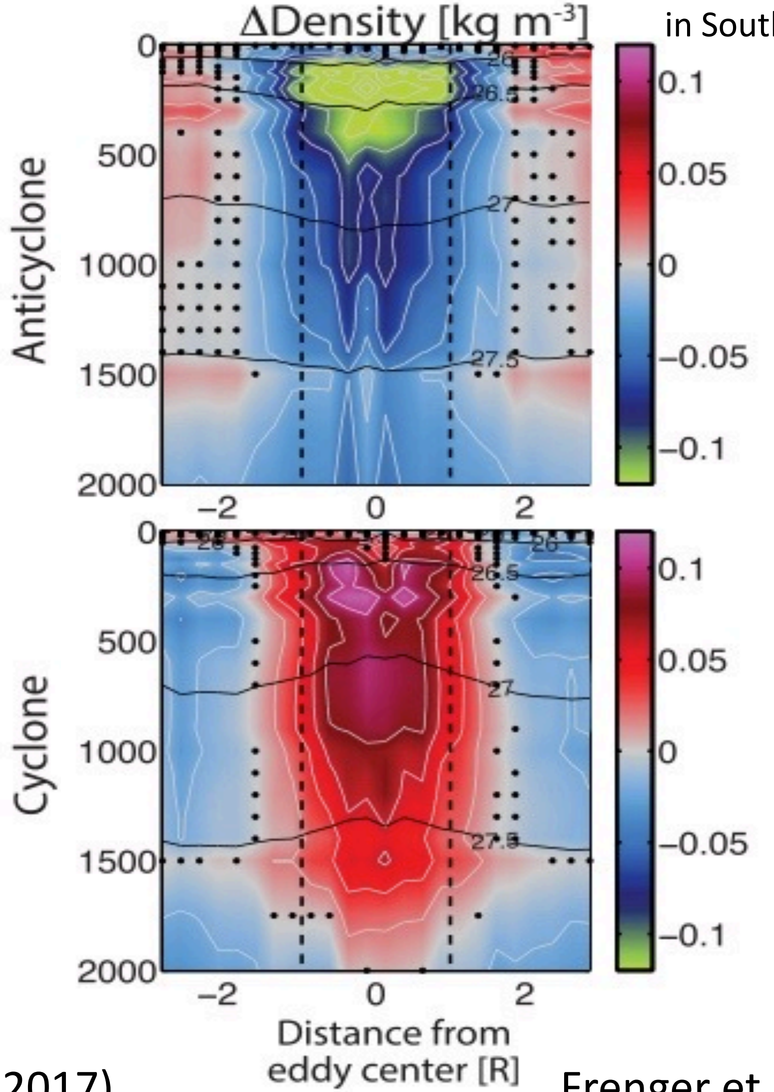
Eddy velocity from mooring observation at Gulf Stream

de La Lama et al. (2016)



Armores et al. (2017)

Composite eddy vertical structure in Southern Ocean



Frenger et al. (2015)

Vertical Normal Modes

- Assuming a wave-like solution to the linear QG PV equation yields an equation for the vertical structure

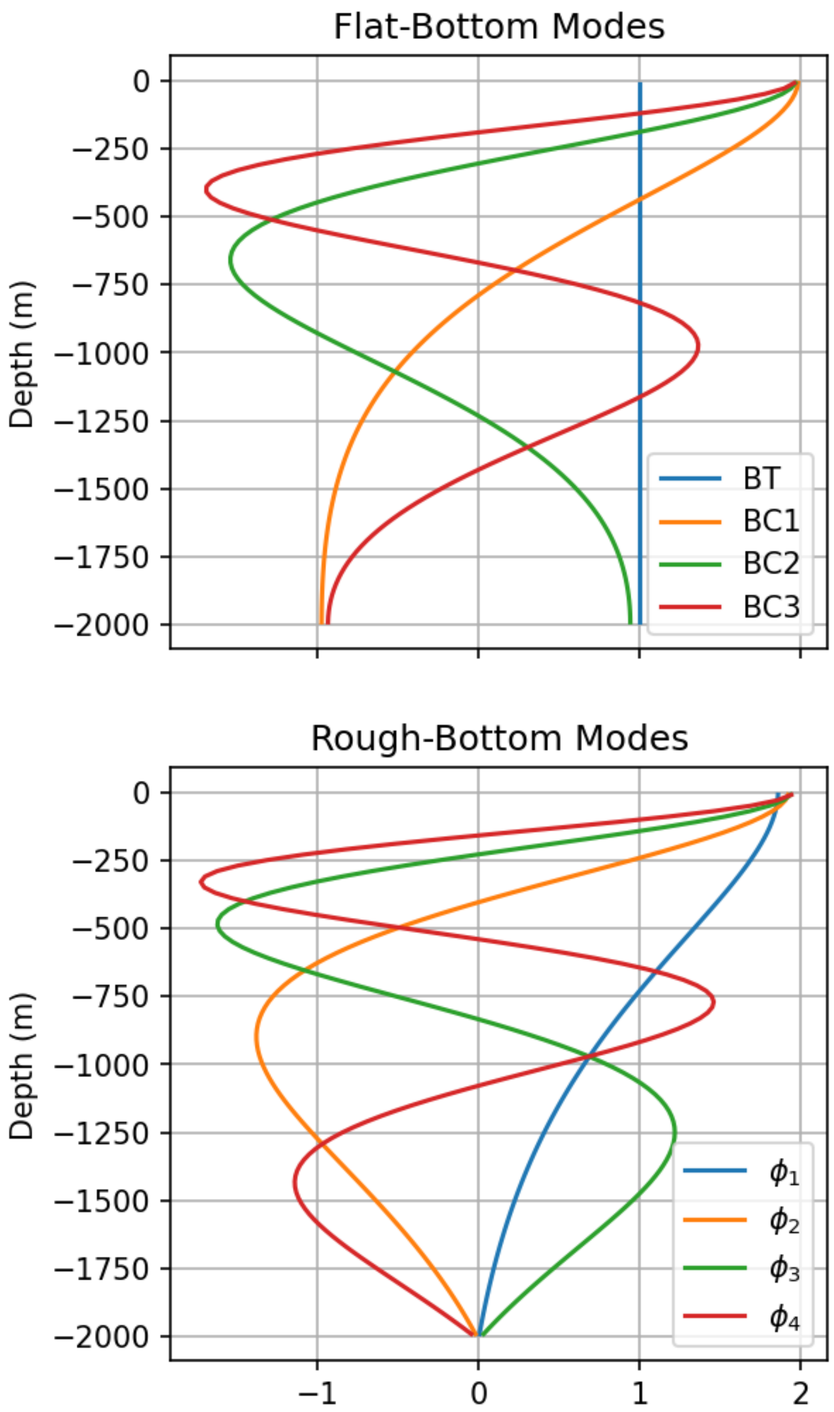
$$\frac{d}{dz}\left(\frac{f_0^2}{N^2}\frac{d\phi}{dz}\right)+\frac{\phi}{L_d^2}=0$$

- Boundary conditions for classical **flat-bottom** modes:

$$\frac{d\phi}{dz}=0 \text{ at z} = 0, -\text{H}.$$

- Boundary conditions for **rough-bottom m**odes:

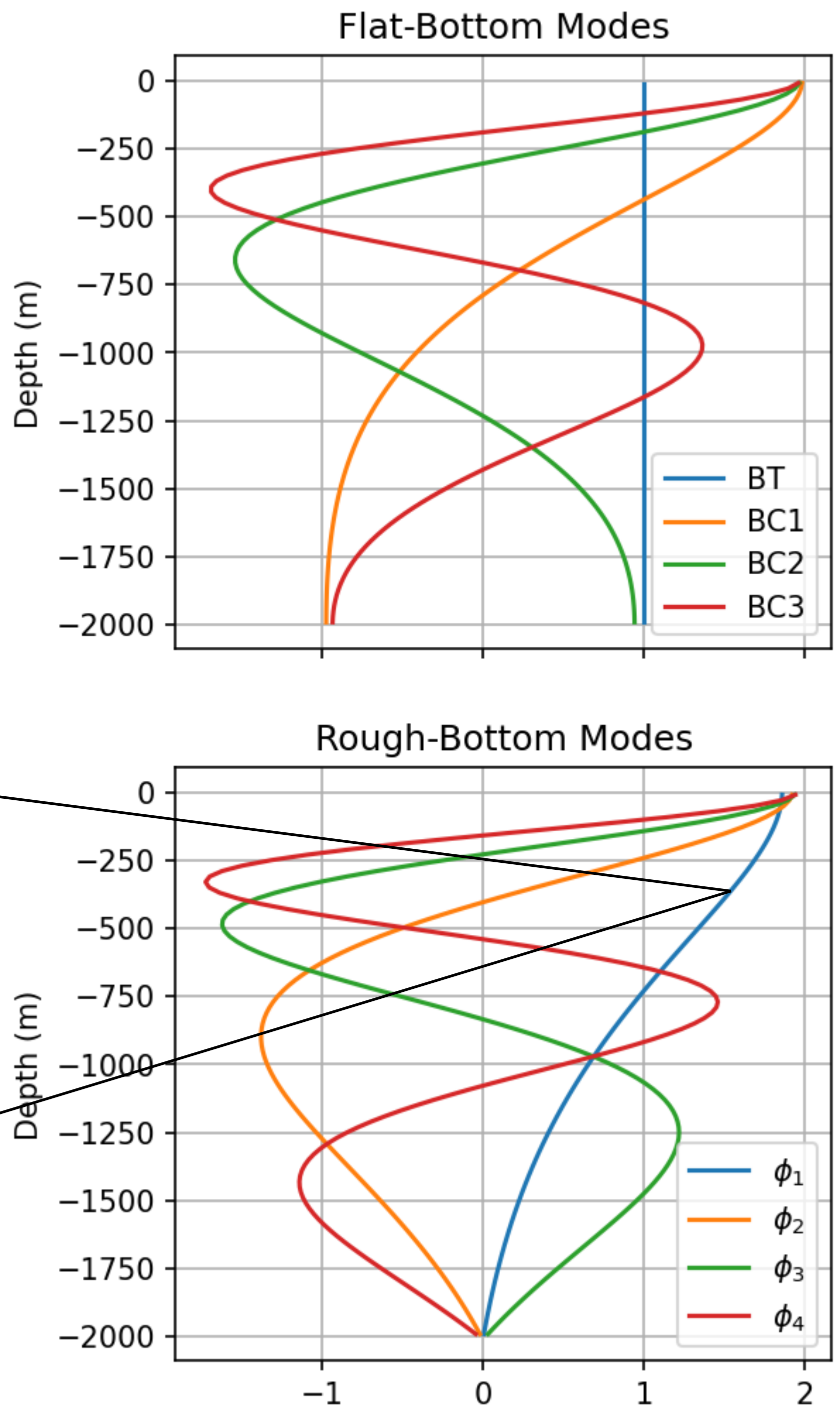
$$\frac{d\phi}{dz}=0 \text{ at z} = 0, \qquad \phi = 0 \text{ at z} = -\text{H}$$



Vertical Normal Modes

- Eddy energy from mooring observations is found to be dominated by the **barotropic (BT)** and **first baroclinic mode (BC1)** (Wunsch, 1997)
- Later studies find the eddy vertical structure is more like the **first rough-bottom mode** (de La Lama et al. 2016), which is also called **equivalent barotropic (EBT)** mode (e.g., Hallberg 1997)
- Parameterization of interface height diffusivity in the coarse-resolution GFDL OM4.0 uses the first rough-bottom mode (ϕ_{EBT}) as its vertical structure (Adcroft et al. 2019):

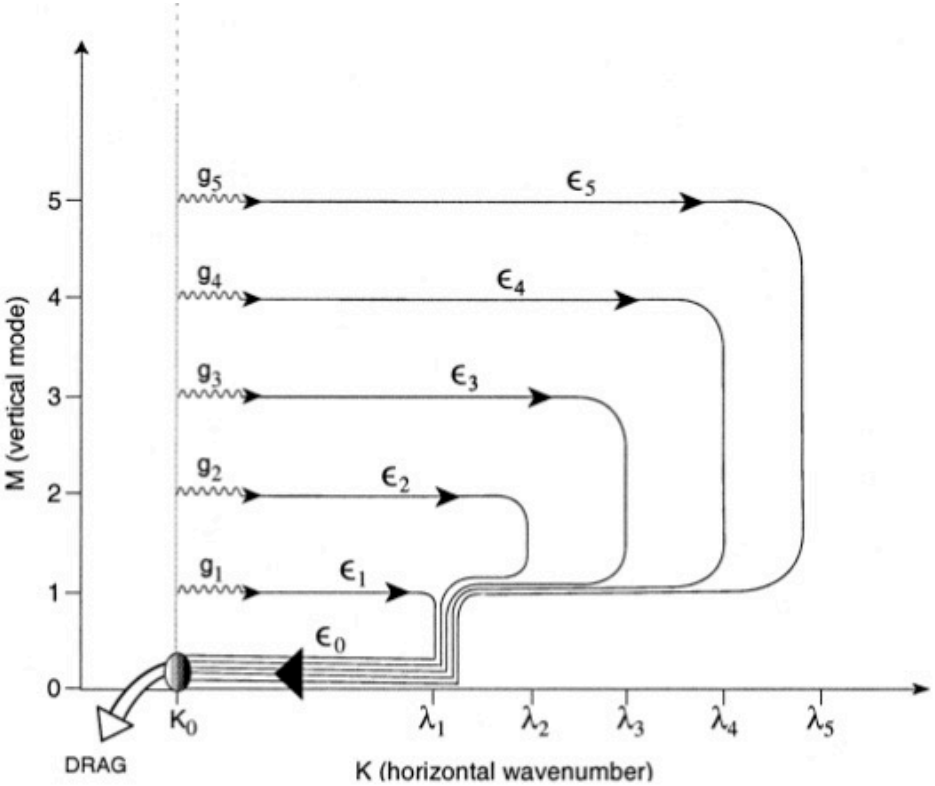
$$\kappa = 0.5 L U_e \phi_{EBT}(z)$$



Barotropization

- Eddy energy tends to transform from higher baroclinic modes to barotropic mode and then undergo inverse cascade (e.g., Salmon 1980)
- The barotropization is inefficient when stratification is surface intensified (Smith and Vallis 2001, 2002)
- Eddy length scale from satellite observations correlates to the first-mode Rossby deformation radius (Stammer, 1997)

Energy schematic in Ocean-like stratification



Smith and Vallis (2002)

Role of SQG "mode"

- Traditional vertical modes assume zero buoyancy anomaly at the surface
- $\psi = \psi_{int} + \psi_{sur}$ (Lapeyre and Klein, 2006), with

$$\frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial \hat{\psi}_{sur}}{\partial z}\right) - k^2 \hat{\psi}_{sur} = 0,$$

$$f\frac{\partial \hat{\psi}_{sur}}{\partial z}\bigg|_{z=0} = b\bigg|_{z=0},$$

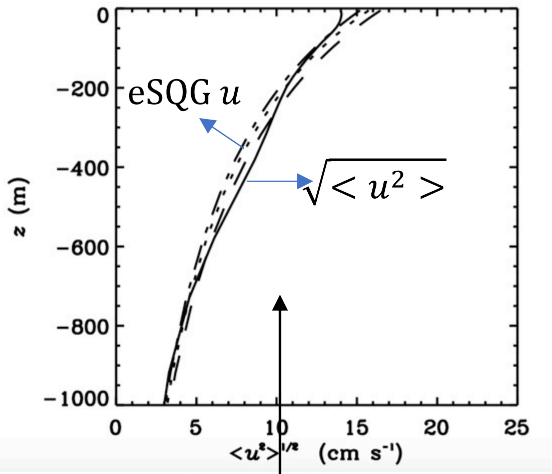
where ψ_{sur} is described by the surface quasigeostrophic (SQG) dynamics (Blumen ,1978; Held, 1995)

- eSQG method (Lapeyre and Klein, 2006):

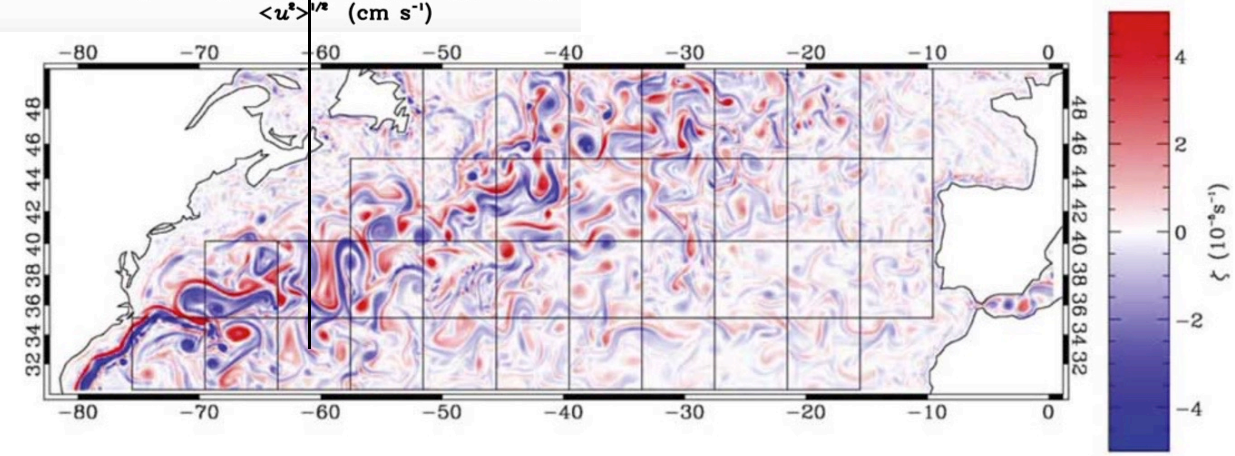
$$\hat{\psi}(\mathbf{k}, z) = \frac{g}{f_0}\hat{\eta}\exp\left(\frac{N_0}{f_0}|\mathbf{k}|z\right)$$

where N_0 is the average of N in upper ocean, $\hat{\eta}$ is the SSH spectrum, and $\mathbf{k}$ is the horizontal wavenumber.

- SQG mode has not been used in parameterization



Isern-Fontanet et al. (2008)



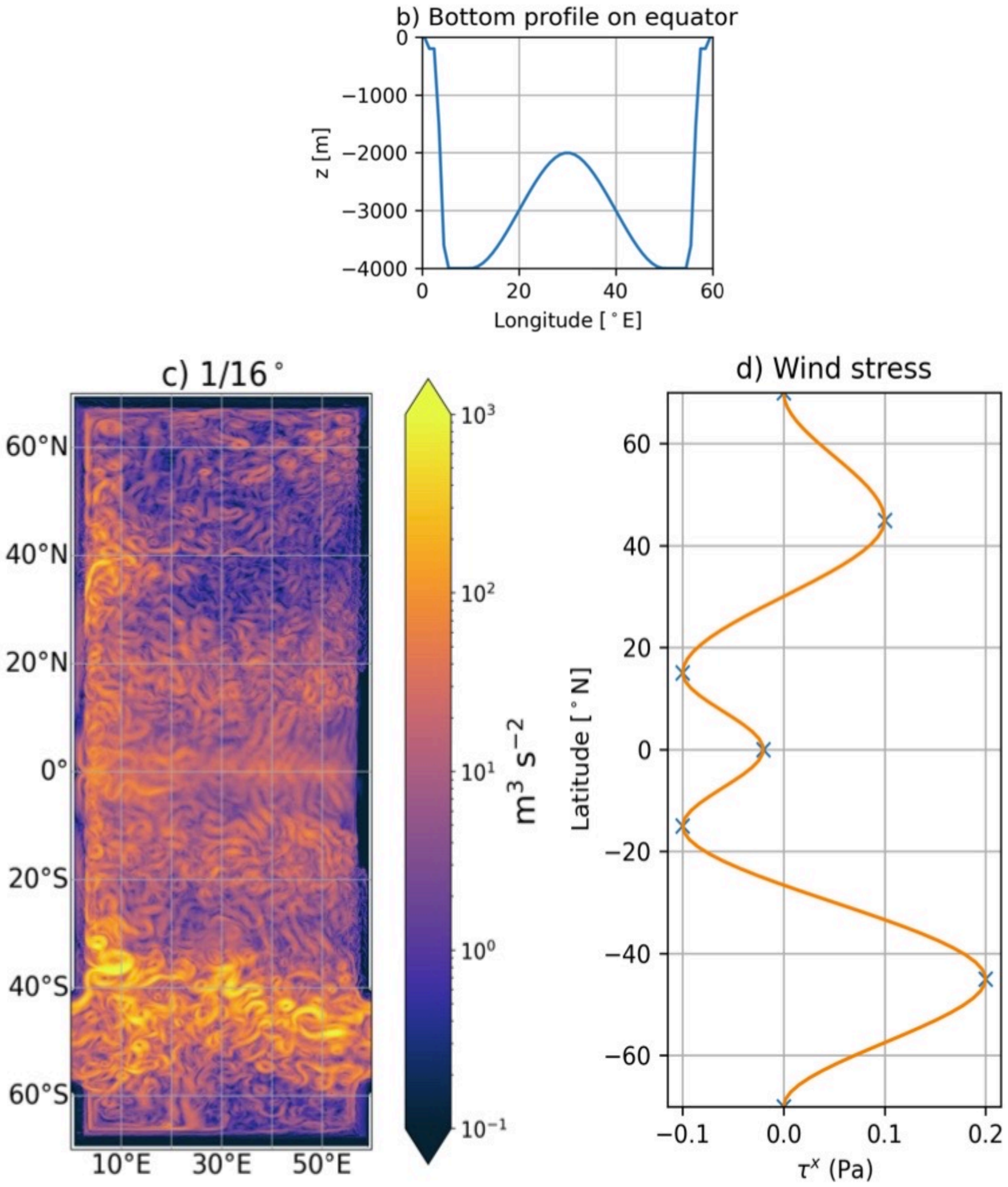
- eSQG method is effective in reconstructing the vertical structure of eddy properties from surface observations (e.g., Isern-Fontanet et al., 2008; Klein et al., 2009; Qiu et al., 2016, 2020)

Questions

- How prevalent is the SQG mode and how does it arise from baroclinic instability?
- What insights can the SQG mode bring for the understanding of energy cascade and parameterization?

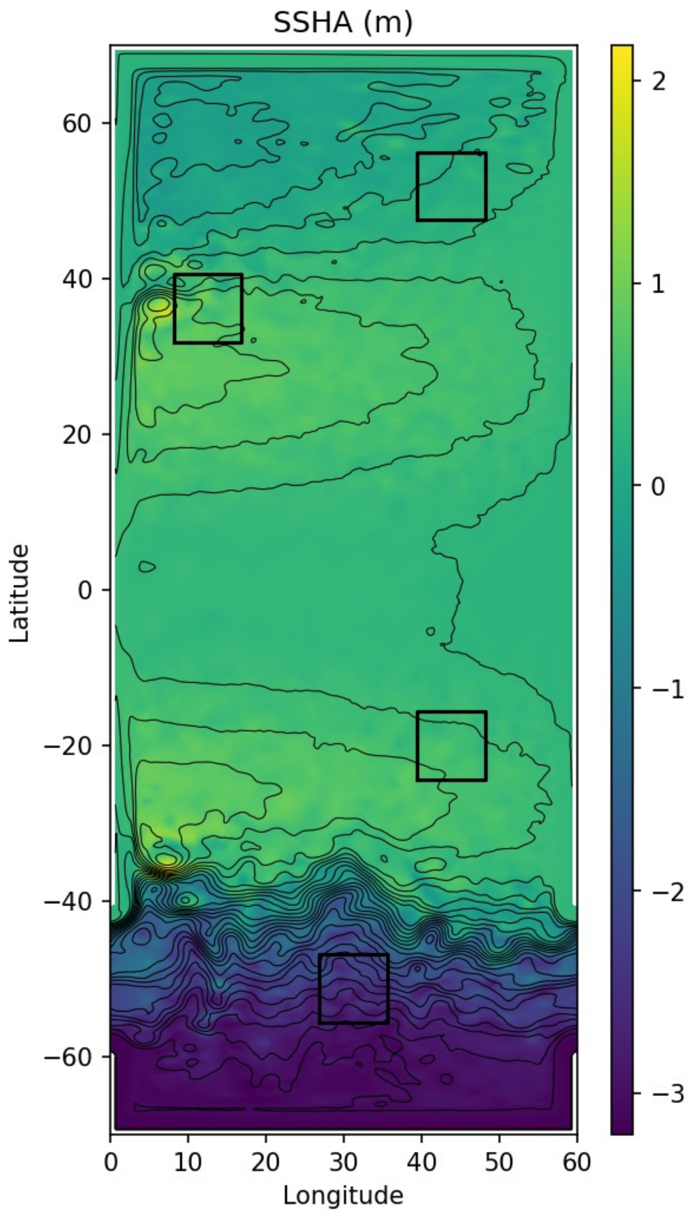
Model: Neverworld2

- Idealized configuration of MOM6
- Double hemisphere
- Isopycnal coordinate, 15 layers
- Horizontal grid spacing: $1/16°$
- Forced by zonally uniform zonal wind stress and no buoyancy forcing
- Abyssal meridional ridge of height 2000 m
- Adiabatic and hydrostatic



Marques et al. (2022)

Vertical Structure of EKE

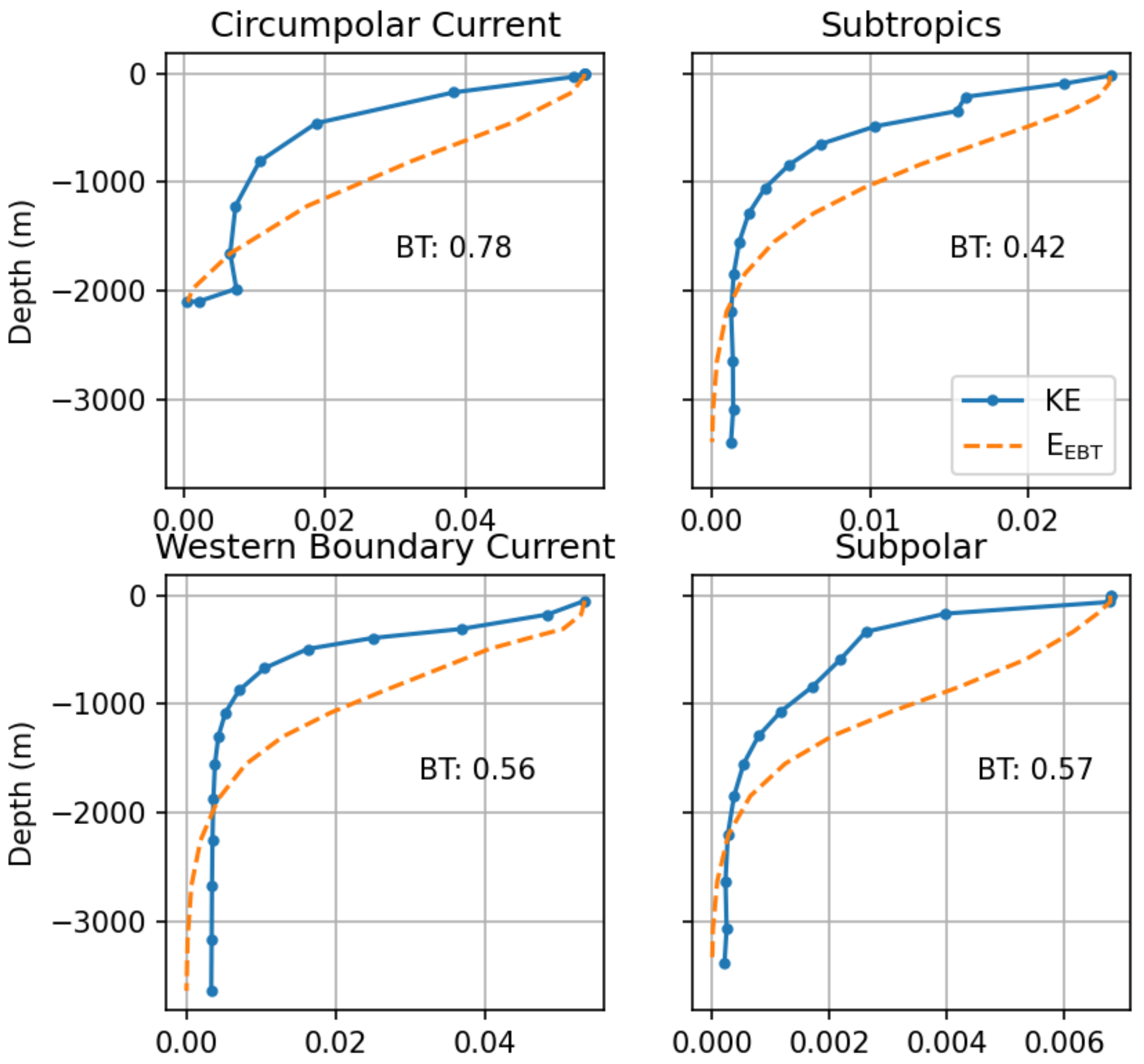


Significant fraction of EKE is in the baroclinic modes

Prediction based on the EBT mode that is diagnosed online:

$$E_{\mathrm{EBT}} = \phi_{\mathrm{EBT}}(z)^2 \mathrm{EKE}_0$$

Can we do better than ϕ_{EBT} at representing the vertical structure?



Surface EKE spectrum in 8.7°×8.7° windows and averaged over 500 days

Role of SQG "mode"

- SQG mode ψ_{sur} is solved from

$$\frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial \hat{\psi}_{sur}}{\partial z}\right) - k^2 \hat{\psi}_{sur} = 0,$$

where k is the horizontal wavenumber

- Numerical solution:

$$\hat{\psi}_{sur}\Big|_{z=0} = \frac{g}{f_0}\hat{\eta}, \qquad \frac{\partial \hat{\psi}_{sur}}{\partial z}\Big|_{z=-H} = 0,$$

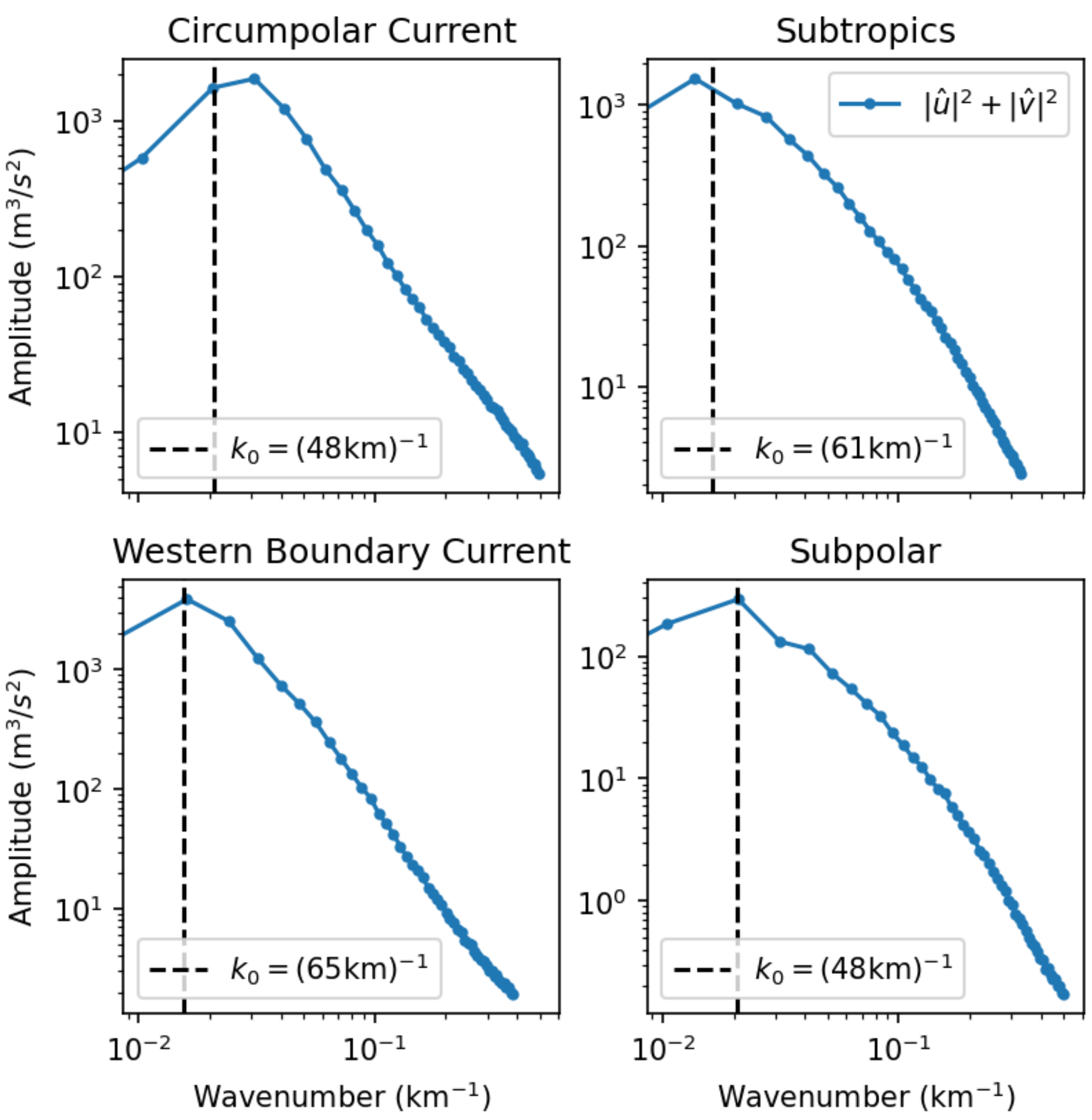
which assumes SQG mode dominates surface pressure anomaly

- WKB approximations (first order) assuming $\hat{\psi}|_{z=-\infty} = 0$:

$$\hat{\psi}_{sur}(k,z) \approx \hat{\psi}(k,0)e^{-kz_s},$$

where $z_s = \int_z^0 \frac{N}{|f|}dz$, $\hat{\psi}(k,0) = \frac{g}{f_0}\hat{\eta}$

- Vertical structure of $\hat{\psi}_{sur}$ is dependent on horizontal scale



Role of SQG "mode"

- SQG mode reproduces the vertical structure of EKE better than the equivalent barotropic mode

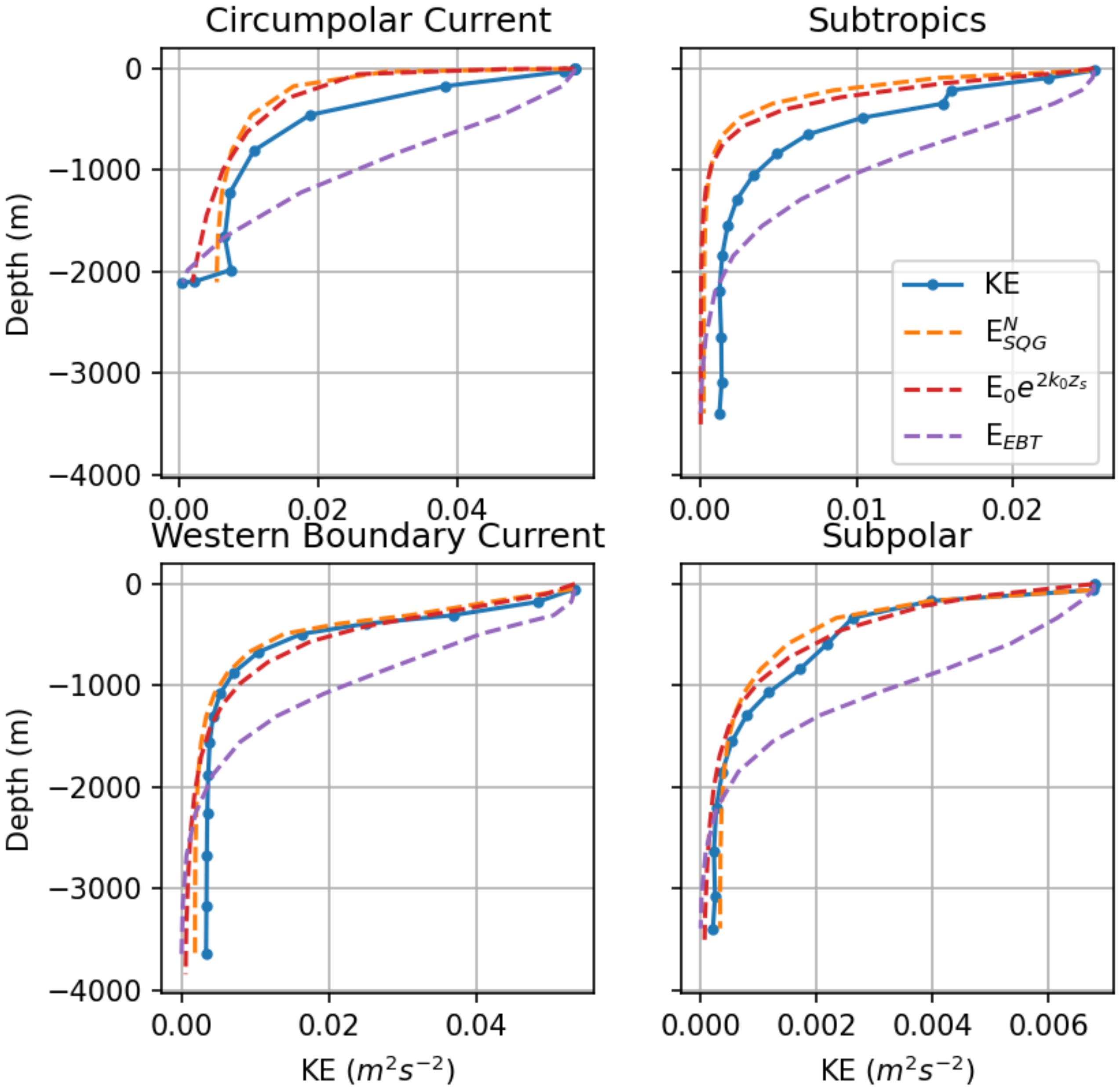
- The vertical structure can even be simply approximated by

$$E(z) \approx E_0 e^{-2k_0 z_s},$$

where k_0 is the **energy-containing wavenumber** estimated following Thompson and Young (2006) :

$$k_0 = \sqrt{\frac{<|\nabla\eta_0'|^2>}{<\eta_0'^2>}} \; ,$$

where η_0' is the SSH anomaly and $<\cdot>$ is a spatial and temporal average.



How does SQG mode arise?

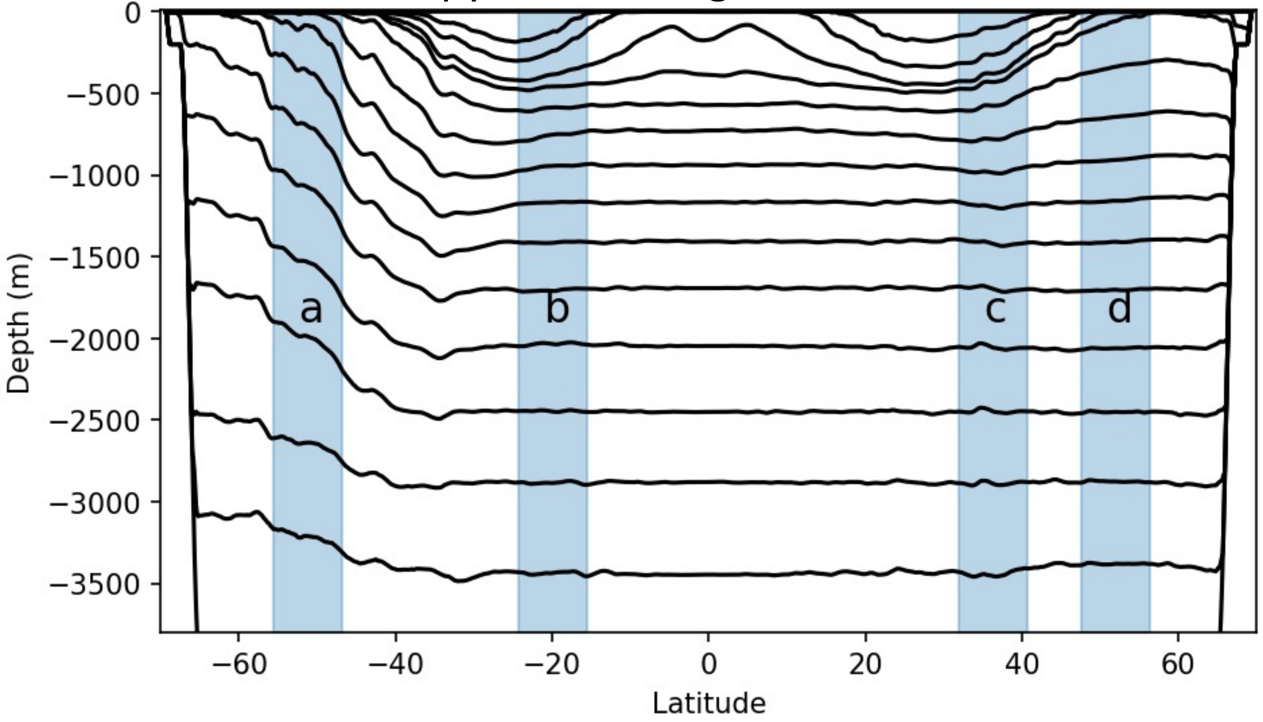
- Meridional QG PV gradient in layer i:

$$Q_{i_y} = \beta + \frac{f_0^2}{H_i}\left(\frac{U_i - U_{i+1}}{g_i'} - \frac{U_{i-1} - U_i}{g_{i-1}'}\right)$$

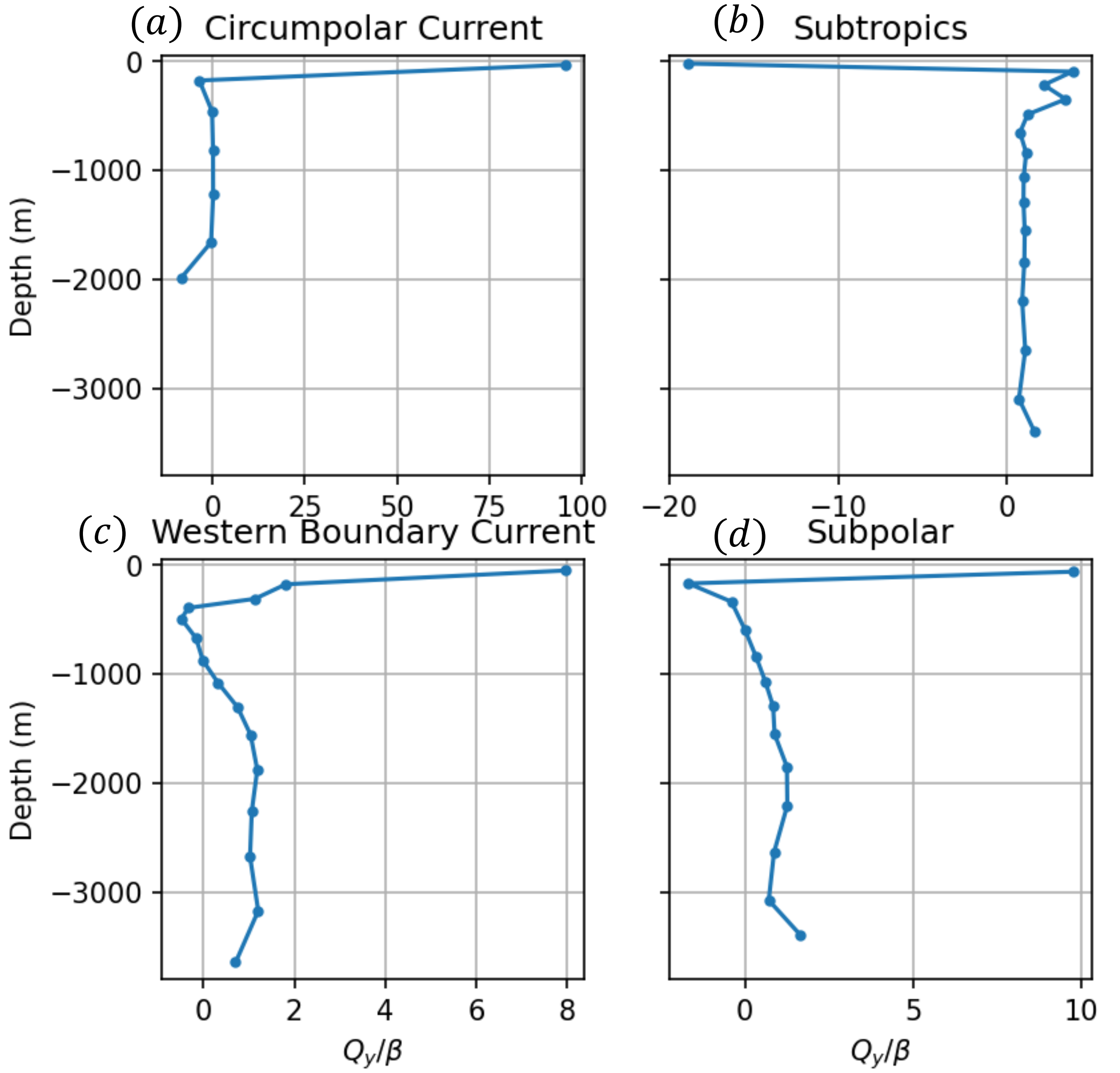
$$Q_{1_y} = \beta + \frac{f_0^2}{H_1}\left(\frac{U_1 - U_2}{g_1'}\right) + \frac{f_0^2}{gH_1}\mathrm{U}_1$$

$$Q_{N_y} = \beta + \frac{f_0^2}{H_N}\left(\frac{U_N - U_{N-1}}{g_{N-1}'}\right) + \frac{f_0}{H_N}\eta_{b_y}$$

Isopycnals at longitude 12.5°

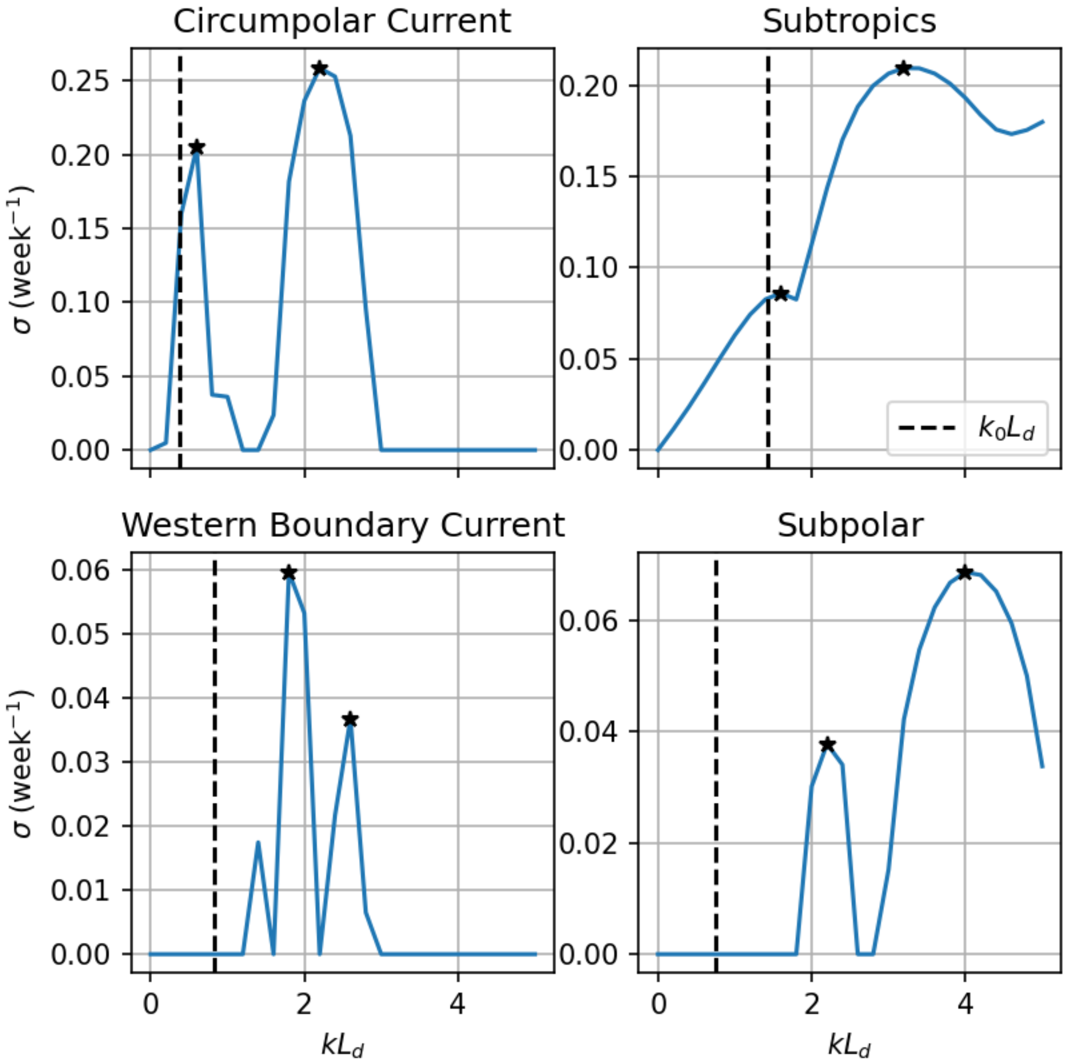


- PV gradient intensifies at surface and changes sign from surface to interior, which can lead to Charney baroclinic instability



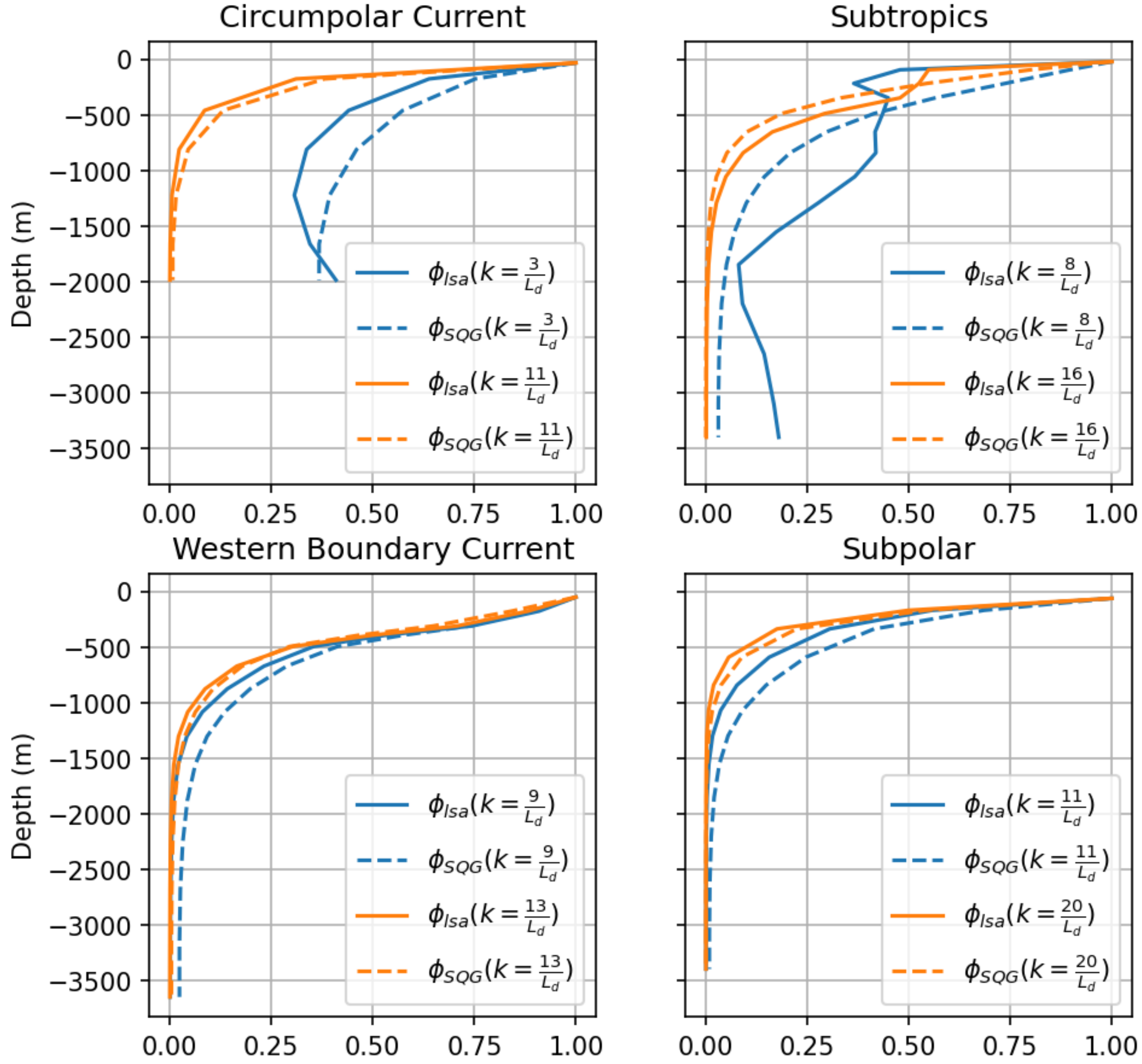
Vertical structure of unstable modes

- The most unstable mode is generally smaller than the deformation radius L_d



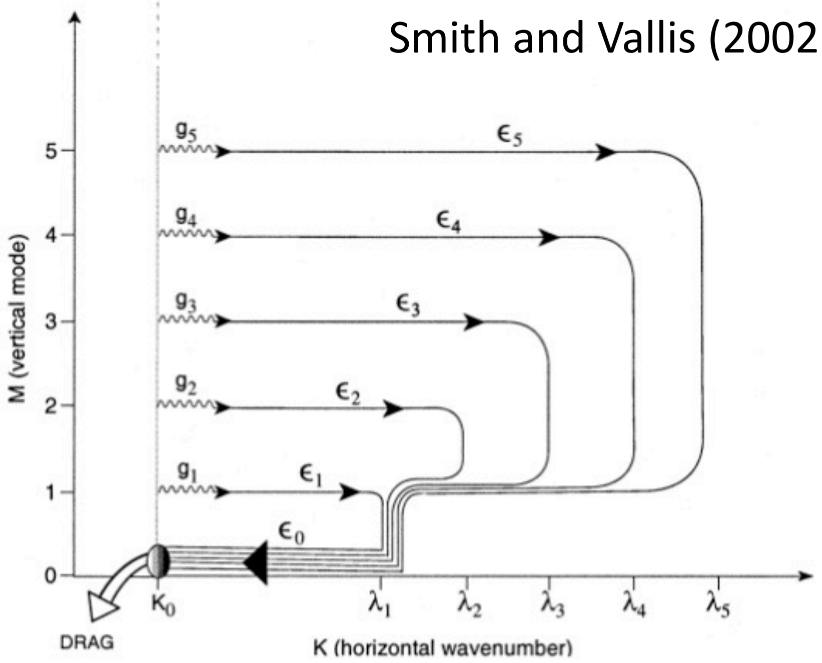
$\omega = \omega_r +$

- Vertical structure of unstable modes are similar to the SQG structure and scale-dependent

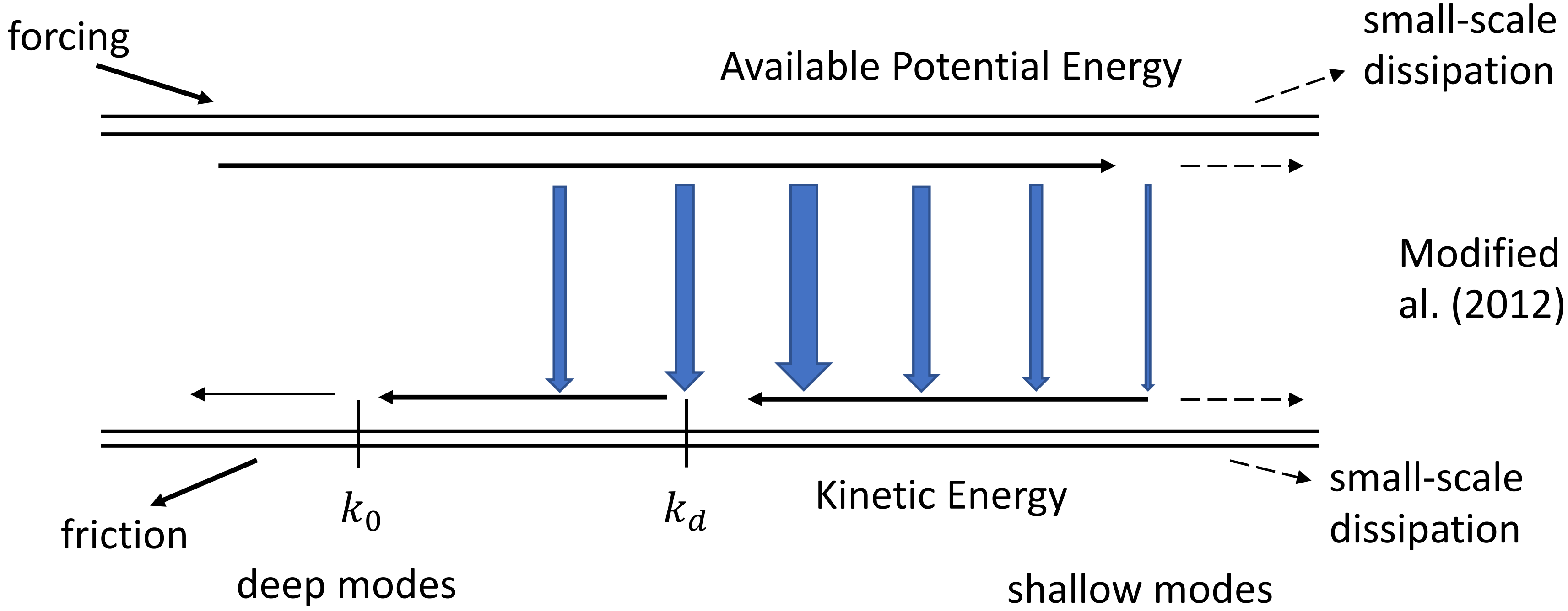


Energy cascade in surface-dominated Charney instability

Smith and Vallis (2002)



- Horizontal and vertical scales are coupled for surface-dominated Charney modes
- Barotropization and inverse cascade become equivalent
- EKE finally concentrates on the energy-containing scale (mode)



Modified from Roullet et al. (2012)

A scale-aware parameterization

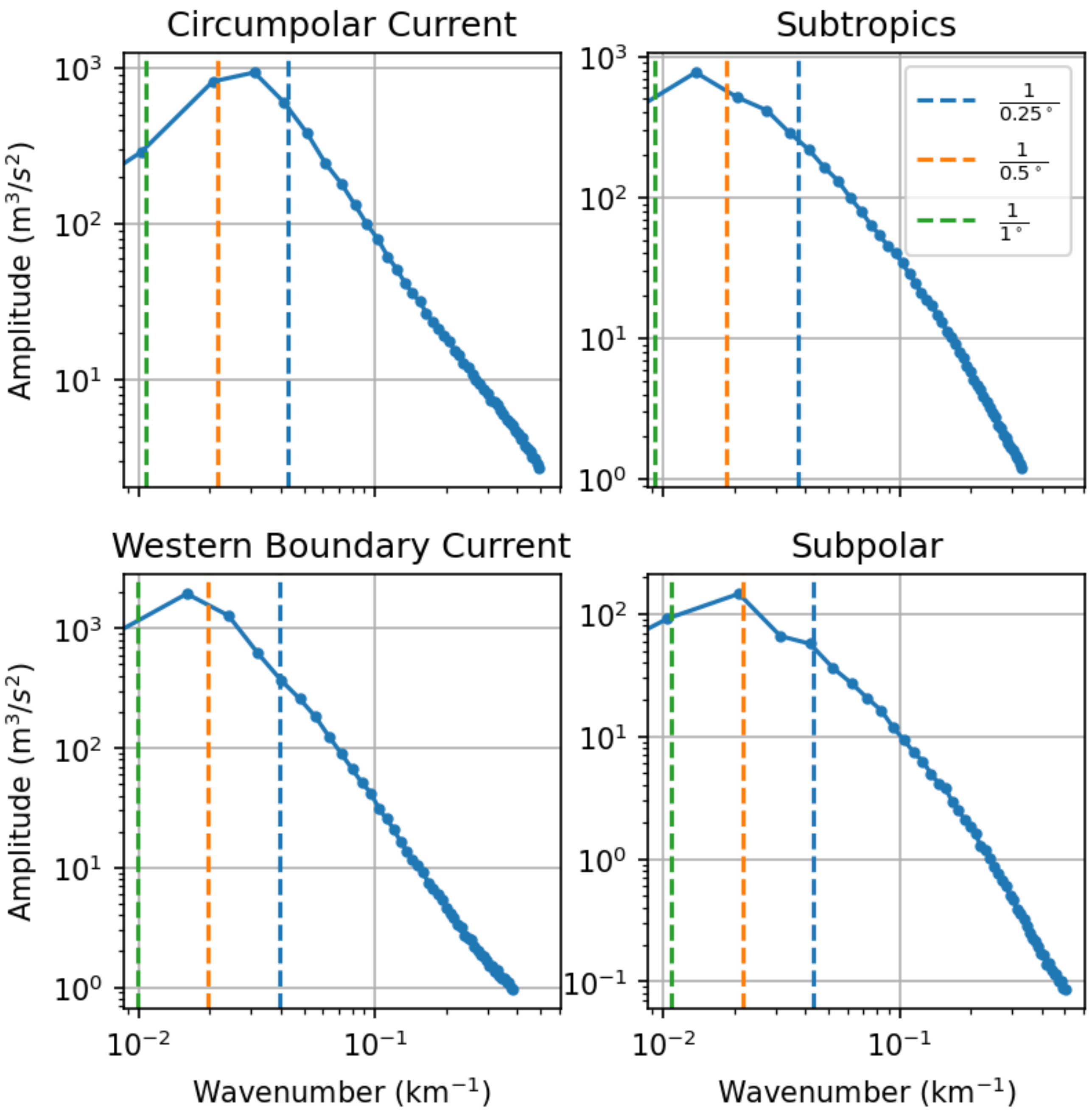
- The largest unresolved eddy by grid spacing Δ has a wavenumber k_s

$$k_s = \frac{2\pi}{6\Delta} \approx \frac{1}{\Delta}$$

- A recipe for the parameterization of vertical structure of EKE:

$$E(z,\Delta) \approx \begin{cases} e^{-2k_0 z_s}, & k_s \le k_0 \\ e^{-2k_s z_s}, & k_s > k_0 \end{cases}$$

where $z_s = \int_z^0 \frac{N}{|f|} dz$



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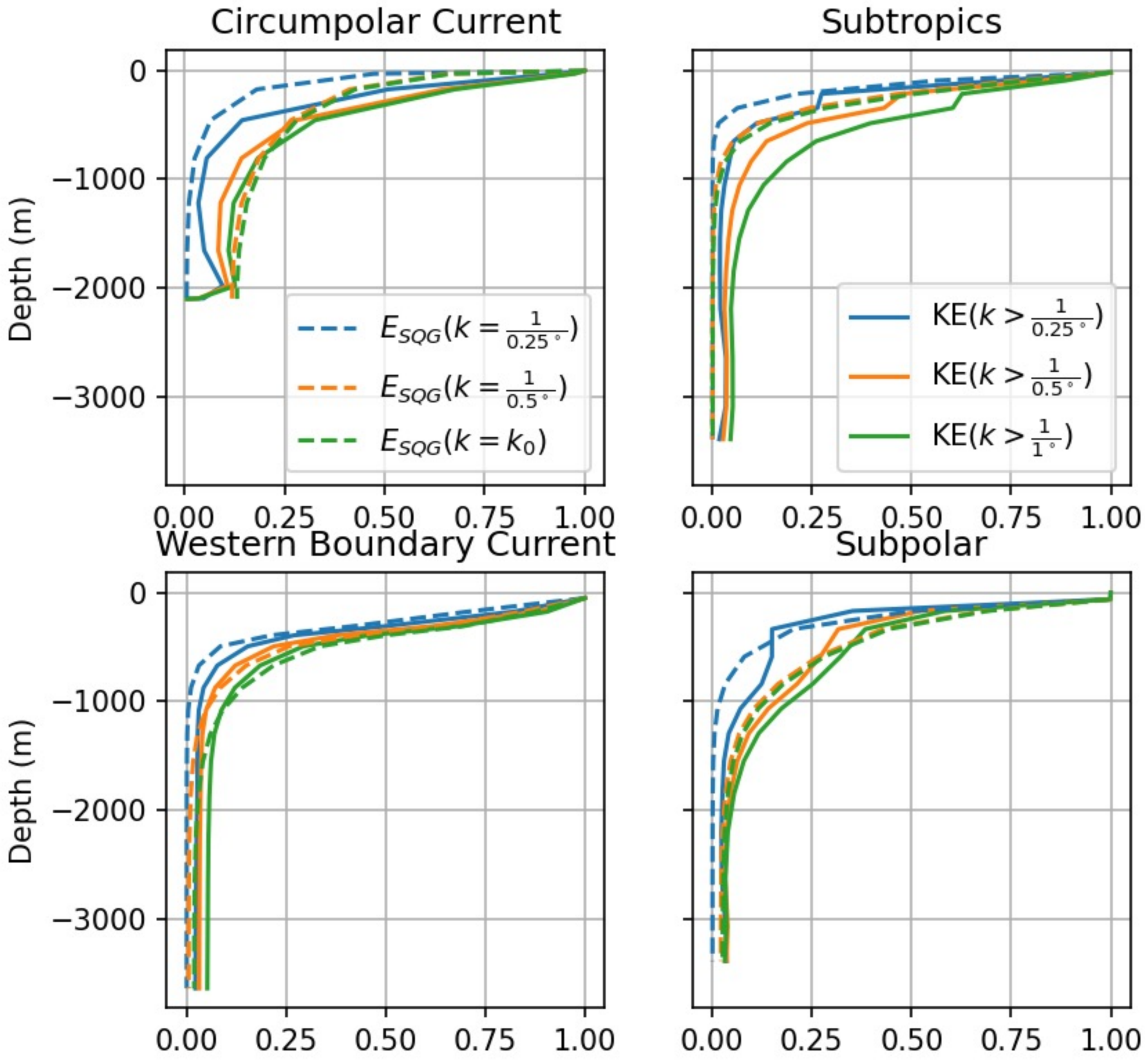
where $k_s = \frac{1}{\Delta}$ and $z_s = \int_z^0 \frac{N}{|f|} dz$

It is mostly k_s that dominates the vertical structure at eddy-permitting resolution



Summary

- The vertical structure of EKE depends on horizontal scale
- SQG mode can efficiently capture the vertical structure of EKE in most of extra-tropical regions and is **scale-dependent**
- The SQG-like vertical structure is likely the result of inverse energy cascade among Charney unstable modes
- A scale-aware parameterization is achieved by combining the largest unresolved scale with the SQG structure

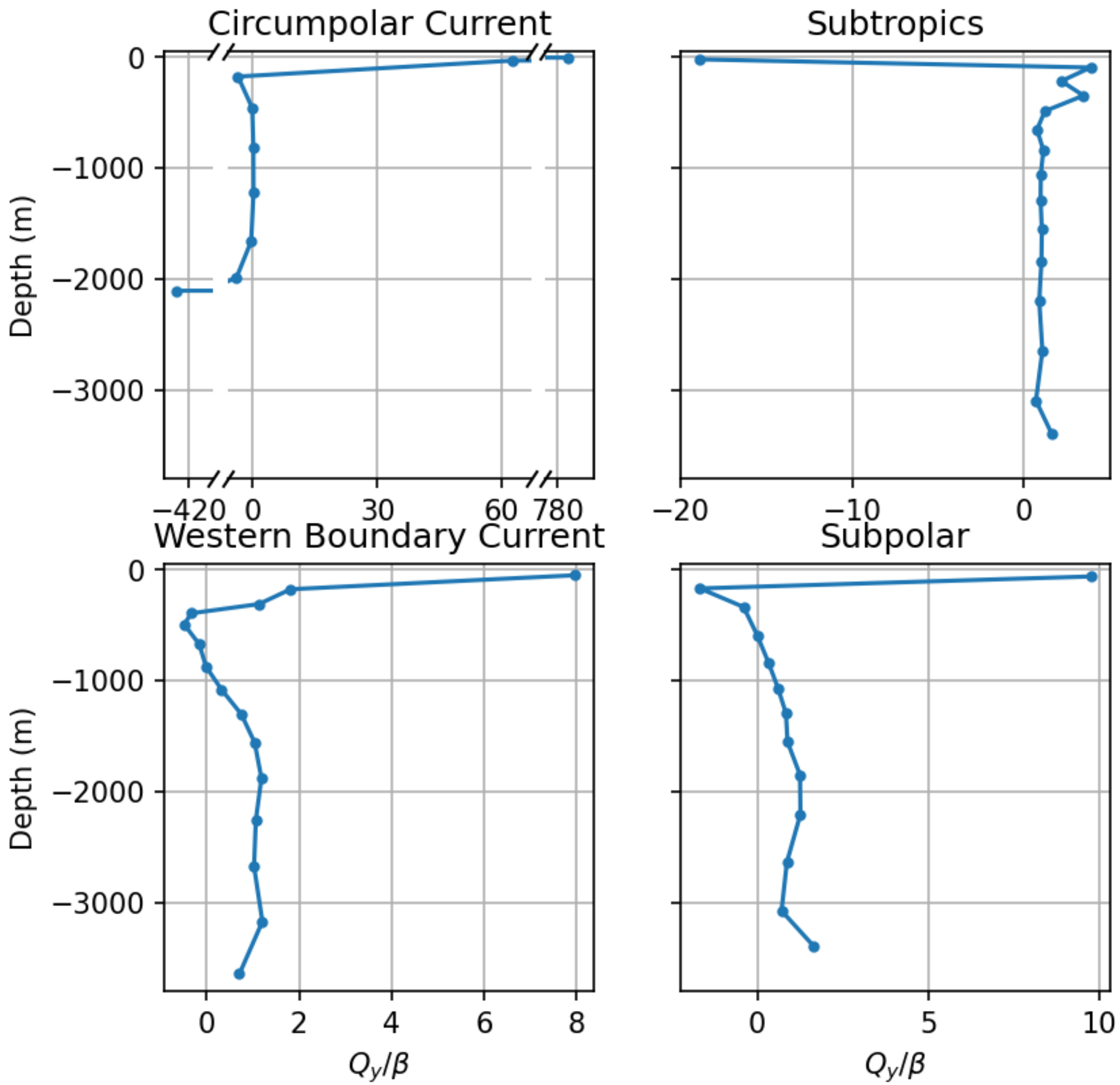
- Charney baroclinic instability has been found prevalent in subtropical gyre, and frontal regions in mid- and high-latitude ocean (Smith 2007; Tulloch et al., 2011; Capet et al., 2016; Feng et al., 2021)

Extra Slides

Linear instability analysis

- Linearized QG PV equation:
$$\frac{\partial q_i}{\partial t} + J(\Psi_i, q_i) + J(\psi_i, Q_i) = 0$$
- QG PV in interior layer i:
$$q_i = \beta y + \nabla^2 \psi_i + \frac{f_0^2}{H_i}\left(\frac{\psi_{i-1} - \psi_i}{g'_{i-1}} - \frac{\psi_i - \psi_{i+1}}{g'_i}\right)$$
$$Q_i = \beta y + \frac{f_0^2}{H_i}\left(\frac{\Psi_{i-1} - \Psi_i}{g'_{i-1}} - \frac{\Psi_i - \Psi_{i+1}}{g'_i}\right)$$
- For the top and bottom layers,
$$q_1 = \nabla^2 \psi_1 + \frac{f_0^2}{H_1}\left(\frac{\psi_2 - \psi_1}{g'_1}\right) - \frac{f_0^2}{gH_1}\psi_1,$$
$$q_N = \nabla^2 \psi_N + \frac{f_0^2}{H_N}\left(\frac{\psi_{N-1} - \psi_N}{g'_{N-1}}\right)$$
$$Q_1 = \beta y + \frac{f_0^2}{H_1}\left(\frac{\Psi_2 - \Psi_1}{g'_1}\right) - \frac{f_0^2}{gH_1}\Psi_1$$
$$Q_N = \beta y + \frac{f_0^2}{H_N}\left(\frac{\Psi_{N-1} - \Psi_N}{g'_{N-1}}\right) + \frac{f_0}{H_N}\eta_b$$

Assume $\psi = \phi(z)e^{i(\mathbf{kx} - \omega t)}$ and solve for the eigenvalue ω and eigenvector $\phi(z)$.

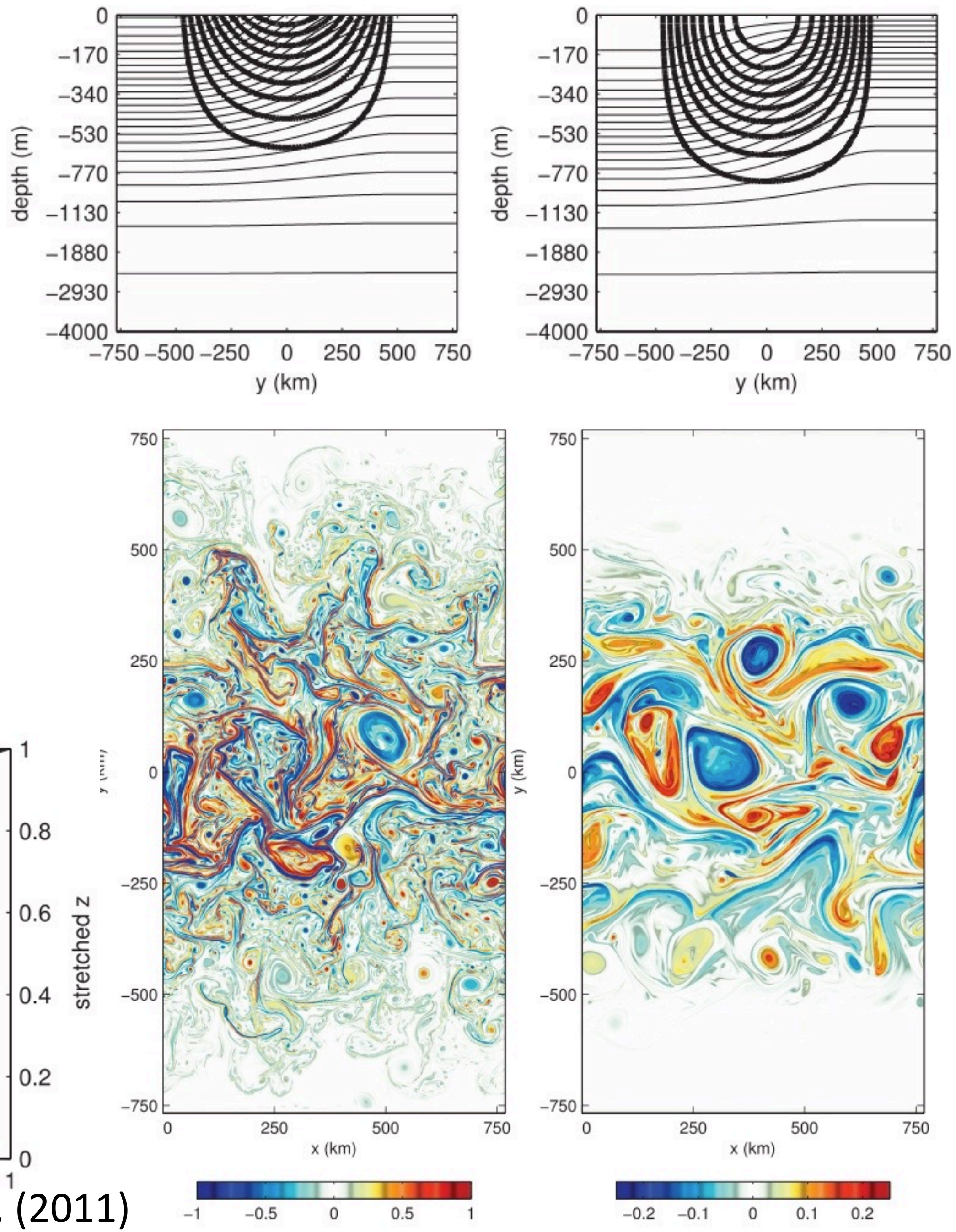


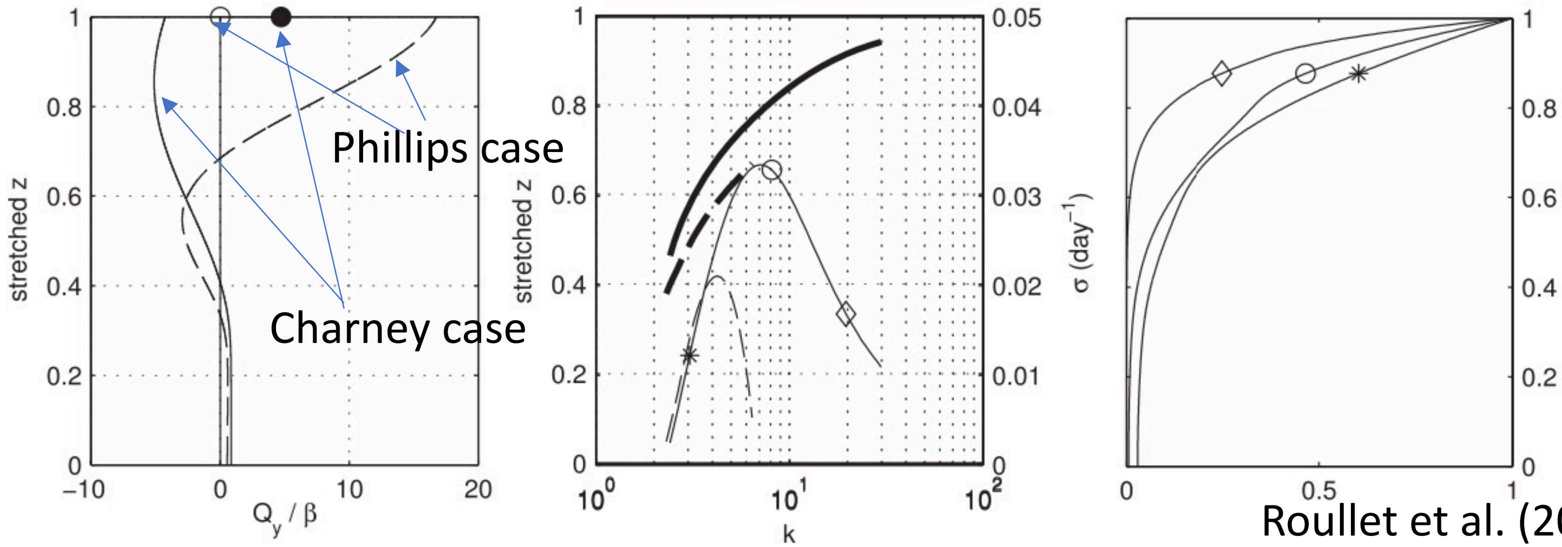
Role of surface buoyancy

- Surface buoyancy gradient can induce surface-intensified Charney baroclinic instability in subtropical and mid-latitude ocean (Smith 2007; Tulloch et al., 2011; Capet et al., 2016; Feng et al., 2021)
- Meridional QG PV gradient:

$$Q_y = \beta + \frac{\partial}{\partial z}\frac{f}{N^2}\bar{b}_y - \frac{f}{N^2}\bar{b}_y\Big|_{z=0}\delta(z) + \frac{f}{N^2}\bar{b}_y\Big|_{z=z_b}\delta(z+H)$$

Roullet et al. (2011)





Roullet et al. (2011)

EKE is averaged in 4°×4° grids

$$\frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial \hat{\psi}^p}{\partial z}\right) - \kappa^2 \hat{\psi}^p = 0$$

$$\hat{\mathrm{E}}_0{}^p = \begin{cases} \left(\frac{k}{k_0}\right)^{-2}, & k \geq k_0 \\ \left(\frac{k}{k_0}\right)^{2}, & k < k_0 \end{cases}$$

$$\mathrm{E}_{SQG}^{p}(z) = \mathrm{EKE}_0 \sum_k \hat{\mathrm{E}}_{Numerical}^{p}(k,z)$$

$$\mathrm{E}_{SQG}^{k_0}(z) = \mathrm{EKE}_0 e^{2k_0 z_s}$$

$$\mathrm{R}^2 = 1 - \frac{\int_{-H}^{0}\left(E_{diag}(z) - E_{pre}(z)\right)^2 dz}{\int_{-H}^{0}\left(E_{diag} - \bar{E}_{diag}^{z}\right)^2 dz}$$