Atmosphere Model, Chemistry Climate & Whole Atmosphere Working Group Meeting 2023

Logarithmic profile of temperature in sheared and unstably stratified atmospheric boundary layers

Yu Cheng (Columbia University; currently Harvard)

Qi Li (Cornell University)

Dan Li (Boston University)

Pierre Gentine (Columbia University)

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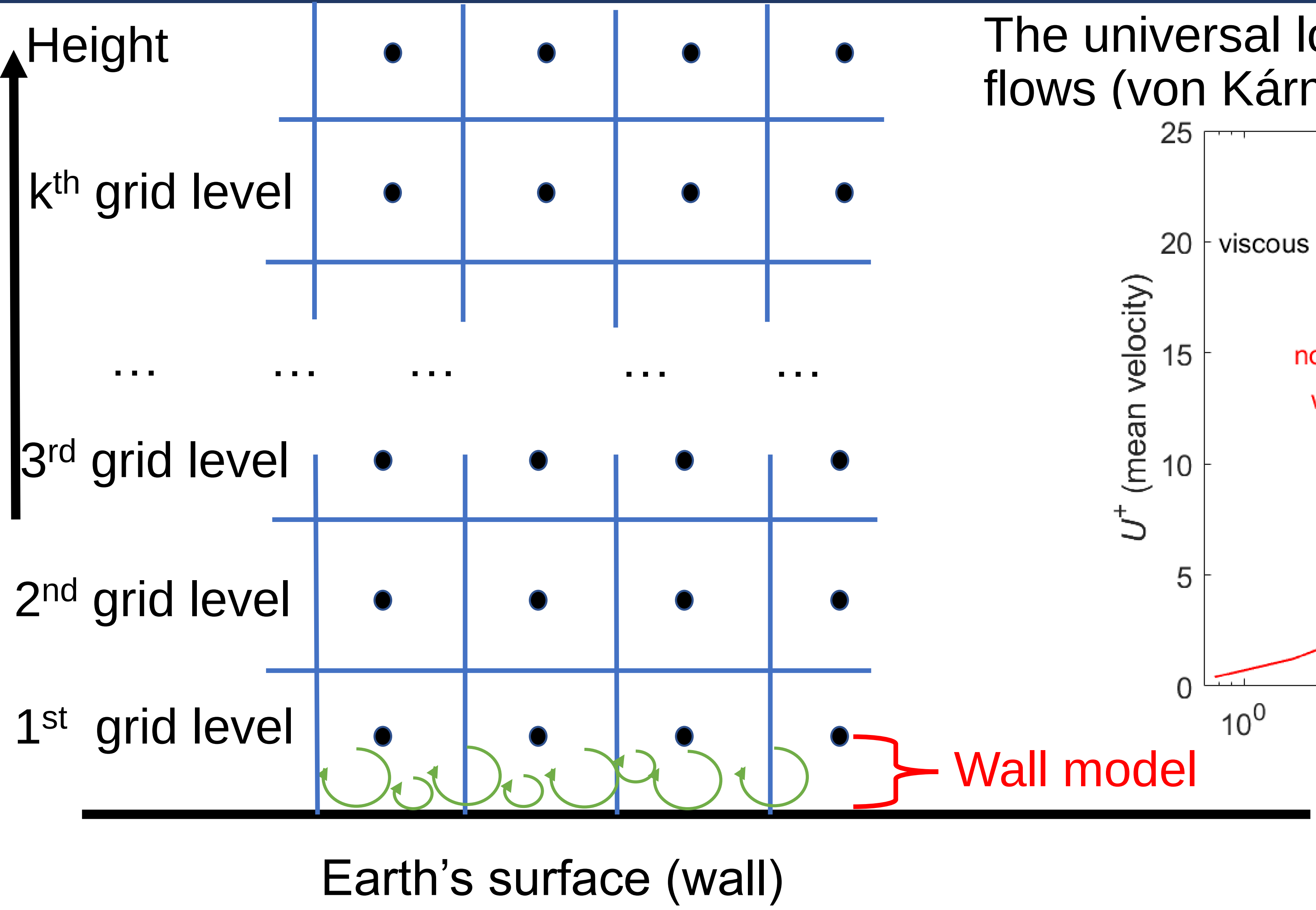
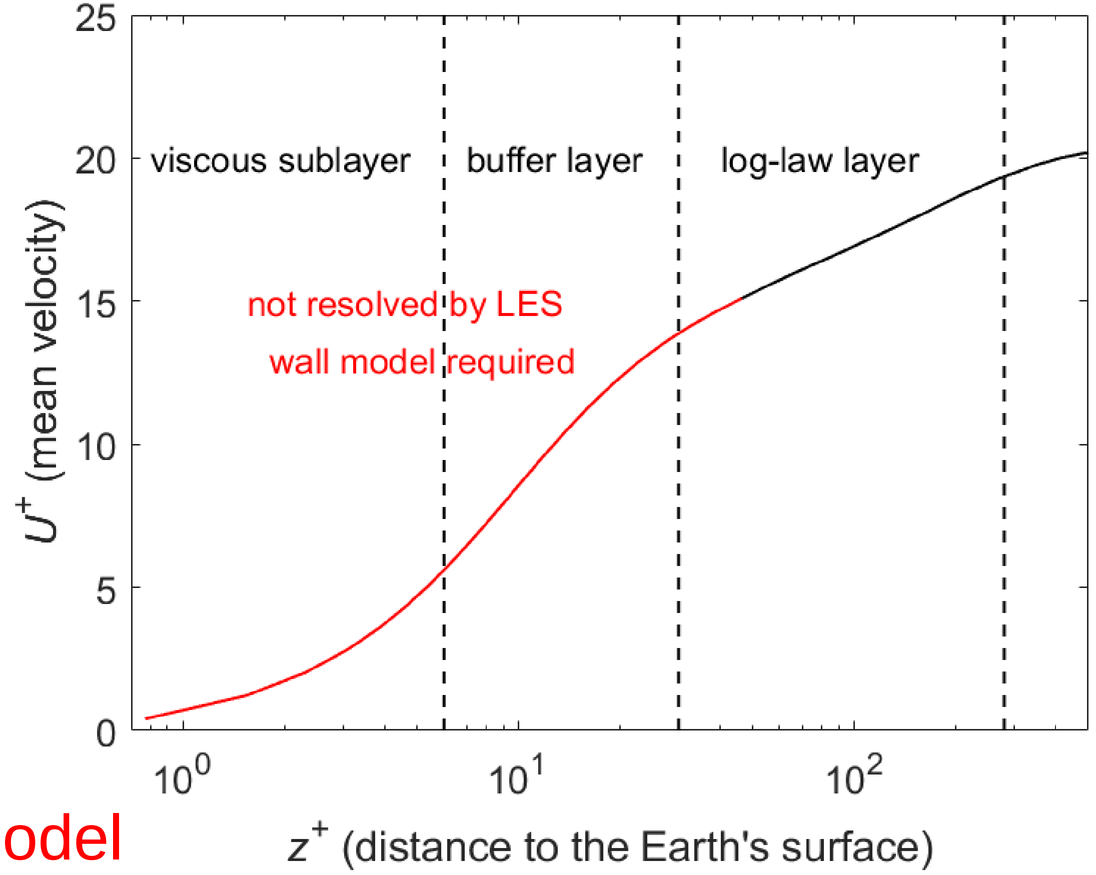
Wall parameterization: log law in the absence of buoyancy



The universal log law of the wall in turbulent flows (von Kármán 1930)



No buoyancy effects: the universal log law

Buoyancy-driven correction of log law: Monin-Obukhov similarity theory

- Wall model: Monin-Obukhov similarity theory (Monin and Obukhov, 1954; MOST):

  log law breaks down due to **buoyancy effects**

  But MOST does not always apply (Panofsky et al., 1977; Kaimal, 1978; Johansson et al., 2011).

- Temperature log profile in Rayleigh-Bénard convection (Ahlers et al. 2012)

➢ Does a temperature log law exist in the atmospheric boundary layer (ABL) driven by both shear and buoyancy?

Direct numerical simulations of convective boundary layers

DNS data	Re_τ	$\frac{z_i}{L}$	Δ_z^+
Sh40	1900	-1.7	1.13
Sh20	1243	-7.1	2.65
Sh5	554	-105.1	1.19
Sh2	309	-678.2	0.71
Microhh ReL	80	-100171.6	0.21

Neutral limit $\frac{z_i}{L} = 0$

buoyancy increases

Free convection limit $\frac{z_i}{L} = -\infty$

Direction numerical simulation (DNS): solving Navier-Stokes equations and resolving all scales

Re_τ is friction Reynolds number, z_i is boundary layer height
L Obukhov length, Δ_z^+ normalized grid size

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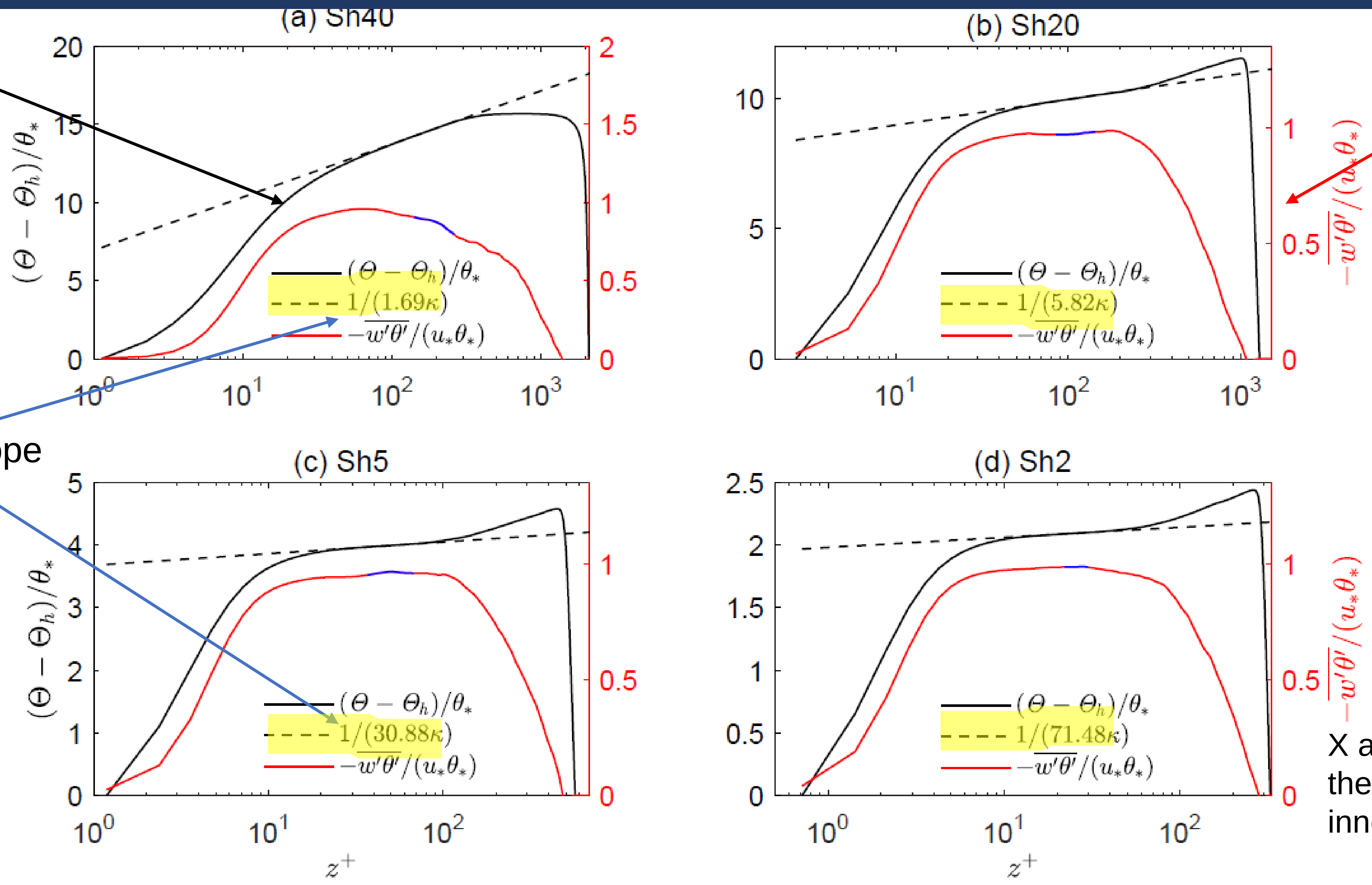
Temperature log profiles in the convective ABL

Y axis: normalized potential temperature

Y axis: normalized heat flux

Log law slope



X axis: distance to the wall denoted by inner units

- Slope of temperature log law not universal

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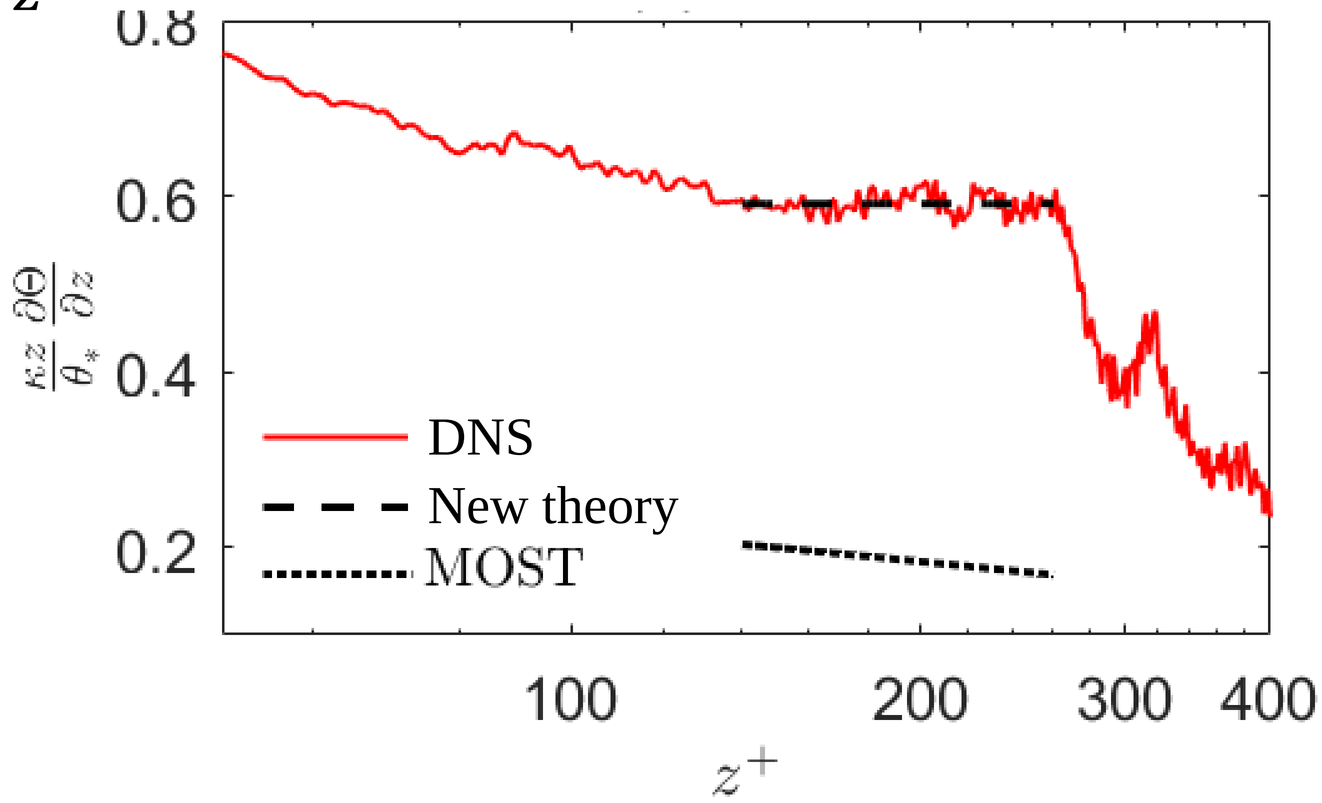
A plateau of $\frac{\kappa z}{\theta_*}\frac{\partial\Theta}{\partial z}$ indicative of a temperature log law in DNS

- DNS: a plateau of $\frac{\kappa z}{\theta_*}\frac{\partial\Theta}{\partial z}$, independent of z
- MOST: $\frac{\kappa z}{\theta_*}\frac{\partial\Theta}{\partial z} = \phi\left(\frac{z}{L}\right)$, a function of z
- Range of temperature log law: $140 < z^+ < 260$ for Sh40

(Marusic et al. 2013, neutral velocity log law range, $3Re_\tau^{1/2} < z^+ < 0.15\, Re_\tau$, $131 < z^+ < 285$)



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Field observations

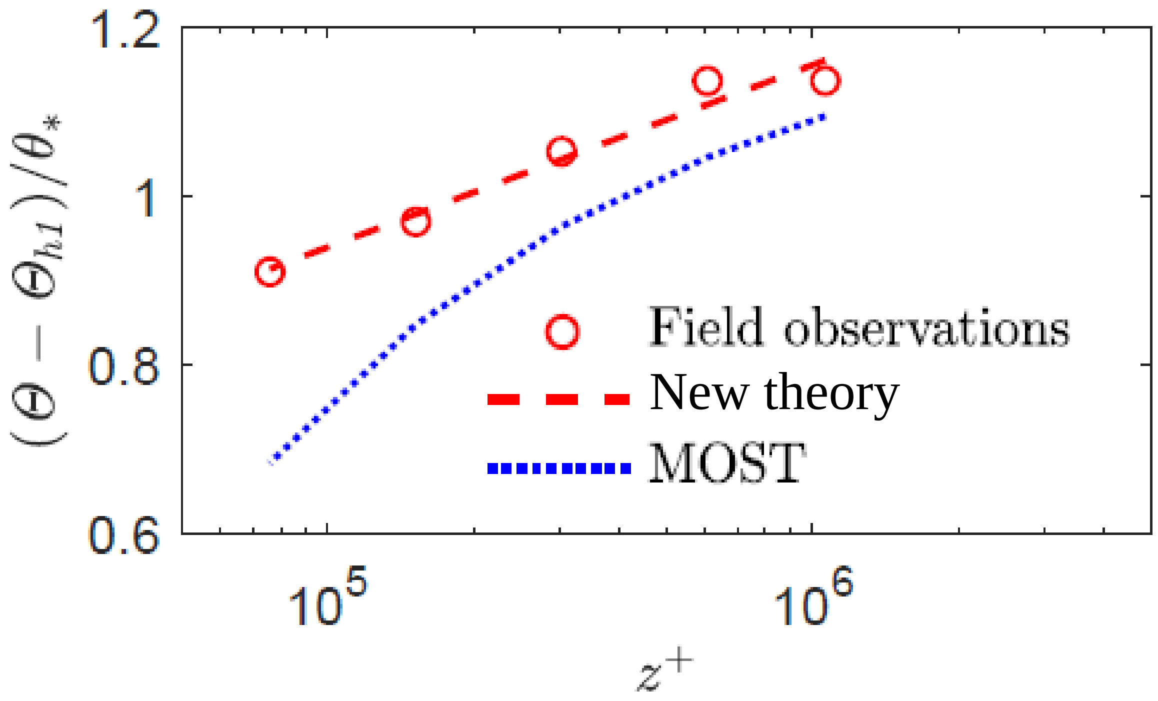
- Temperature measurements at 10 m, 20 m, 40 m, 80 m, and 140 m above the land surface

- $\frac{\Theta - \Theta_{h1}}{\theta_*}$ fits a linear relation with $\log(z^+)$

- Why log law not reported before?

(1) Sparse measurements and uncertainties

(2) MOST regarded as the fundamental theory of ABL turbulence



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Dimensional analysis

- Fully developed convective boundary layer flows: $\nu, z_i, z, \theta_*, u_*$
  forming 3 non-dimensional groups: $\frac{z}{z_i}$, $Re_\tau = \frac{z_i}{\delta_v}$, $\frac{z_i}{L}$

- In the log law region, $\frac{\partial \Theta}{\partial z}$ can be written as

$$\frac{\partial \Theta}{\partial z} = \frac{\theta_*}{z} \Phi\left(\frac{z_i}{\delta_v}, \frac{z_i}{L}\right).$$

- Integrating from a reference height z_r to z and denoting $\frac{1}{\kappa_\theta} \equiv \Phi\left(\frac{z_i}{\delta_v}, \frac{z_i}{L}\right)$,

$$\frac{\Theta - \Theta_r}{\theta_*} = \frac{1}{\kappa_\theta} \log\left(\frac{z}{z_r}\right).$$

- $\frac{1}{\kappa_\theta}$ is a function of $\frac{z_i}{\delta_v}$ (Reynolds number) and $\frac{z_i}{L}$ (buoyancy)

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Asymptotic analysis at sufficiently high Reynolds number

- Neutral limit, $\theta_* \to 0$ and $\frac{z_i}{L} \to 0$

$$\Phi\left(\frac{z_i}{\delta_v}, \frac{z_i}{L}\right) = \Phi\left(\frac{z_i}{L}\right) = \Phi(0) = c_1, \text{ for } \frac{z_i}{L} \to 0.$$

$$\kappa_\theta = \frac{1}{c_1}, \text{ for } \frac{z_i}{L} \to 0.$$

$c_1 = 1/\kappa$ if turbulent Prandtl number is 1 in the neutral limit

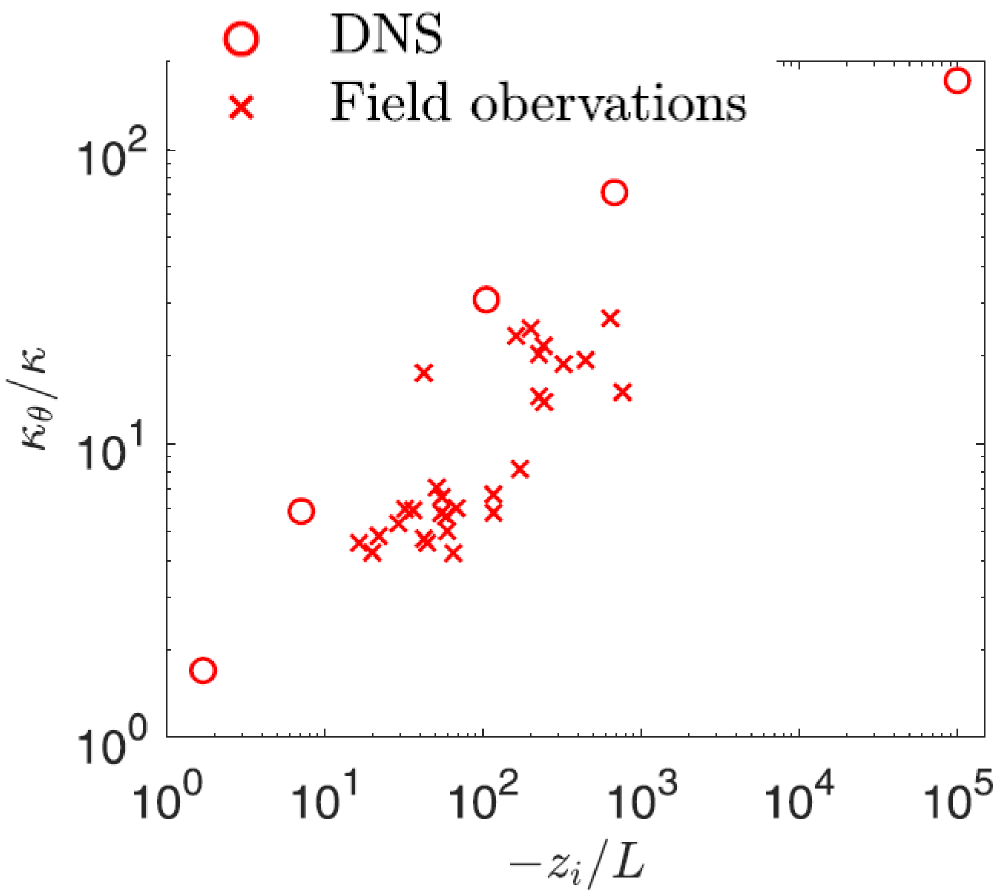
- Free convection limit, $\theta_* \to -\infty$ and $\frac{z_i}{L} \to -\infty$,

$$\frac{\partial \Theta}{\partial z} = \frac{\theta_*}{z} \Phi\left(\frac{z_i}{\delta_v}, \frac{z_i}{L}\right).$$

$$\Phi\left(\frac{z_i}{\delta_v}, \frac{z_i}{L}\right) = \Phi\left(\frac{\kappa g z_i}{\Theta_r} \frac{\theta_*}{u_*^2}\right) = \mathrm{c}_2 \left(-\frac{\kappa g z_i}{\Theta_r} \frac{\theta_*}{u_*^2}\right)^{-1}, \text{ for } \frac{z_i}{L} \to -\infty$$

$$\kappa_\theta = \frac{1}{c_2}\left(-\frac{z_i}{L}\right), \text{ for } \frac{z_i}{L} \to -\infty$$

The slope κ_θ increases as $-\frac{z_i}{L}$ increases.



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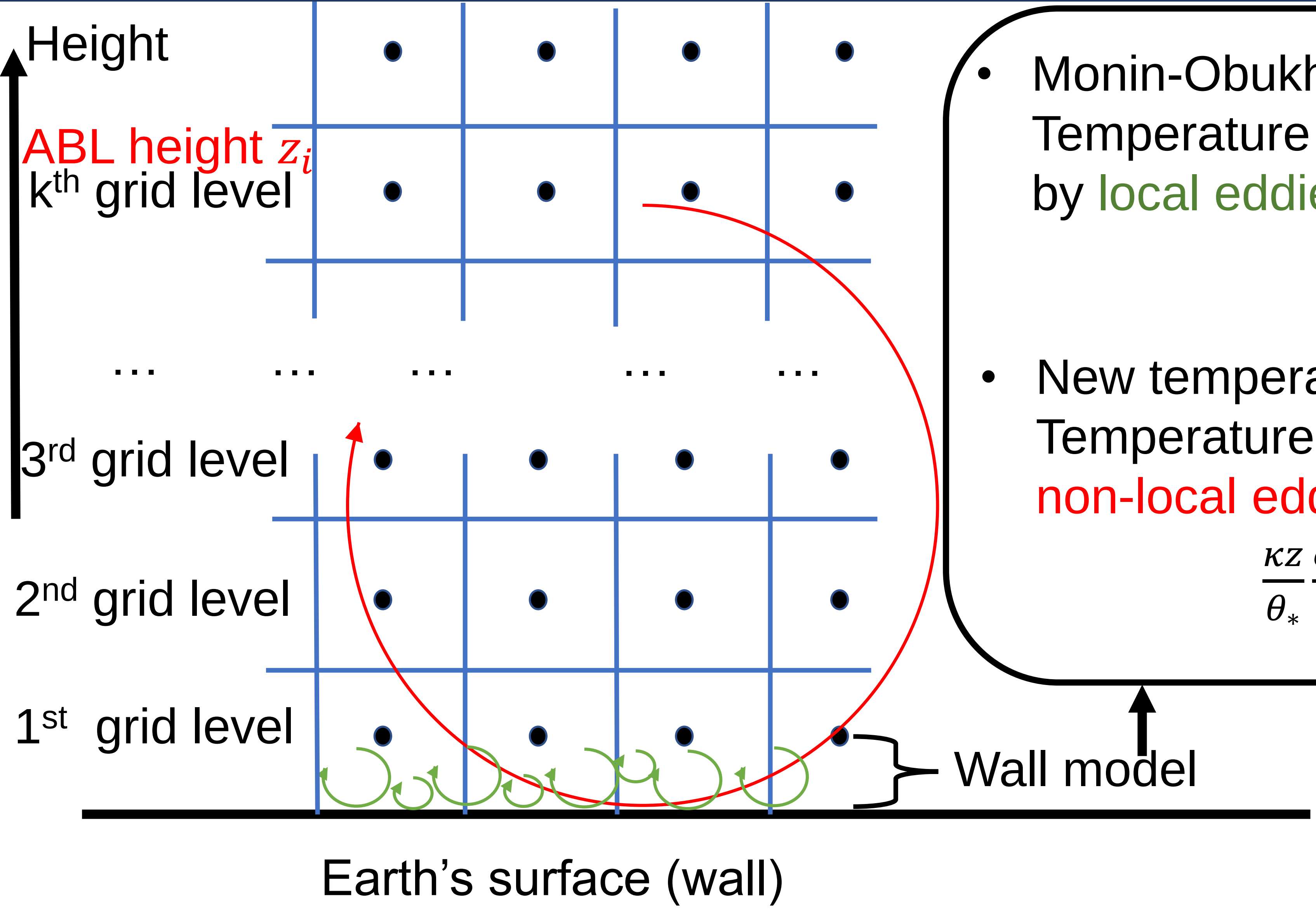
New temperature log law vs MOST



- Monin-Obukhov similarity theory (MOST): Temperature (Θ) gradient only determined by local eddies of height z

$$\frac{\kappa z}{\theta_*}\frac{\partial \Theta}{\partial z} = \phi\left(\frac{z}{L}\right).$$

- New temperature log law (Cheng et al. 2021): Temperature (Θ) gradient influenced by non-local eddies of height z_i

$$\frac{\kappa z}{\theta_*}\frac{\partial \Theta}{\partial z} = \Phi\left(\frac{z_i}{\delta_v}, \frac{z_i}{L}\right).$$

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Conclusion

- New temperature log law in convective ABL driven by both shear and buoyancy
- The slope is a function of Reynolds number and buoyancy effects
- Physically based temperature boundary conditions in climate models

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