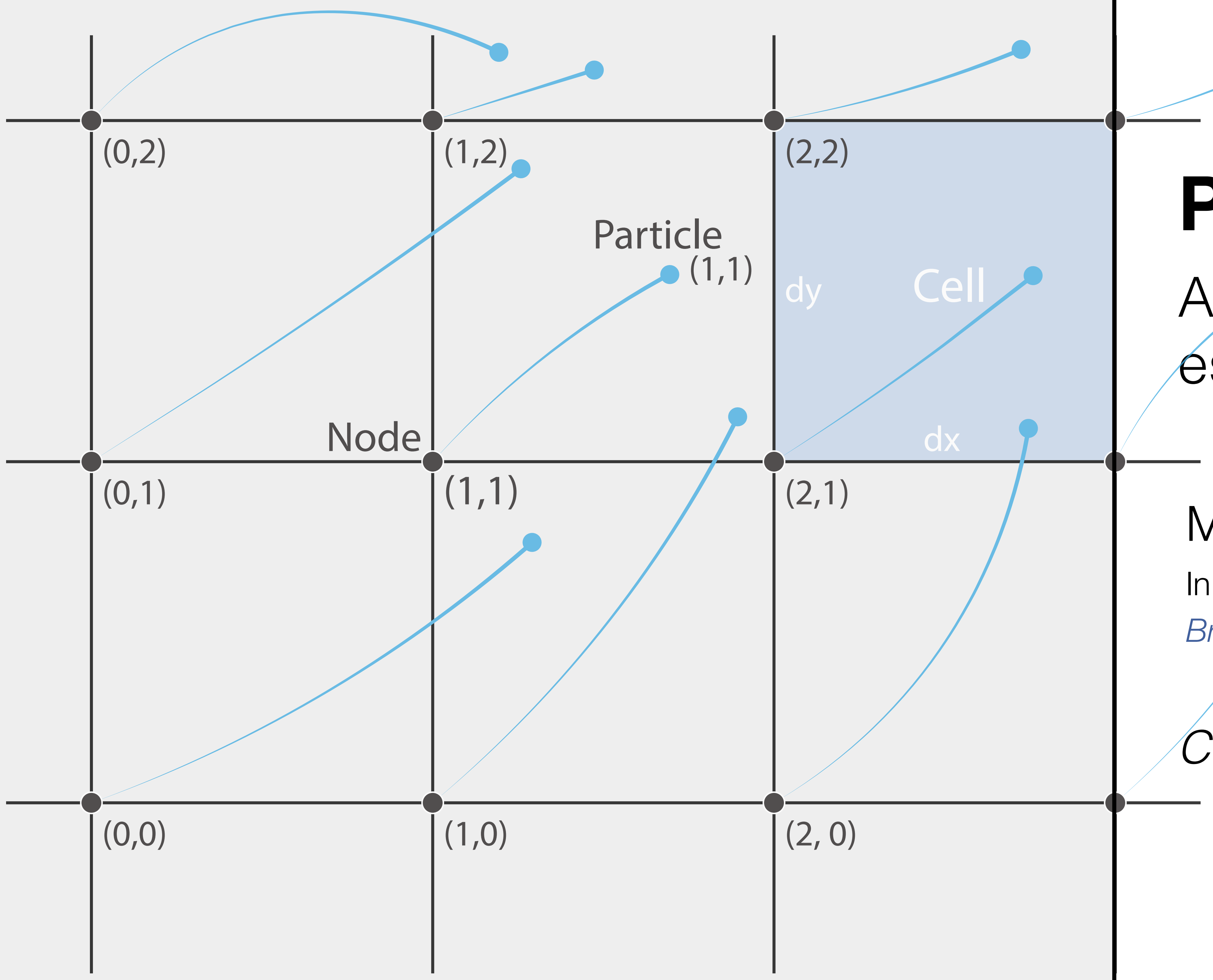


Particle-In-Cell for Efficient Swell *PiCLES*

A wave model for efficient sea-state and swell estimates in coupled models

Momme Hell and Baylor Fox-Kemper

In collaboration with Bertrand Chaperon

Brown University, Providence, RI, USA, mhell@brown.edu

CESM Ocean Working Group Meeting, February 2023

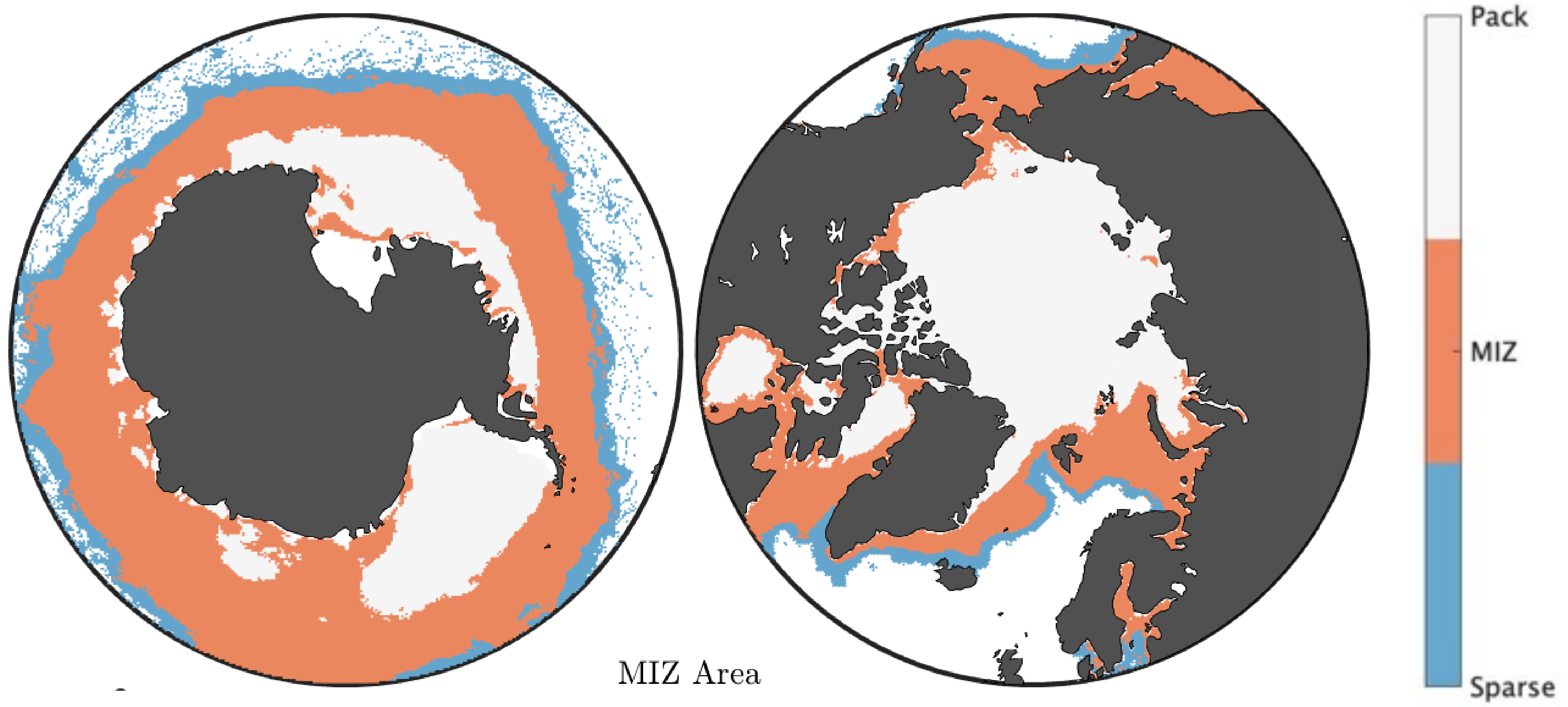
Wave modeling for Earth System Models

Main objectives and goals

Top 3 targets for a coupled modeling context

1) **Non-local swell impact on the MIZ**
2) **Gradients in the wave field on scales $O(Ro_{atm})$** effect drag, white capping, sea spray, …
3) **Wave-current interaction on scales $O(Ro_{ocean})$** may effect Langmuir turbulence …

4) (There are more, but they are not on the scale of ESMs … yet)



(From Chris Horvat)

Arctic MIZ
15-25% of ice extent

Southern Ocean MIZ
20-60% of ice extent

- Large discrepancy in the MIZ between CMIP6 models

Wave modeling for Earth System Models

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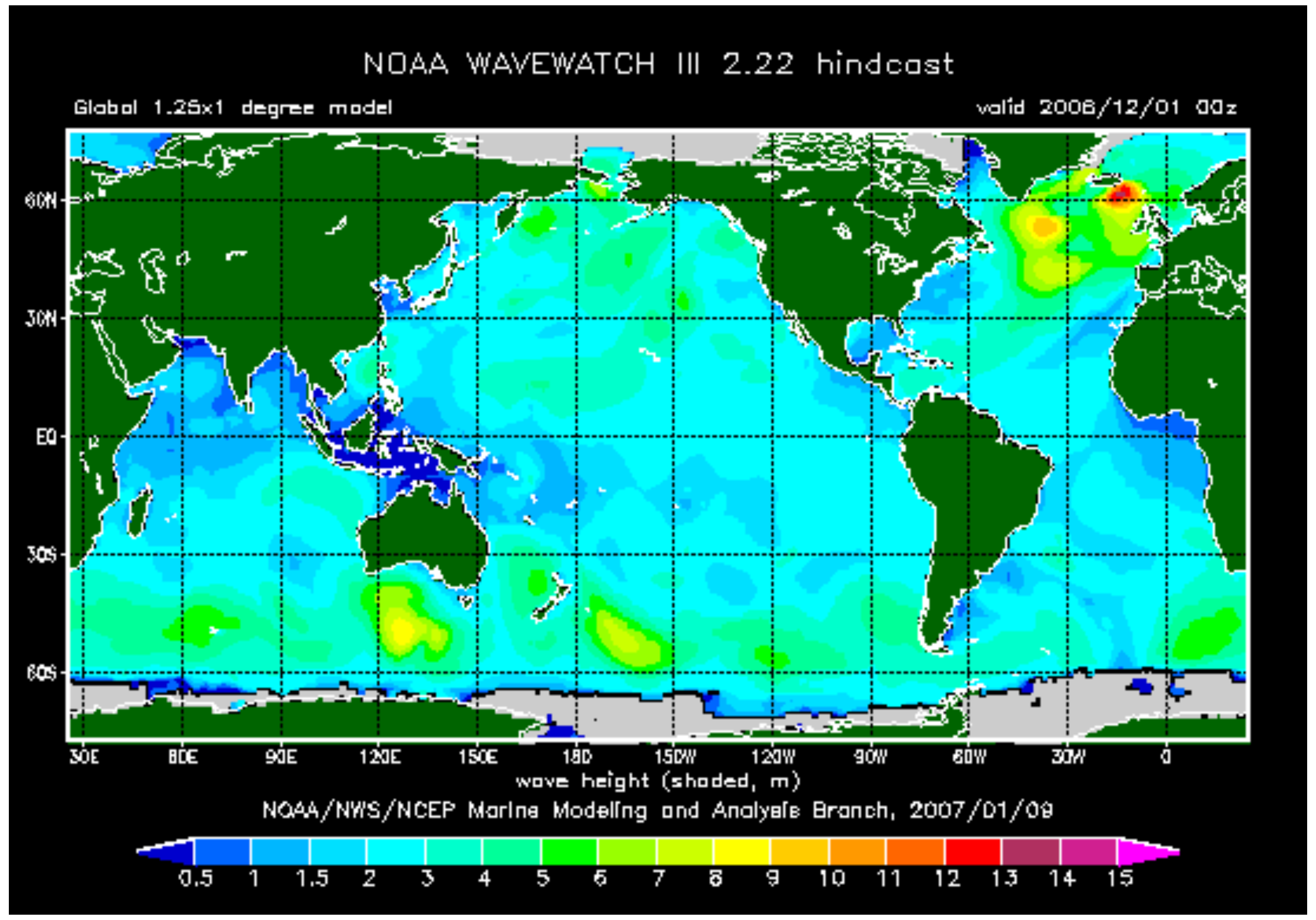

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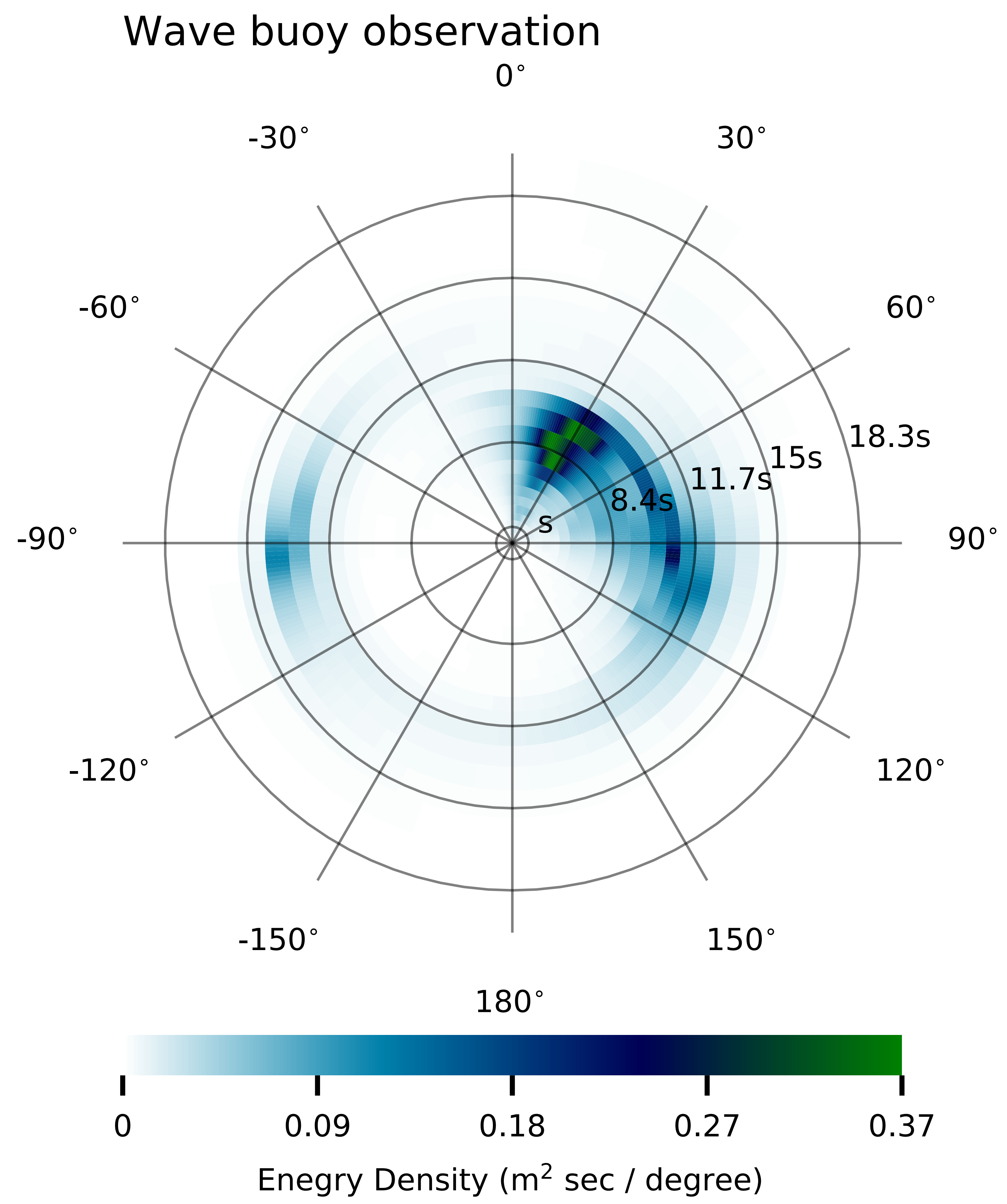
Directional wave spectra at Ocean Station Papa

Surface stress balances the excess of atmospheric angular momentum



Typical wave observations

- Wind sea & 1-3 Swell fields
- Each of these wave partition have **a direction, peak frequency, and energy**
- The total wave spectrum can be *approximated* by **9 variables**

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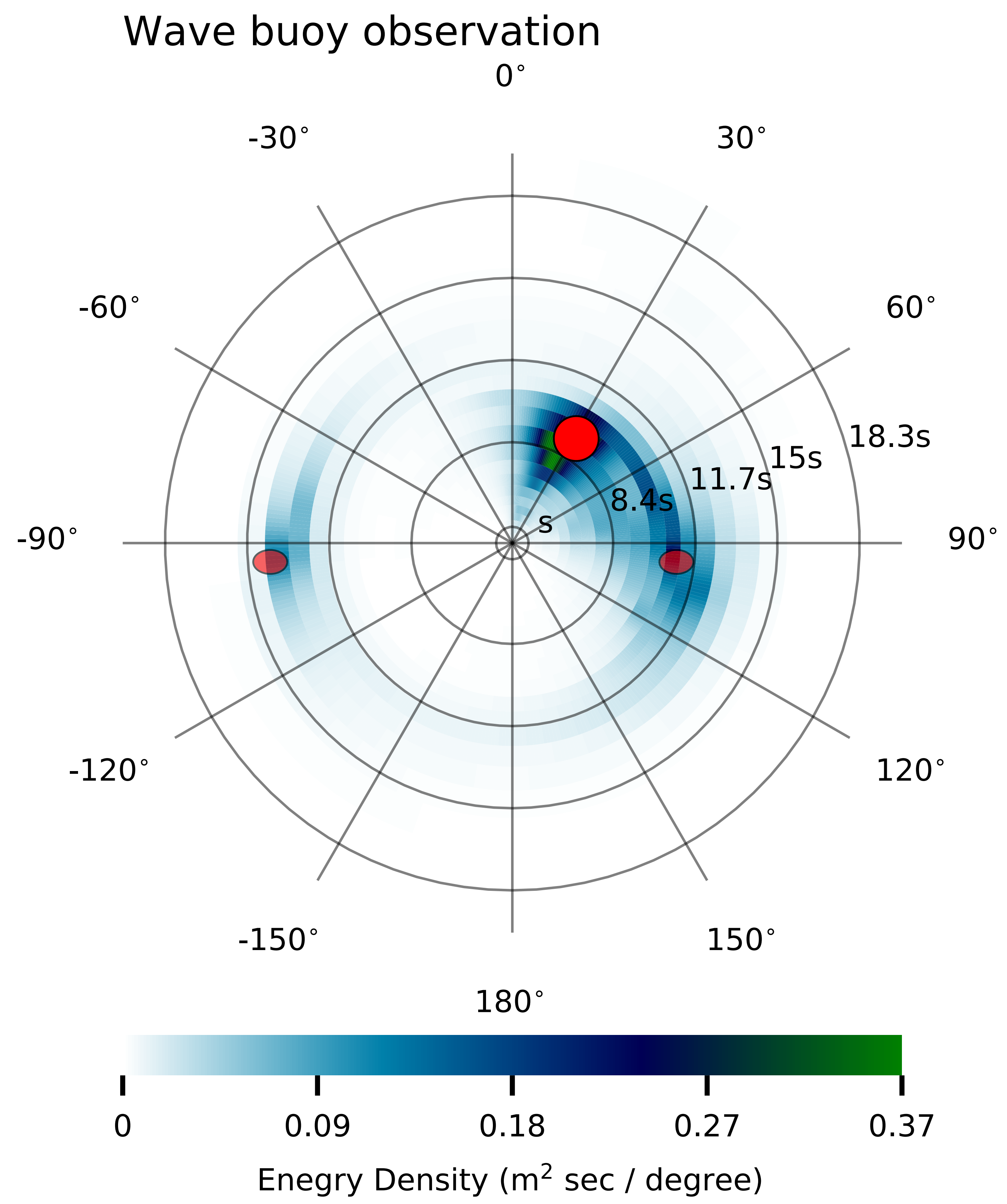

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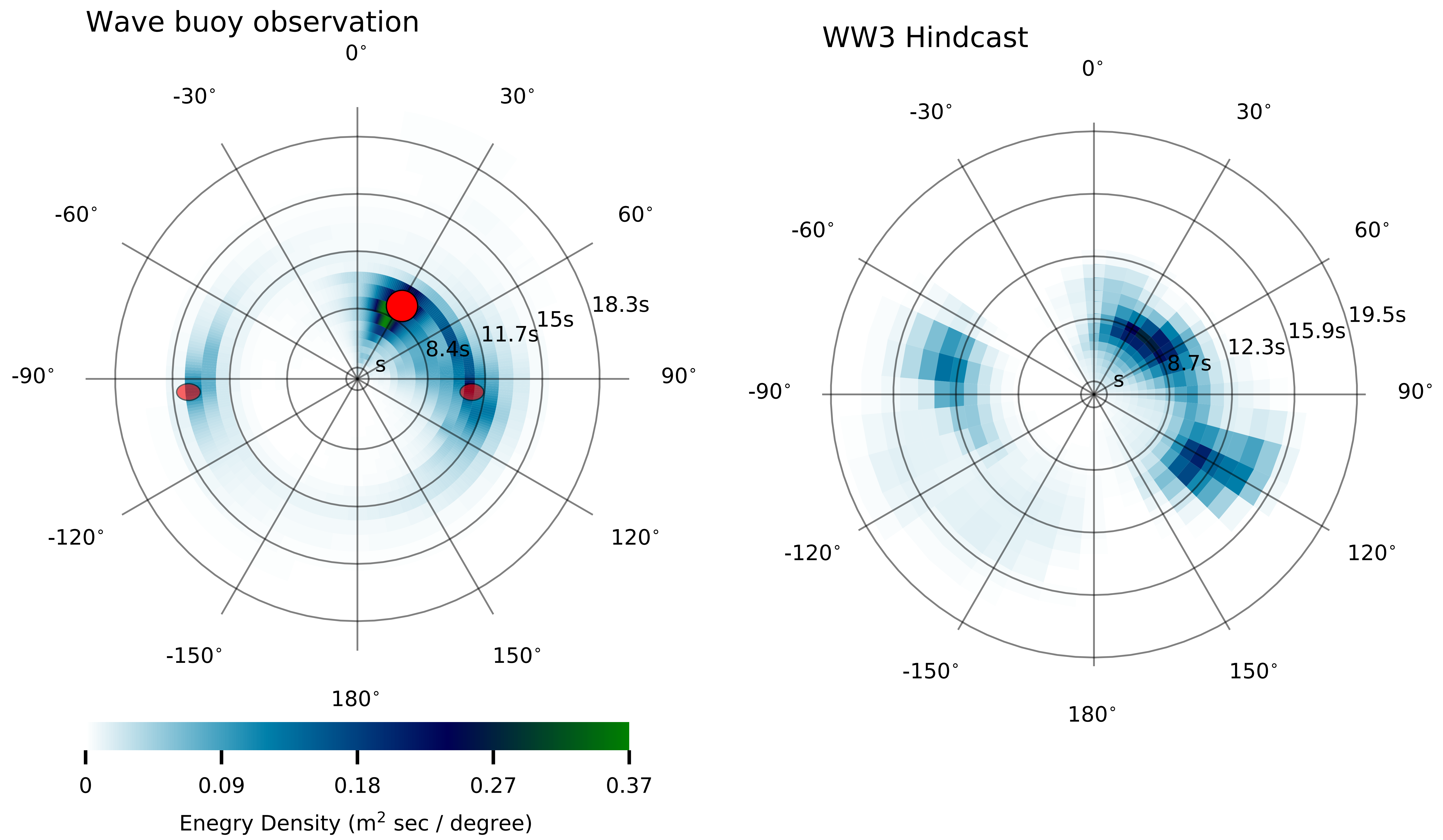

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- discretize the wave action in frequency and direction
- needs **about 600 variables** to describe nearly the same information

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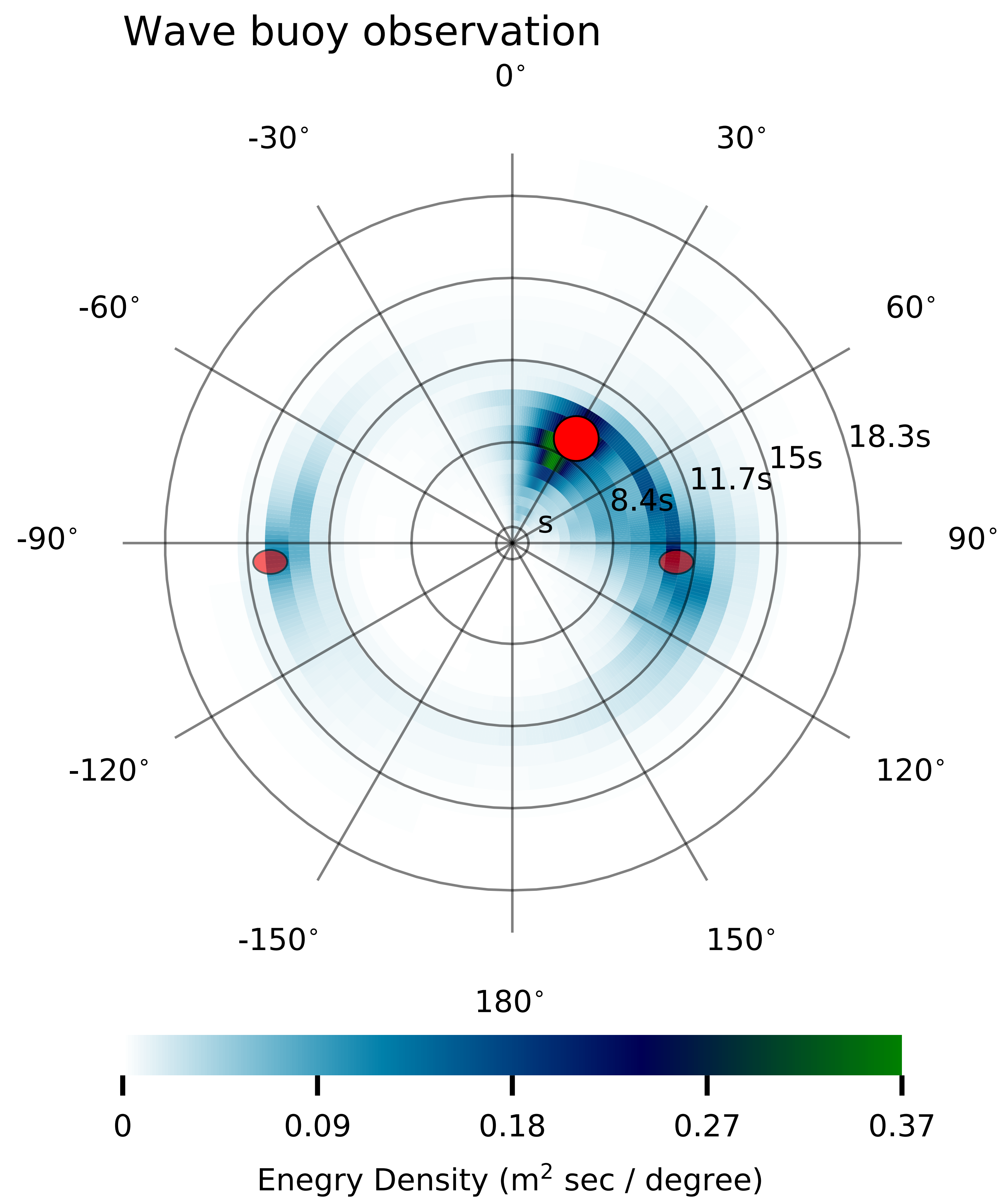

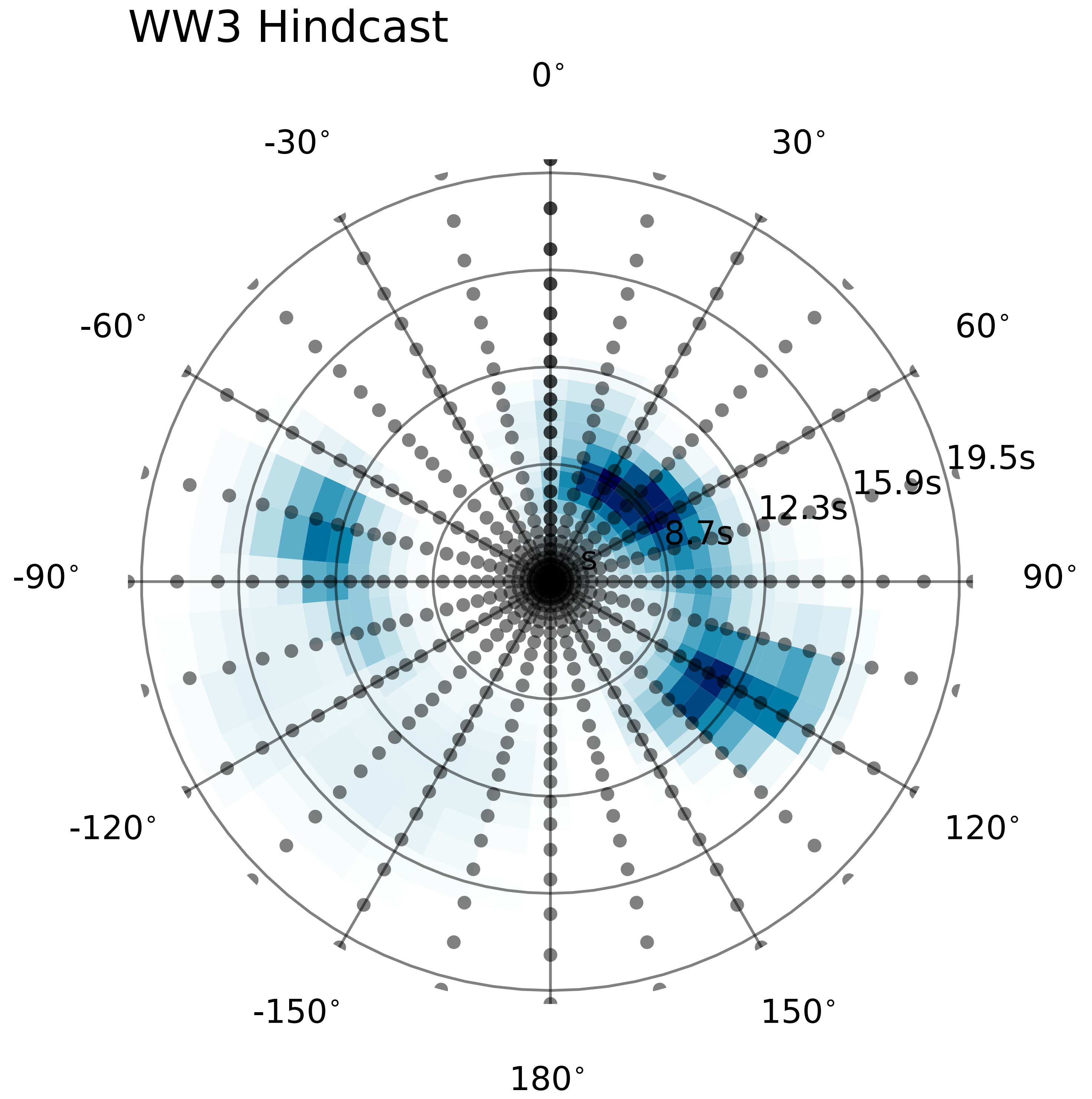

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Wave action equation

$$\frac{\partial}{\partial t}N + \nabla \cdot (c_g N) = S_{in} + S_{ds} + S_{nl}$$

$$N(x,\ y,\ t,\ k,\ l)$$

A hierarchy of surface wave models

Time travel to simpler models?

Increasing level of complexity

3rd generation wave models

The wave modeling project (WAM)
International effort that let to the modern wave modeling methods (1984-1994)

WAM, WW3, SWOM, SWAN
can model non-linear interactions, but often parametrizes
Susan Hasselman & Hasselmann, 1985

space (2D), time, frequency, direction

- Solves wave action equation for each frequency and direction
- provides 2D spectral at each grid point

Lagrangian Wave modeling
Parameterized non-linear interactions in a moving system
Kudryavtsev, et al. 2015, 2021, Hell et al. 2021, Ardhuin et al. 2000, ..

space (1D) *and* time
Lagrangian wave growth along a particle trajectory

2nd generation

2nd generation wave models — *Fetch relation*
Pierson-Moskowitz, GONO, HYPA, UKMO, JONSWAP, ..
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space (1D) *or* time
simulates wave growth for a given fetch

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2nd generation+

Particle-in-Cell for Efficient Swell - **PiCLES**
Lagrangian Wave source terms with an integrative remeshing

space (2D) *and* time

- wave growth along particle trajectories, and re-meshes
- provides output on a required grid and tilmestep

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Parameterized non-linear interactions in a moving system
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2nd generation+ wave model

PiCLES

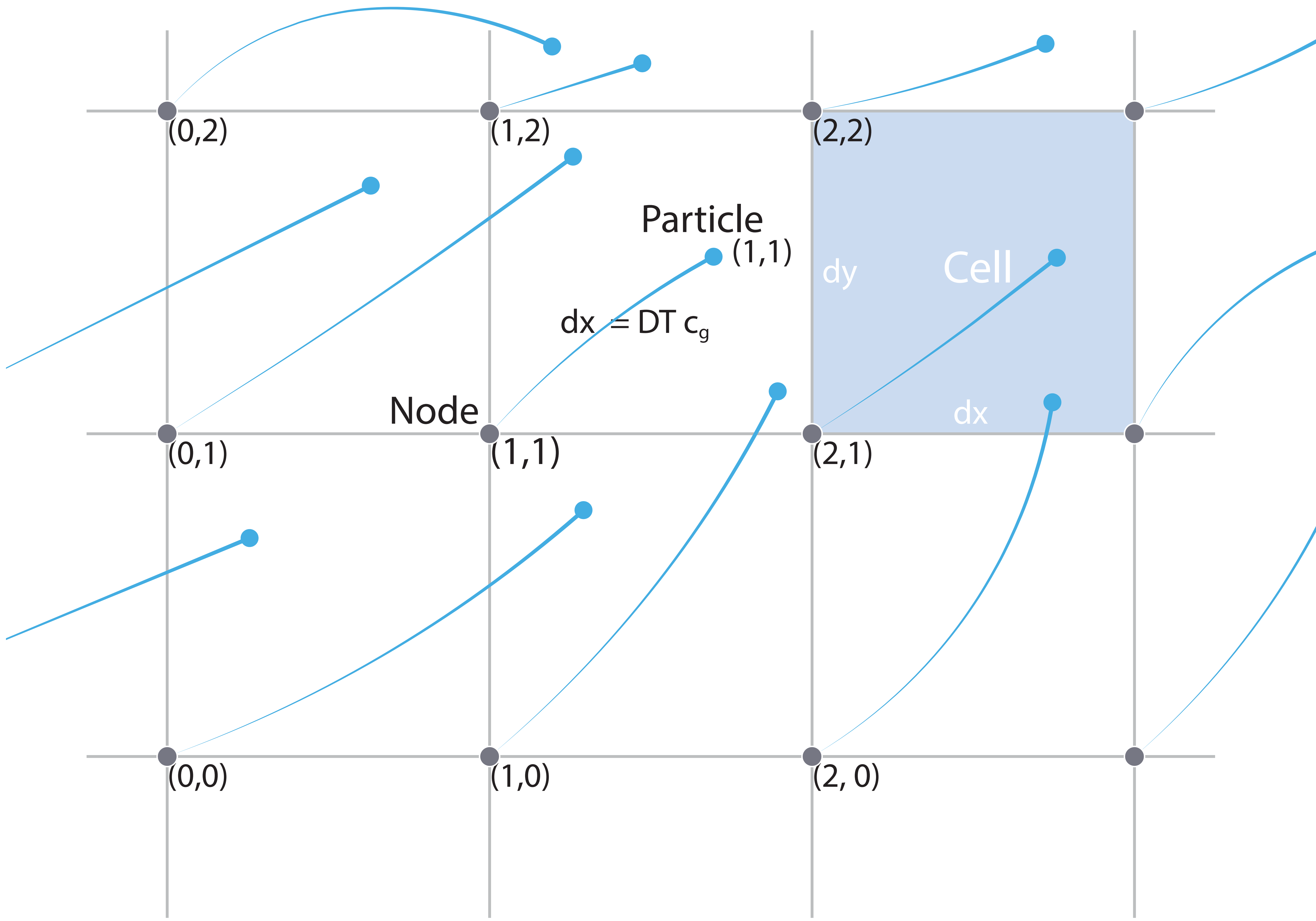
Main objective:

Trade accuracy for speed and convenience!

- Find alternative to reduce the high-dimensionality to **improve efficiency**
- Describe **sufficiently accurate** surface statistics for air-sea interaction in Earth System Models.

Key requirements

- Minimize particle interaction
- Designed to be parallel on GPUs
- Partition between wind sea and swell
- Written in julia
- Focus on open-ocean waves



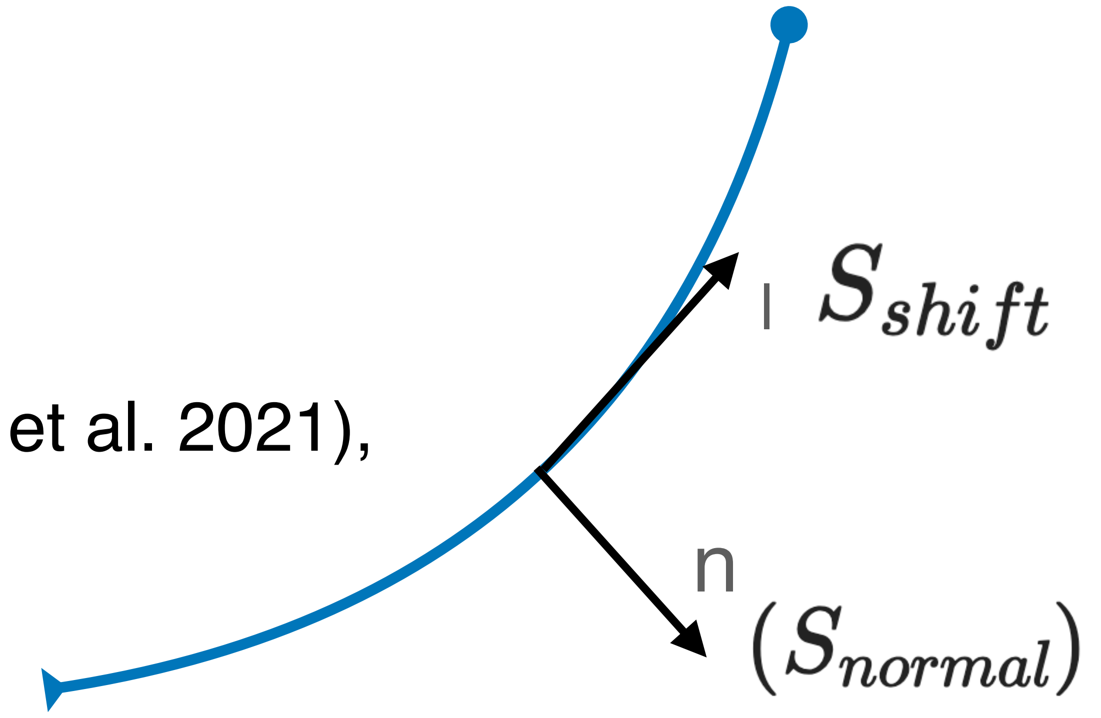
Equations to solve along a trajectory

Conservation of wave energy (~ wave action)

$$\frac{\partial}{\partial t}E = -\nabla(c_g E) + S_{in} - S_{dis} + S_{nl}$$

Splitting the divergence term in along and across track component (Kudryavtsev et al. 2021),

$$\frac{\partial}{\partial t}E = -\partial_l\left(c_g E\right) - \partial_n\left(c_g E\right) + S_{in} - S_{dis} + S_{nl}$$



Similar to WW3

parameterized wave-wave interaction

Particle equations

$$\frac{\partial}{\partial t}ln(e) = S_{input} - S_{dis.} - S_{shift} - (S_{normal})$$

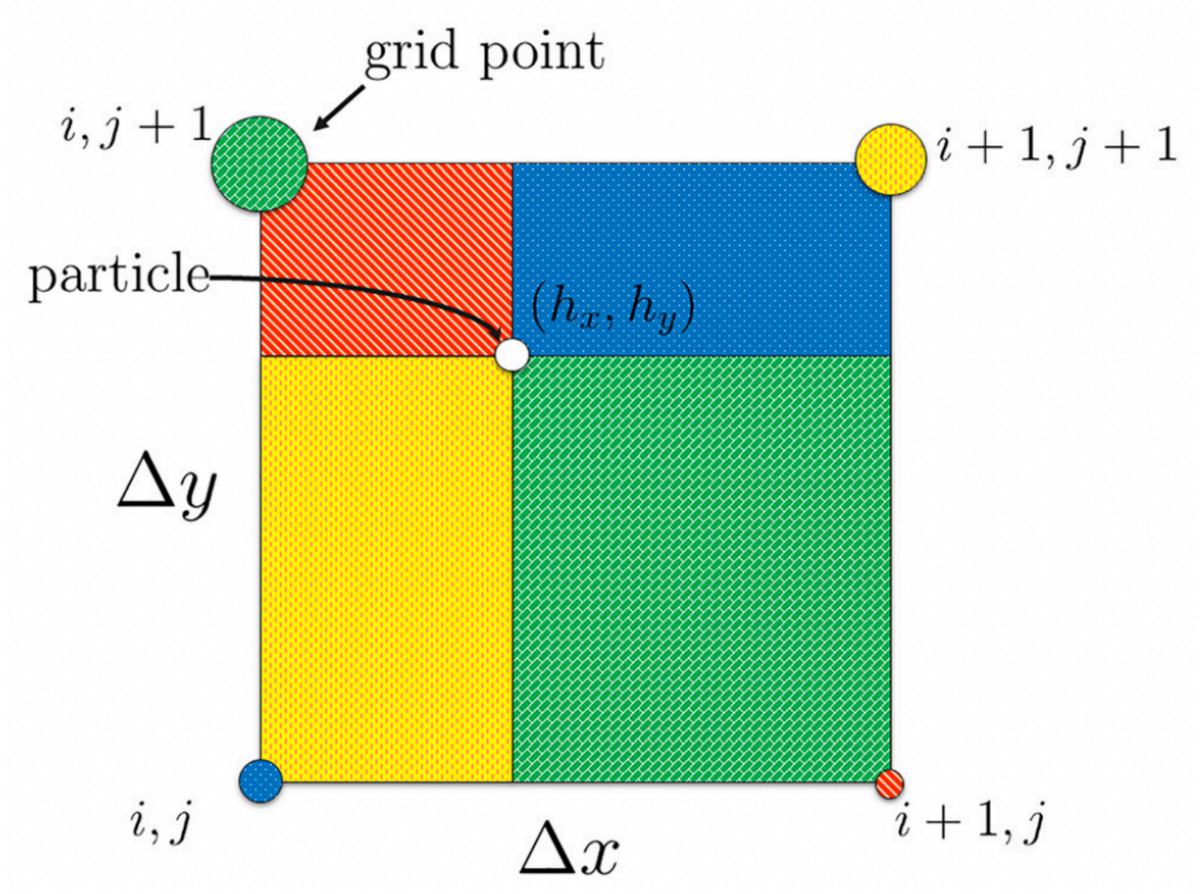
$$\frac{\partial}{\partial t}\mathbf{c_g} = S_{shift} - \mathbf{S}_{direction}$$

$$\frac{\partial}{\partial t}\mathbf{x} = \mathbf{c_g}$$

- Wave-wave interaction along the trajectory is parametrized
- Wave-wave interaction normal to the particle trajectory are often small and
    a) modeled in the re-meshing step, or,
    b) parameterized by cross-interaction term

Re-meshing
based on Particle-in-Cell

- Originally developed in Los Alamos (Evans 57, Harlow, Brackbill et al. 86, 88, …)
- now widely used in plasma physics, electro-magnetics, and geophysical applications
- can model strong gradients and shocks well



Weight factors

$$\rho_{i,j} = \rho_c \frac{(\Delta x - h_x)(\Delta y - h_y)}{\Delta x \Delta y},$$

$$\rho_{i,j+1} = \rho_c \frac{(\Delta x - h_x) h_y}{\Delta x \Delta y},$$

$$\rho_{i+1,j} = \rho_c \frac{h_x (\Delta y - h_y)}{\Delta x \Delta y},$$

$$\rho_{i+1,j+1} = \rho_c \frac{h_x h_y}{\Delta x \Delta y},$$

(from Rodríguez-Patiño et al. 2020, AJP)

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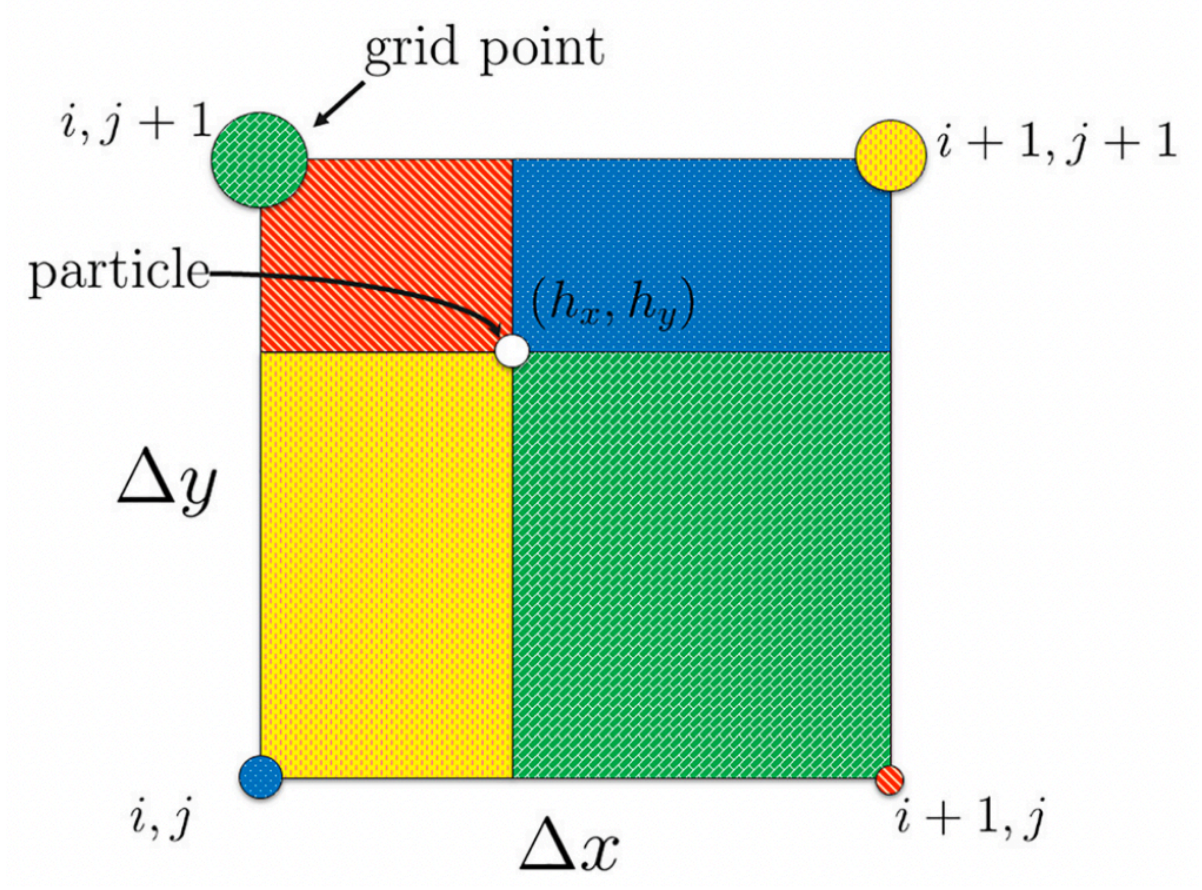

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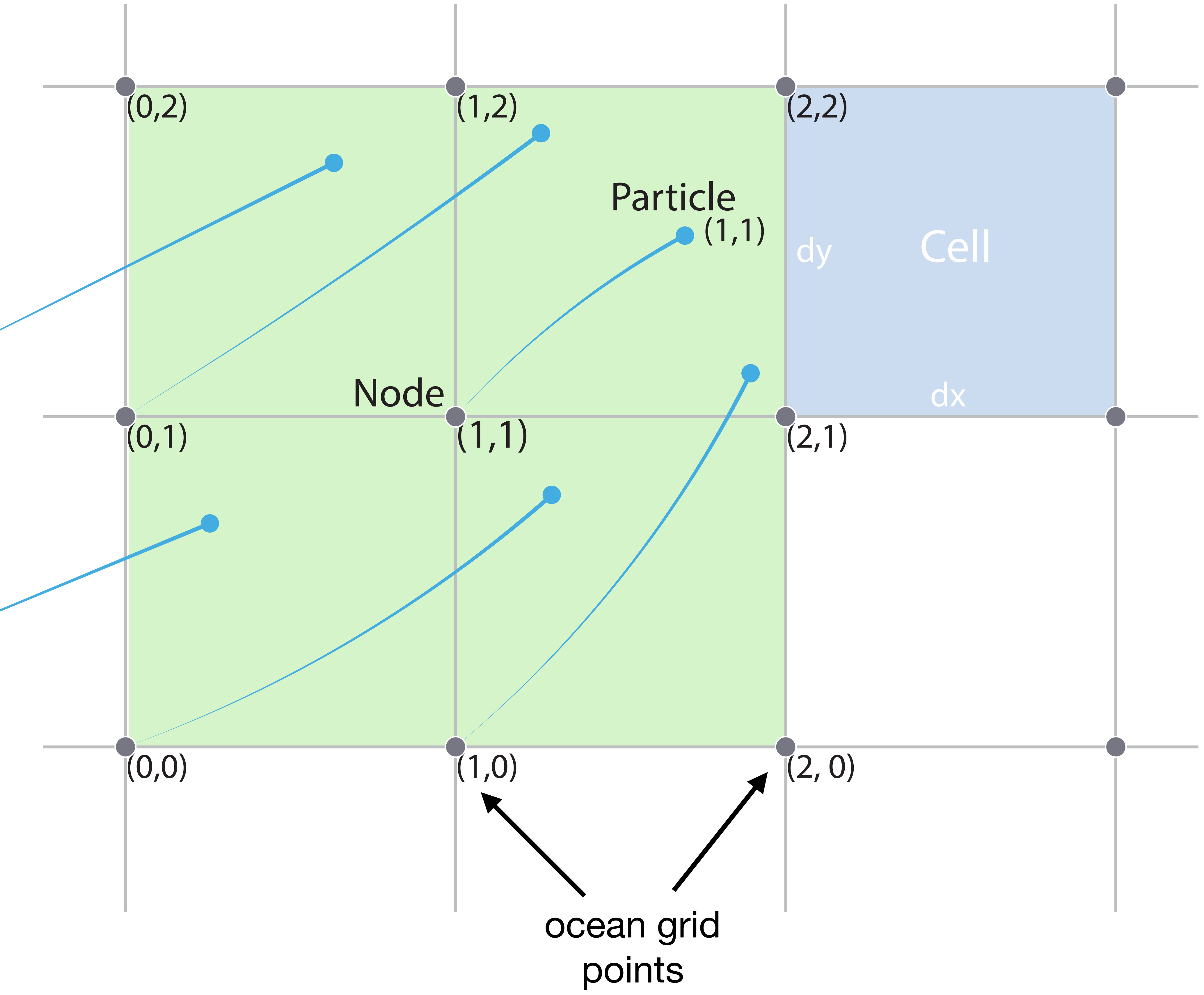
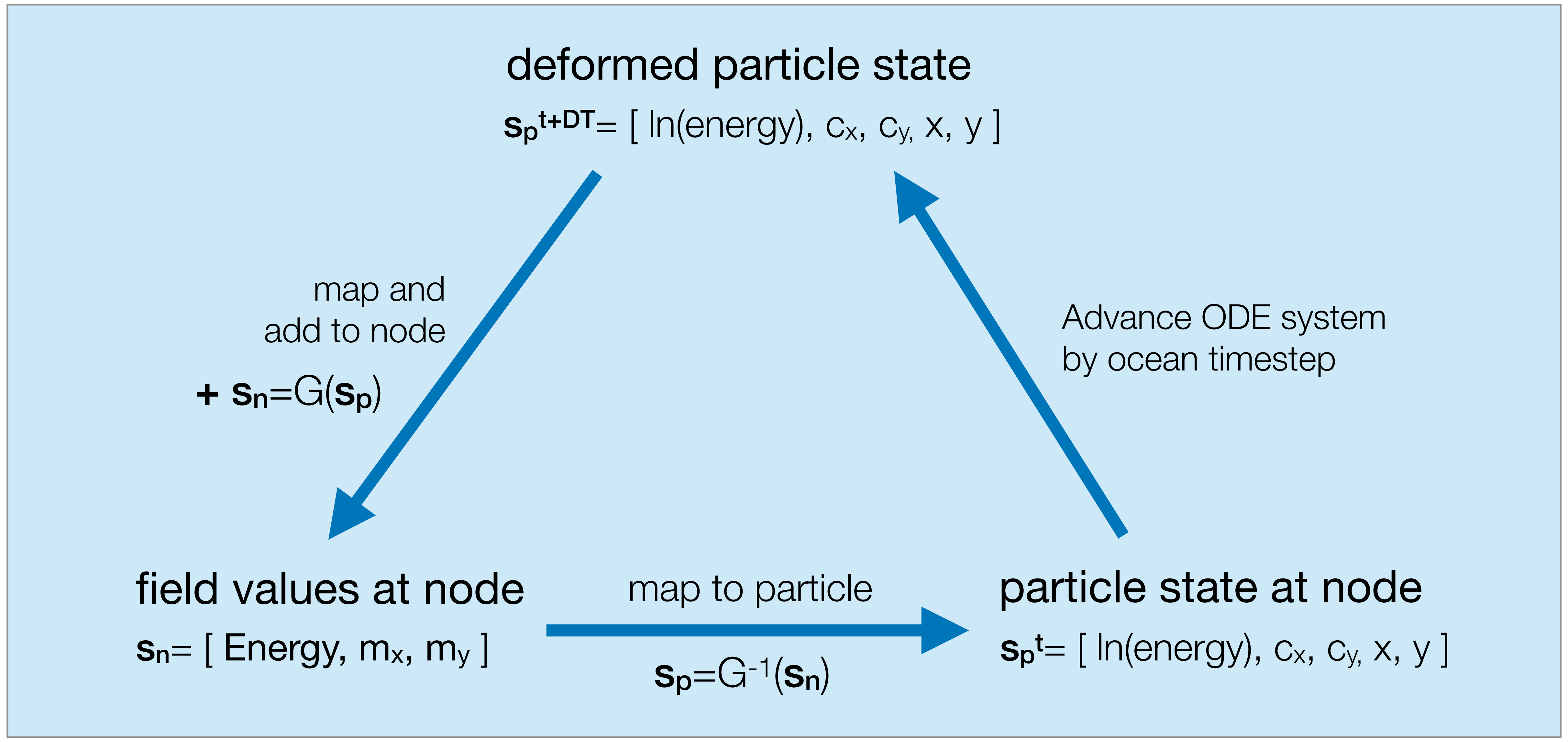
Advance and re-mesh

Lagrangian wave growth + Particle-in-Cell = PiCLES



Model cycle



$$\hat{\boldsymbol{m}} = \sum_{n}^{N} w_n \boldsymbol{m}_n,$$

$$\hat{e} = \sum_{n}^{N} w_n e_n,$$

Particle-In-Cell weighting function

- re-meshing conserves energy and momentum
- only additive operation to minimize particle interaction

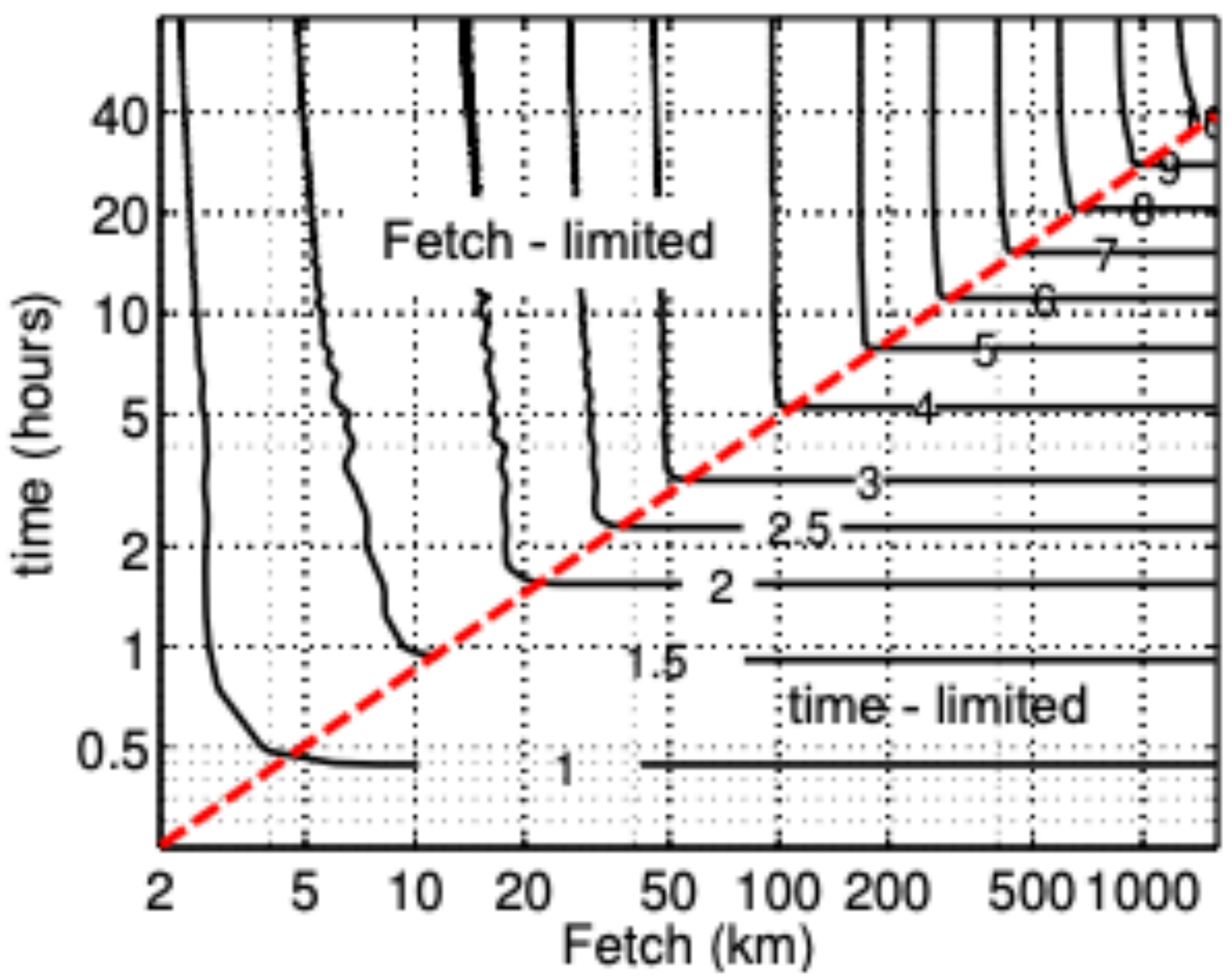
Test Case I: Static Fetch

Reproducing 2nd generation models

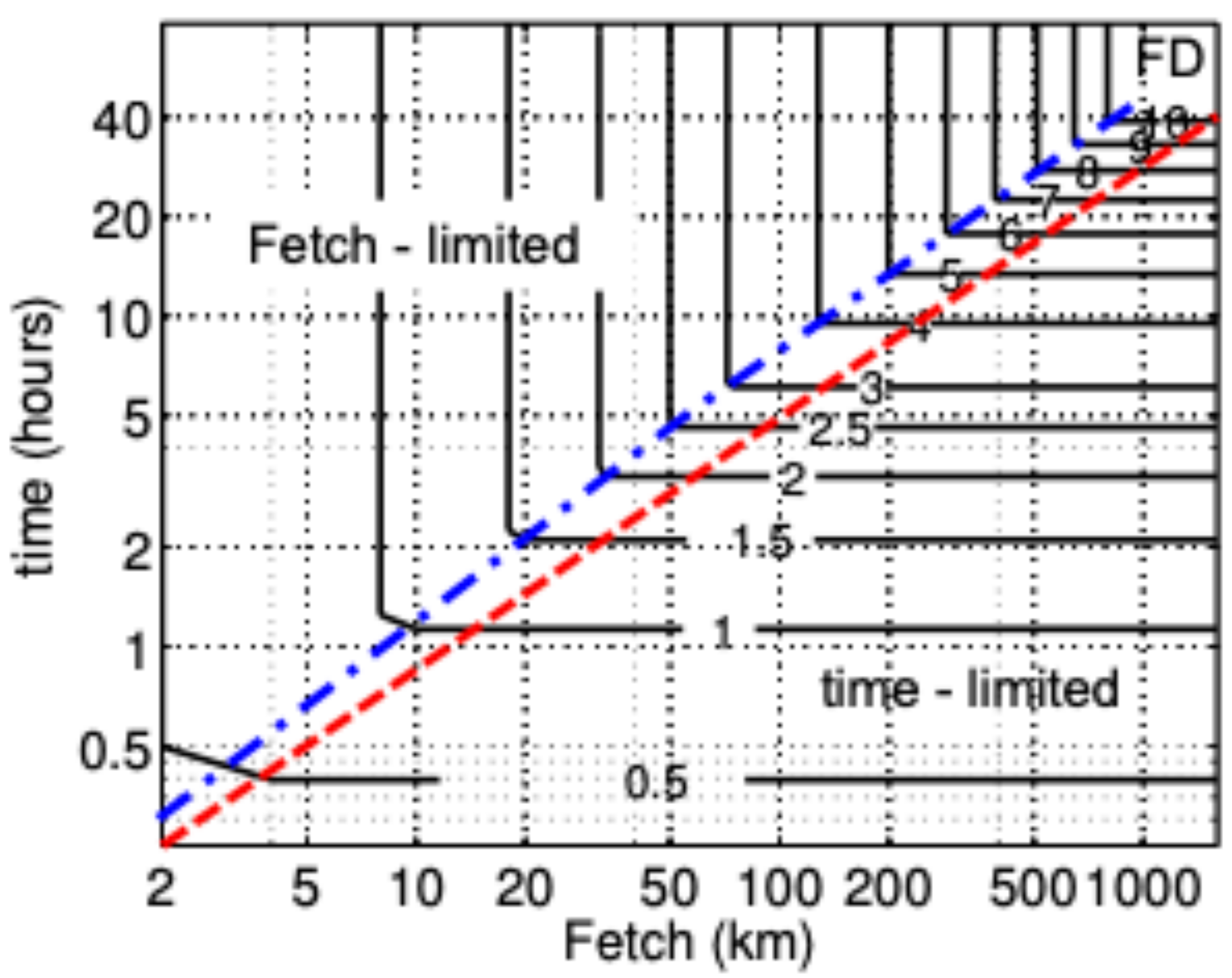
The model qualitatively reproduces the fetch relation well

- Numerical diffusion needs tuning of wave growth and dissipation
- We plan to optimize using ensemble Kalman sampling
  (*Calibrate, Emulate, Sample*, Cleary et al. 2020)

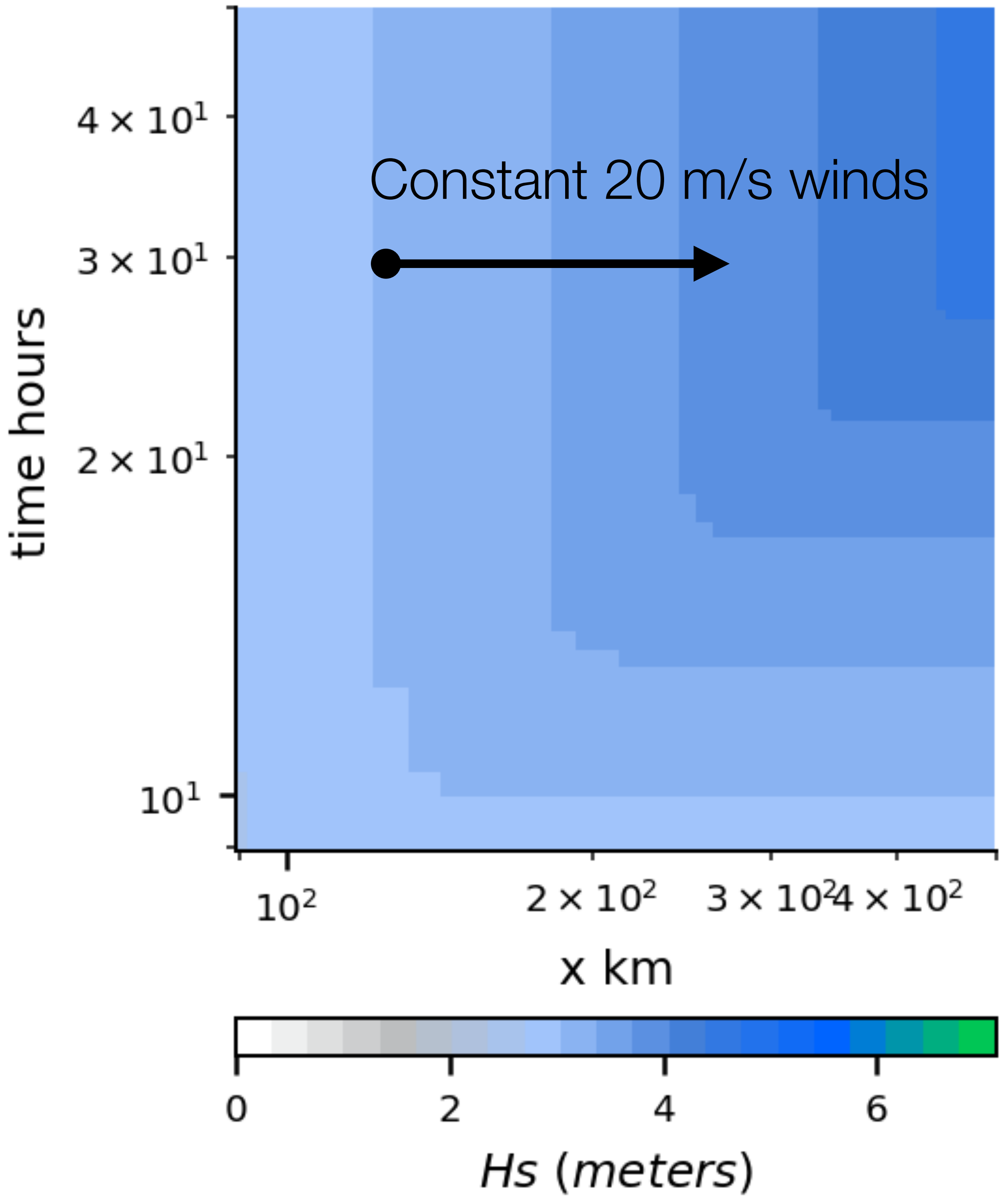
Wave Watch III test

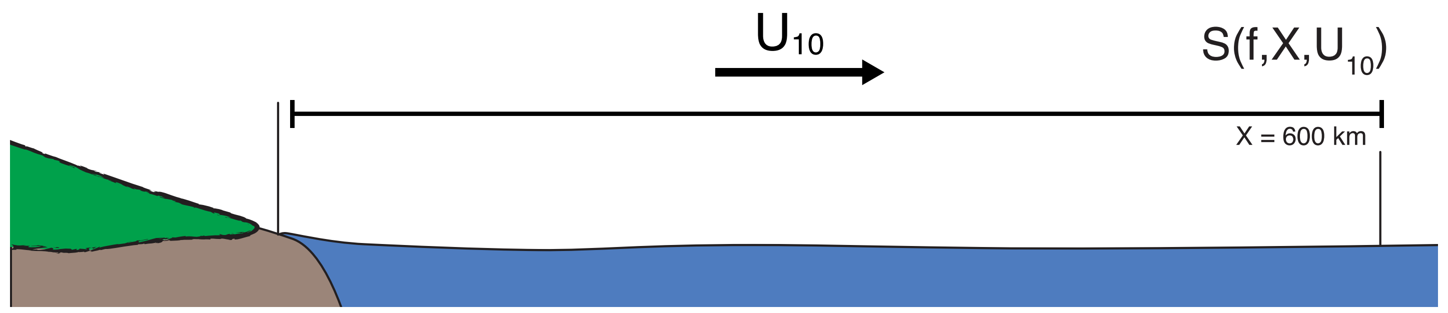


Static fetch laws (from F. Ardhuin's Book)



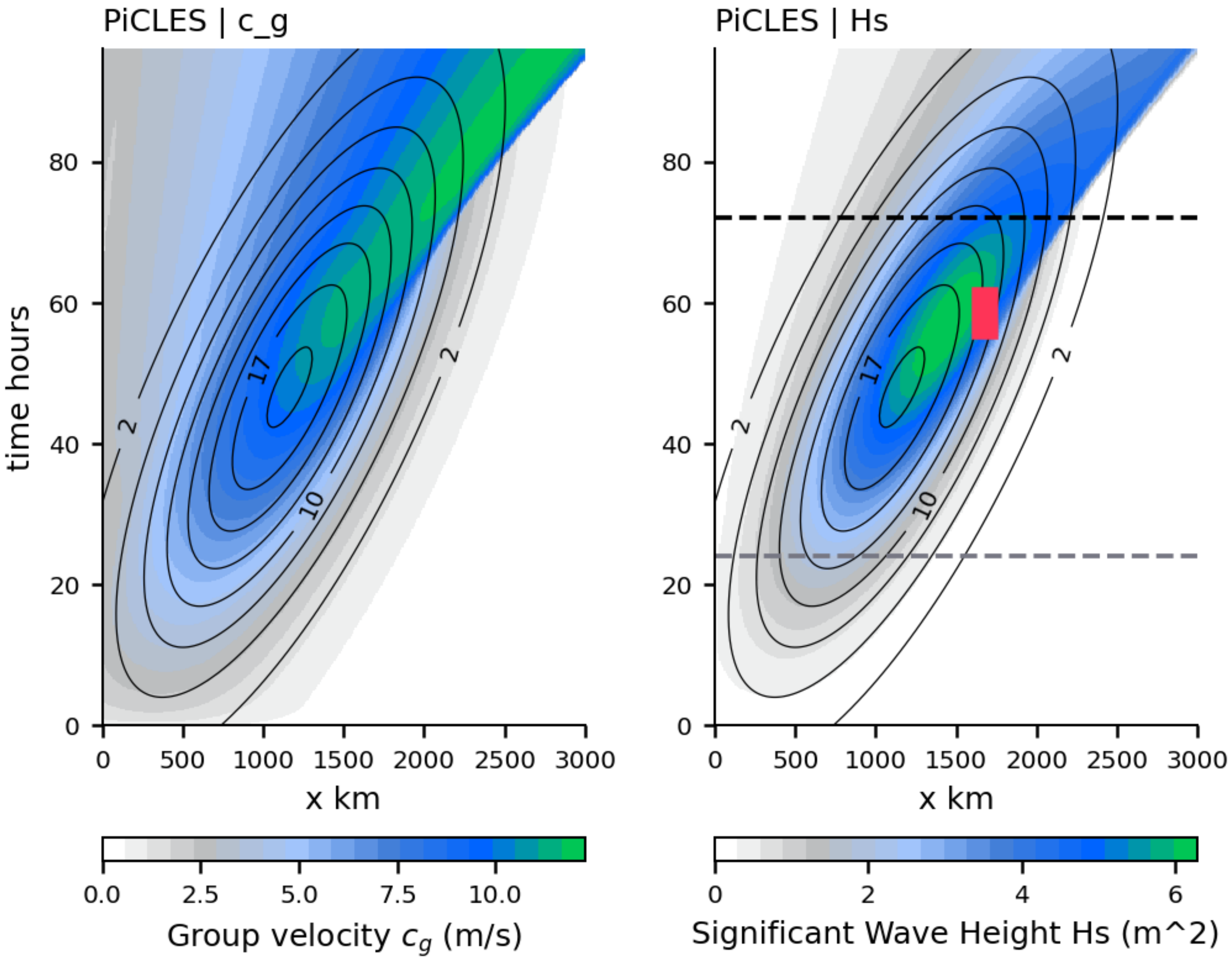
PiCLES model





Test Case II: Dynamic Fetch

Growing waves under a moving fetch



Qualitatively reproduces results from Hell et al 2021:

- highest wave speeds and energy ahead of the high-test wind speeds.
- non-local effects under wave-growth conditions
- frequency and geometric dispersion not included yet.

Test Case II: Dynamic Fetch

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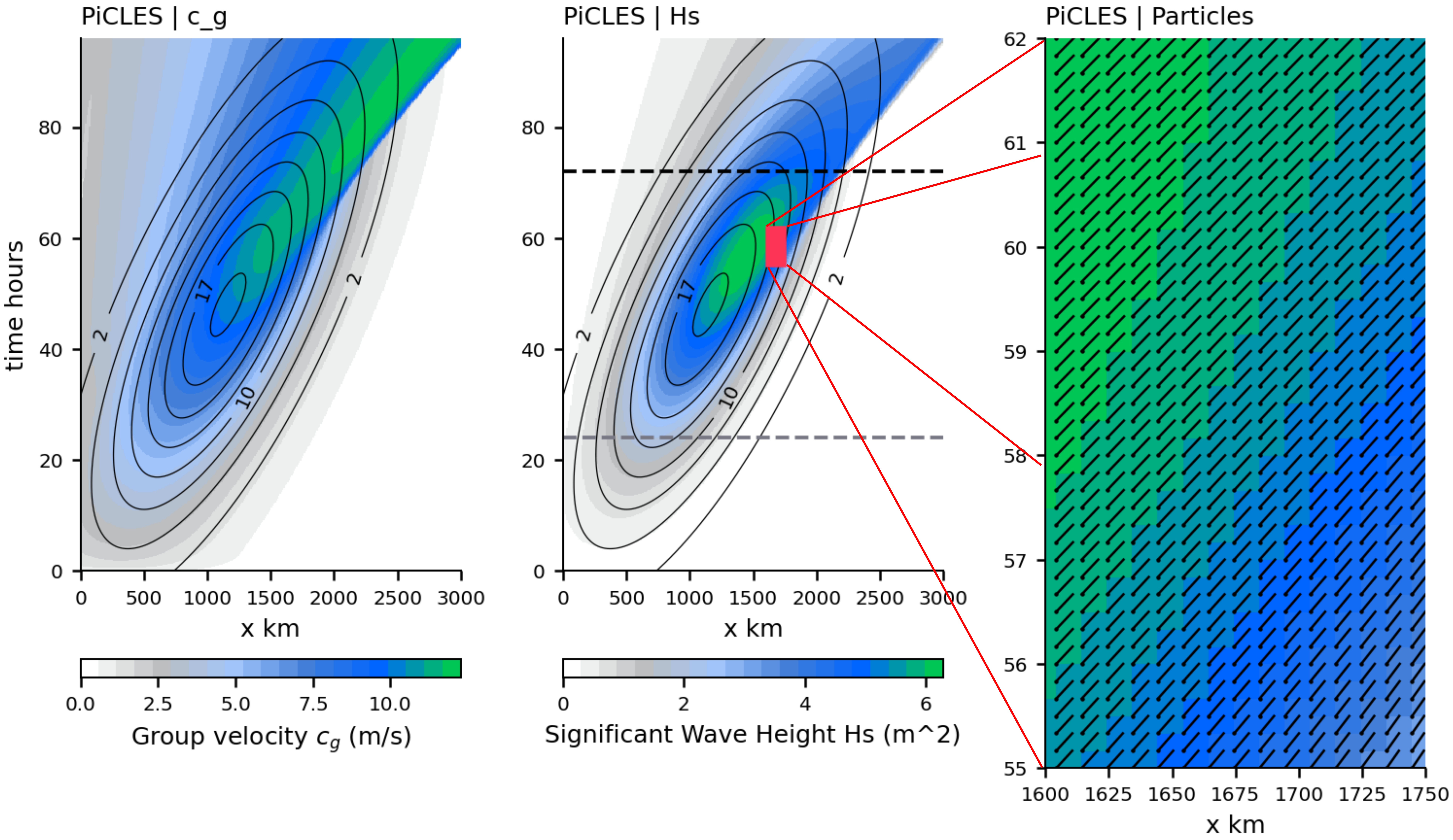

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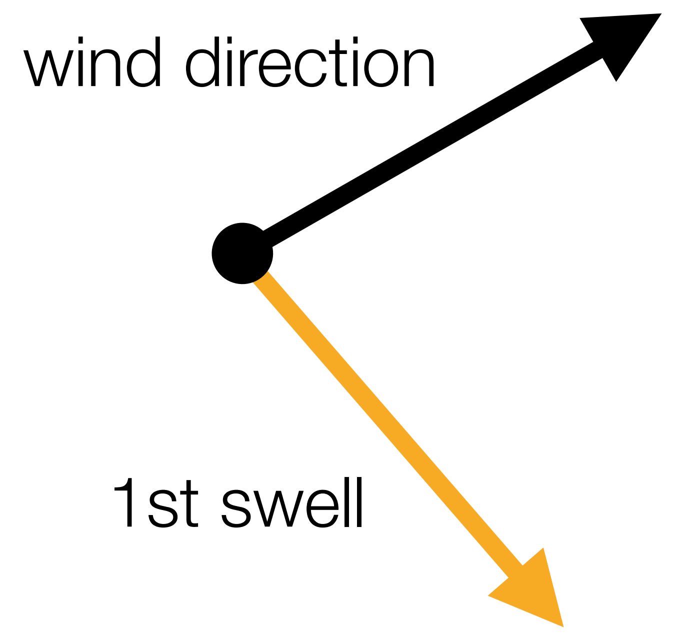
Whats next: Propagating swell

How? We take the model x 4!



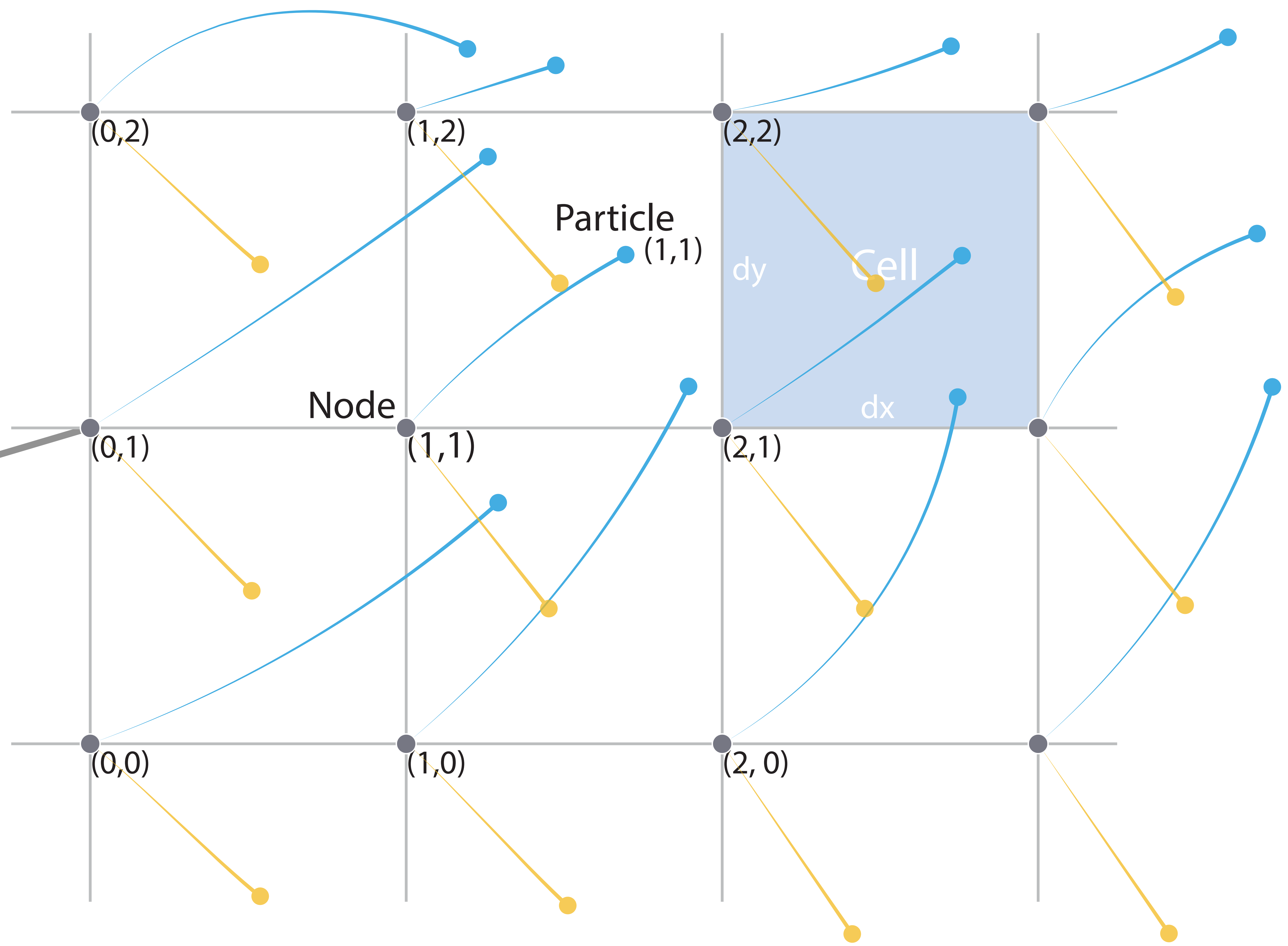
Each node has multiple particles

Wind sea: 1 x 5 energy, cg_x, cg_y, x, y

Swell I: 1 x 5 energy, cg_x, cg_y, x, y, + age or travel time

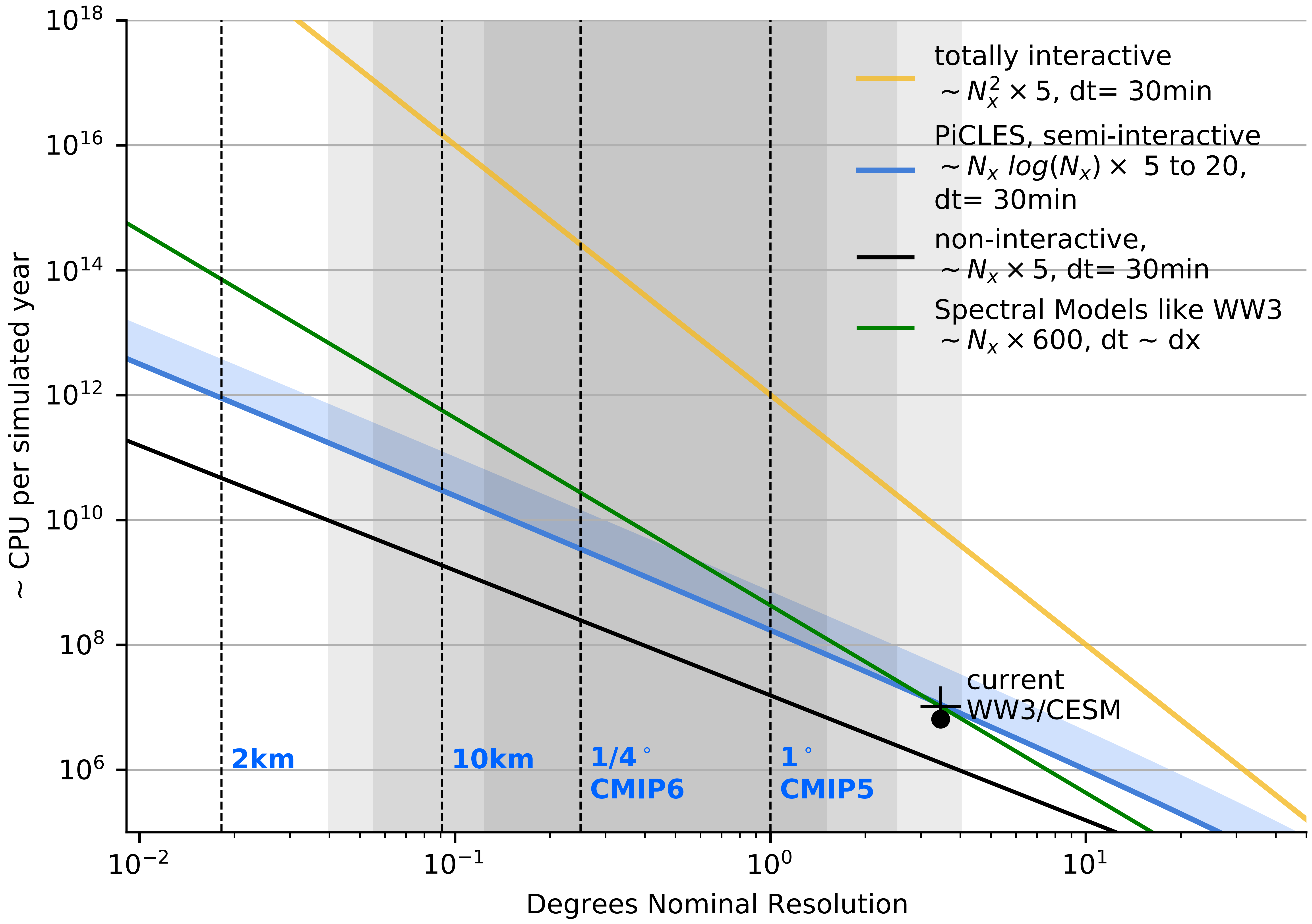
Swell II: 1 x 5 energy, cg_x, cg_y, x, y, + age or travel time

Swell III: 1 x 5 energy, cg_x, cg_y, x, y, + age or travel time



Scaling of the computational effort with future resolutions

A semi-interactive wave model is **more efficient** on resolutions of current and future Earth System Models



CPU per simulated year for a given ocean grid

- For CMIP6, a semi-Lagrangian model is about an order of magnitude faster then spectral models
- PiCLES is expected to scale with N log(N)
  - small state vector
  - designed to be highly parallel with minimal overhead
  - has no strict CFL condition for regrinding

Spectral Models in ESMs

- Special models are large (state vector ~600) and do not parallelize well within CESM
- current WaveWatch3 resolution in CESM is reduced to save computational cost

Future work

1) Implement swell propagation and dispersion
2) Test against WW3
3) Test against observations and optimize
4) Implement in ESMs

Thanks and stay swell!

contact me if you have more question

mhell@brown.edu

Sea Ice

Grid based model

Sea Ice Model component in coupled earth system model

SIC, FSD, thickness, damage, …

↑ Inform

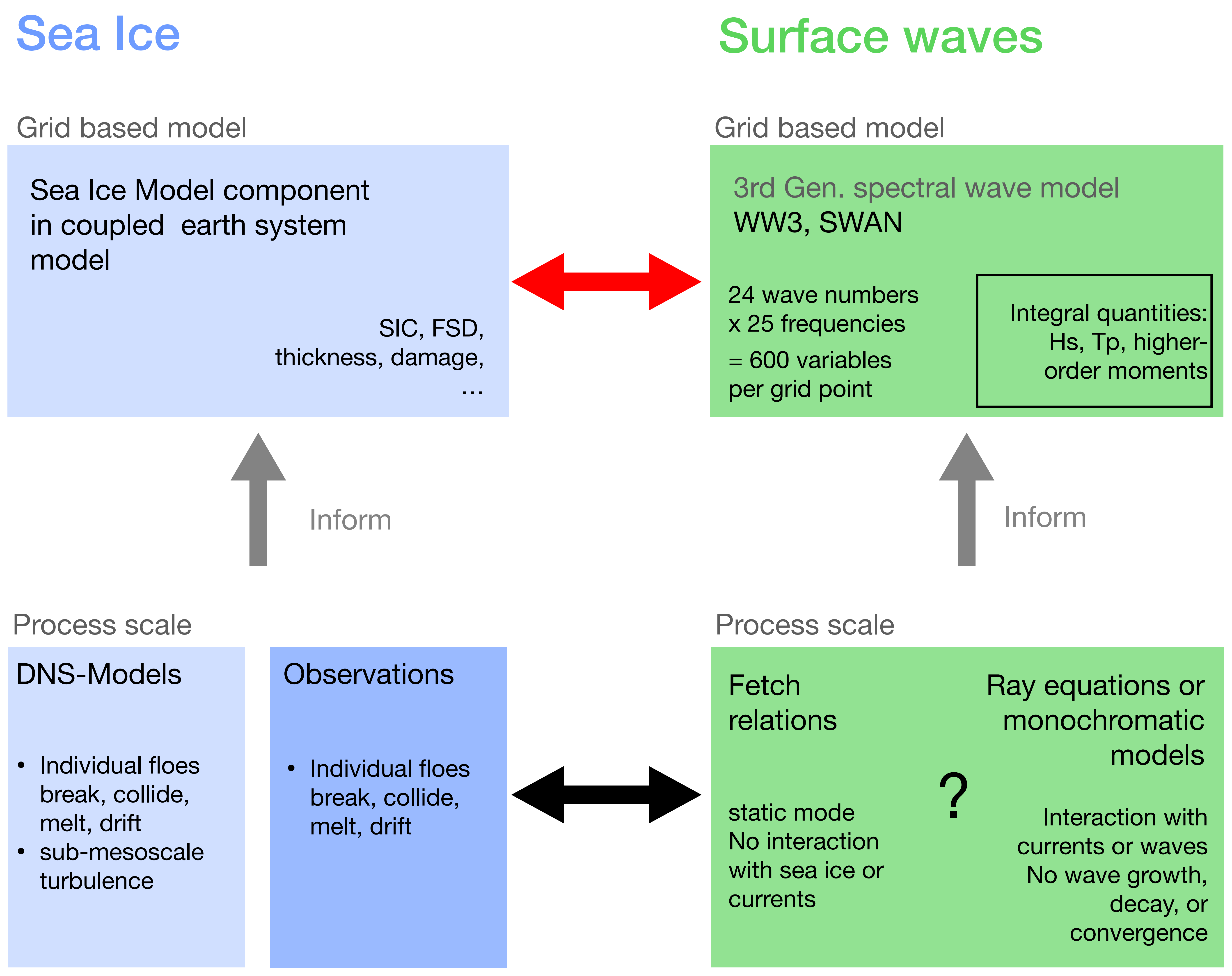
Process scale

DNS-Models

- Individual floes break, collide, melt, drift
- sub-mesoscale turbulence

Observations

- Individual floes break, collide, melt, drift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Surface waves
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Grid based model
3rd Gen. spectral wave model
WW3, SWAN
24 wave numbers x 25 frequencies = 600 variables per grid point
Integral quantities: Hs, Tp, higher-order moments
Inform
Inform
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Process scale
Fetch relations
static mode
No interaction with sea ice or currents
?
Ray equations or monochromatic models
Interaction with currents or waves
No wave growth, decay, or convergence

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3rd Gen. spectral wave model
WW3, SWAN

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= 600 variables
per grid point

Integral quantities: Hs, Tp, higher-order moments

Particle Wave Model

Provide integral quantities efficiently for fully coupled model.

Provides non-local waves for the MIZ

Inform

Process scale

Fetch relations

static mode
No interaction with sea ice or currents

?

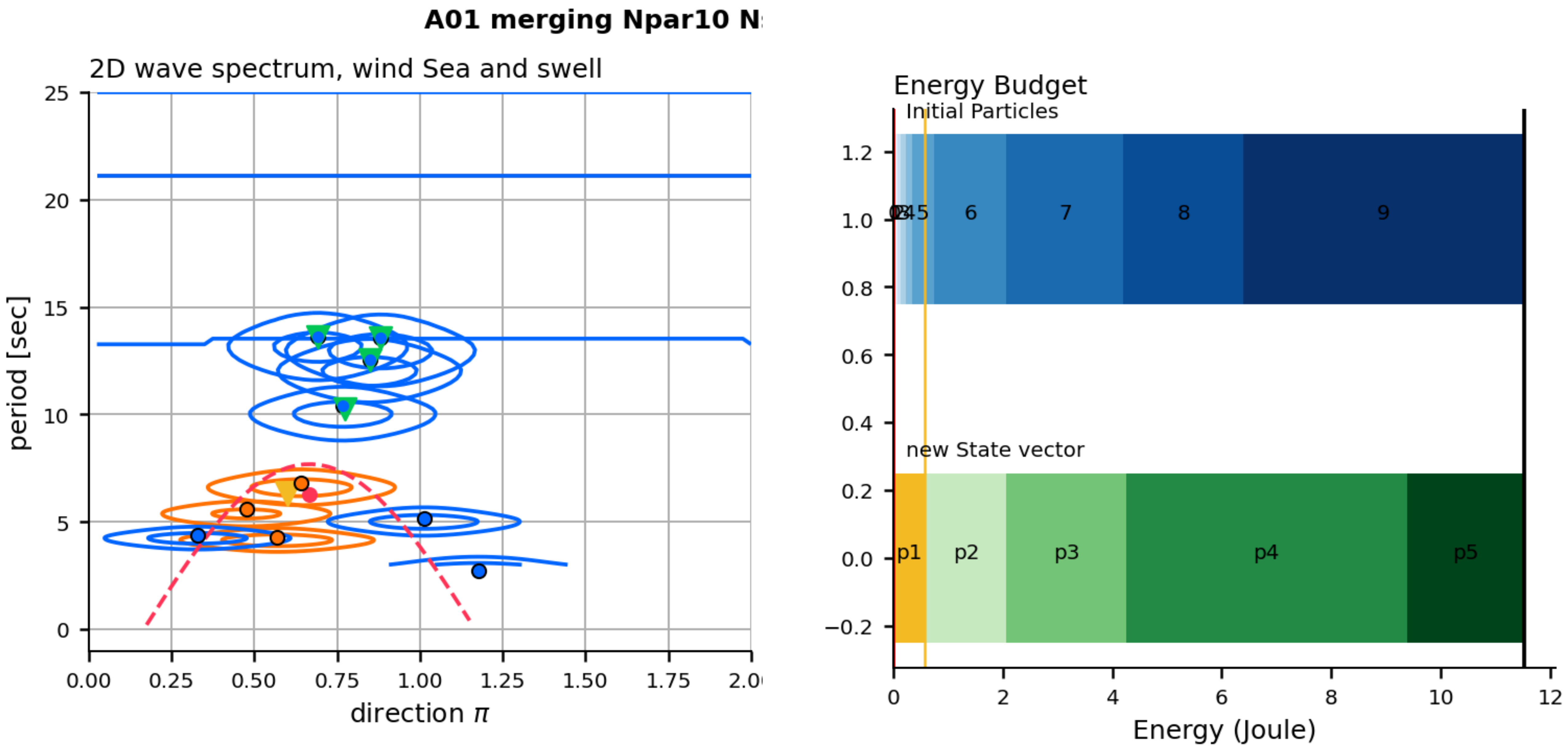
Ray equations or monochromatic models

Interaction with currents or waves
No wave growth, decay, or convergence

do both!

Particle Wave Model
With particle-particle interaction

Could be configured for DNS-like simulations (thats a hope)


A01 merging Npar10 N
2D wave spectrum, wind Sea and swell
period [sec]
direction π
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
0
5
10
15
20
25
Energy Budget
Initial Particles
new State vector
6
7
8
9
p1
p2
p3
p4
p5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
2
4
6
8
10
12
Energy (Joule)