A new measure of predictability and preliminary results from CESM large ensemble data

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Outline

- Predictability: two kinds
- Prognostic Potential Predictability PPP
- Desirable properties of predictability measures
- Our multidimensional predictability measure
- Our measure applied to a simple case
- Summary
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Predictability: first and second kinds
(Lorenz, 1975)

First Kind
Compare $P_e(t)$ to $P_e(t)$
Refers to “initial-value predictability”

Second Kind
Compare $P_c(t)$ to $P_c(0)$
Arises from predictable changes in the external forcing.
“Forced predictability”

(Figure adapted from Branstator and Teng, 2010)
**Prognostic Potential Predictability PPP**

(Pohlmann et al. 2004)

\[
PPP_x(t) = 1 - \frac{\sigma_{x,ens}^2(t)}{\sigma_{x,clim}^2(day(t))}
\]

- \( PPP_x(t) = 1 \) when \( \sigma_{x,ens}^2(t) = 0 \)
- \( PPP_x(t) \to 0 \) as \( \sigma_{x,ens}^2(t) \to \sigma_{x,clim}^2(day(t)) \)

(Figure adapted from Branstator and Teng, 2010)
Desirable properties of a multidimensional predictability measure $P$

- $P$ should be a non-dimensional scalar quantity
- $P$ should be zero when the ensemble variance equals the climatological variance
- $P$ should be one when the ensemble variance is zero
- $P$ should reduce to $PPP$ in the one-dimensional case
- $P$ should naturally divide into “predictability components” when the system is partitioned into subsystems
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- $P$ should naturally divide into “predictability components” when the system is partitioned into subsystems.
Multidimensional Predictability measure

- Adapted from Delsole and Tippett, 2009
- Based on global average temperature $T_g$
  - Could use other variables
- Partition $T_g$ into temperatures of subsystems
  - Land, Ocean
    e.g. $T_g = f_{\text{Land}}T_{\text{Land}} + f_{\text{Ocean}}T_{\text{Ocean}}$
  - Nino and non-Nino regions, Latitude bands etc.
- In general, $T_g = \vec{f} \cdot \vec{T}$
  e.g. $\vec{f} = (f_{\text{Land}}, f_{\text{Ocean}})$; $\vec{T} = (T_{\text{Land}}, T_{\text{Ocean}})$
Multidimensional Predictability measure

For global variable $T_g$, the measure is identical to PPP

$$P_{T_g} = P P P_{T_g} = \frac{\sigma^2_{T_g,\text{clim} - \sigma^2_{T_g,\text{ens}}}}{\sigma^2_{T_g,\text{clim}}}$$

Expresses $P_{T_g}$ in terms of the predictabilities of its partitioned subsystems

$$P_{T_g} = \frac{\vec{f}[C_{\text{clim}} - C_{\text{ens}}] \vec{f}^T}{\sigma^2_{T_g,\text{clim}}}$$

where $C$ are covariance matrices
Multidimensional Predictability measure

- Diagonal elements of $C$ are variances of individual subsystems.

- Off-diagonal elements are covariances between pairs of subsystems. 
  
  “Teleconnections”

- For $n$ partitions, $P_{Tg}$ naturally divides into $\frac{n(n+1)}{2}$ predictability components.

\[
\frac{n(n+1)}{2} \quad \frac{n(n-1)}{2}
\]

$n$ from individual subsystems

From teleconnections
Application to a 2D System: Daily Land, Ocean Surface Averaged T

\[ T_g = f_L T_L + f_O T_O \]

\[ f_L = 0.2875, f_O = 0.7125 \]

\[ P_{T_g} = \frac{\sigma_{T_g,clim}^2 - \sigma_{T_g,ens}^2}{\sigma_{T_g,clim}^2} = p_{Land} + p_{Ocean} + p_{Land\_Ocean} \]

- Component due to land
- Component due to ocean
- Component due to land, ocean teleconnection

• The 1850 control run is used to estimate climatological quantities
Land-Ocean Decomposition: Initial Value Predictability

\[ P_{T_g} \]

Days

Global

1
10
20
30
40
50
60
70
80
90
100

1
0.8
0.6
0.4
0.2
0
-0.2
-0.4

-0.4
Land-Ocean Decomposition: Initial Value Predictability

\[ P_{\mathcal{T}_g} \]

\[ P_{\mathcal{T}_L} \]

Global

Land
Land-Ocean Decomposition: Initial Value Predictability

Global

Ocean

Land

$P_{T_g}$

$P_{T_o}$

$P_{T_L}$
Land-Ocean Decomposition: Initial Value Predictability

Global

$P_{T_g}$

$P_{T_o}$

Days

Ocean

$P_{T_g}$

$P_{T_o}$

Days

Land

$P_{T_L}$

$P_{T_L,T_o}$

Days

Teleconnection

$P_{T_L}$

$P_{T_L,T_o}$
Land-Ocean Decomposition: Initial Value Predictability
Application to a 2D System: 1920 - 2100
Annual-running-mean Land, Ocean Surface Averaged T

\[ T_g = f_L T_L + f_0 T_0 \]

\[ f_L = 0.2875, f_0 = 0.7125 \]

\[ P_{T_g} = \frac{\sigma_{T_g,clim}^2 - \sigma_{T_g,ens}^2}{\sigma_{T_g,clim}^2} = P_{Land} + P_{Ocean} + P_{Land \_ Ocean} \]

- The 1850 control run is used to estimate climatological quantities
Application to a 2D System: Annual-running-mean Land, Ocean Surface Averaged $T$

- On longer time scales, $P_{T_g}$ represents the magnitude of the ensemble variance of $T_g$ with respect to the climatological variance.
  - Cannot be interpreted as “initial-value predictability”

\[
\begin{align*}
  P_{T_g} &> 0, \quad \sigma^2_{T_g,ens} < \sigma^2_{T_g,clim} \\
  P_{T_g} &> 0, \quad \sigma^2_{T_g,ens} > \sigma^2_{T_g,clim}
\end{align*}
\]
Land-Ocean Decomposition: 1921 - 2100

Global

\[ \sigma_{ens}^2 < \sigma_{clim}^2 \]

\[ P_{Tg} \]

\[ \sigma_{ens}^2 > \sigma_{clim}^2 \]
Land-Ocean Decomposition: 1921 - 2100

Global

Land

$\sigma_{ens}^2 < \sigma_{clim}^2$

$P_{Tg}$

$\sigma_{ens}^2 > \sigma_{clim}^2$

$P_{TL}$

Years

Land

Global

$\sigma_{ens}^2 < \sigma_{clim}^2$

$\sigma_{ens}^2 > \sigma_{clim}^2$
Land-Ocean Decomposition: 1921 - 2100

\[ \sigma_{ens}^2 < \sigma_{clim}^2 \]

\[ P_{T_g} \]

\[ \sigma_{ens}^2 > \sigma_{clim}^2 \]

\[ PT_L \]

\[ \sigma_{ens}^2 < \sigma_{clim}^2 \]

\[ PT_0 \]

\[ \sigma_{ens}^2 > \sigma_{clim}^2 \]
Land-Ocean Decomposition: 1921 - 2100

Global

Ocean

Land

Teleconnection

\[ \sigma_{ens}^2 < \sigma_{clim}^2 \]

\[ \sigma_{ens}^2 > \sigma_{clim}^2 \]

\[ P_{T_L} \]

\[ P_{T_0} \]

\[ \sigma_{ens}^2 < \sigma_{clim}^2 \]

\[ \sigma_{ens}^2 > \sigma_{clim}^2 \]
Land-Ocean Decomposition: 1921 - 2100

Global

Ocean

Land

Teleconnection

\[
\frac{\sigma_{T_g, ens}^2}{\sigma_{T_g, clim}^2}
\]

\[
\frac{\sigma_{T_O, ens}^2}{\sigma_{T_O, clim}^2}
\]

\[
\frac{\sigma_{T_L, ens}^2}{\sigma_{T_L, clim}^2}
\]

\[
\frac{\text{cov}_{ens}}{\text{cov}_{clim}}
\]
Summary

- We have defined a new measure of predictability with desirable properties
  - Unique Property: Naturally splits into predictability of subsystems

- Measure captures the role of teleconnections to predictability

- Teleconnections are important in predictability studies

- Measure gives us insight into the behavior of variances of variables on climate change time scales
Nino, Non-Nino Decomposition: 1921 - 2100

Global

\[ \frac{\sigma_{Tg,ens}^2}{\sigma_{Tg,clim}^2} \]

Non-Nino

\[ \frac{\sigma_{T_{NN,ens}}^2}{\sigma_{T_{NN,clim}}^2} \]

Years

Nino

\[ \frac{\sigma_{T_{N,ens}}^2}{\sigma_{T_{N,clim}}^2} \]

Teleconnection

\[ \frac{\text{COV}_{ens}}{\text{COV}_{clim}} \]

Years

0 0.5 1 1.5 2 2.5 3 3.5 4

0 0.5 1 1.5 2 2.5 3 3.5 4

0 0.5 1 1.5 2 2.5 3 3.5 4

0 0.5 1 1.5 2 2.5 3 3.5 4

1921 1940 1960 1980 2000 2020 2040 2060 2080 2100

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