Offline simulations of the marine P and O2 cycles using tracer transport matrices from CESM-POP2

Francois Primeau and Weiwei Fu
UCI

funded by DOE BER SciDAC award
Fast Newton Krylov Solver

- Tracers in the deep ocean take thousands of years to spin-up with respect to surface forcing.

- Computationally expensive to calibrate biogeochemical parameters with respect to hydrographic tracer observations.

- Implicit Newton-Krylov solvers can dramatically decrease the computational time needed to spin-up ocean tracers.
The model: \[ \frac{dx}{dt} = F(x, t) \]

such that \( F(x, t + T) = F(x, t) \) with \( T = 1 \) year

Look for periodic solution: \( x(t + T) = x(t) \implies G(x(t_0)) = 0 \)

\[ G(x(t_0)) \equiv \int_{t_0}^{t_0+T} F(x(t'), t') dt' \]

Use Newton method with iterative linear solver to find \( x(t_0) \).
Newton Krylov Method

Solve $G(x) = 0$ iteratively using Newton’s method.

$$x_{i+1} = x_i + \delta x$$

where $\delta x$ is the solution to $J\delta x = -G(x_i)$ with $J \equiv \frac{\partial G}{\partial x}$.

Nested iterative linear solver for $J\delta x = -G$.

Uses a coarse-grained and time averaged approximation of $J^{-1}$ to precondition the solver and accelerate convergence.
P-cycle model

\[
\begin{align*}
\left[ \frac{\partial}{\partial t} + \mathbf{T} \right] DIP &= -\Gamma DIP + DOP/\tau_d + \left( \frac{DIP^{\text{obs}} - DIP}{\tau_g} \right) \\
\left[ \frac{\partial}{\partial t} + \mathbf{T} \right] DOP &= \sigma \Gamma DIP - DOP/\tau_d + POP/\tau_p \\
\left[ \frac{\partial}{\partial t} + \mathbf{S} \right] POP &= (1 - \sigma) \Gamma DIP - POP/\tau_p
\end{align*}
\]

dissolved tracer transport operator: \( \mathbf{T}[..] \equiv \nabla [u[..] - \mathbf{K}\nabla[..]] \)

particulate tracer transport operator: \( \mathbf{S}[..] \equiv \partial_z [w_p[..]] \)

biological P-uptake operator: \( \Gamma[..] \equiv \text{diag}(\gamma)[..] \)

\[
\gamma = \alpha \frac{NPP^\beta}{[PO_4]^{\text{obs}}}
\]
Approach to Equilibrium: P-cycle

 CESM-POP2
 ~1x1 deg
 60 levels
Approach to Equilibrium: Phosphate

last 10 years of 300 year spin-up

10 year run initialized with NK solution
**O₂-cycle model**

\[
\frac{\partial}{\partial t} + \mathbf{T} \left[ \left[ O_2 \right] \right] = +r_{O_2;P} \Gamma DIP - r_{O_2;P} \Theta([O_2]) DOP / \tau_d + \mathbf{L} \left( [O_2]_{\text{sat}} - [O_2] \right)
\]

*nonlinear* operator to shut off $O_2$ consumption:

\[
\Theta([\cdot]) \equiv \frac{1}{2} \left( 1 + \tanh \left[ \frac{[\cdot] - [O_2]_c}{\lambda} \right] \right)
\]

air-sea gas exchange operator:

\[
\mathbf{L}[\cdot] \equiv \text{diag} \left( \frac{k_2}{\Delta z} \right) [\cdot]
\]

\[
k_w = \begin{cases} 
 a(1 - f_{\text{ice}}) \left[ \frac{Sc}{660} \right]^{1/2} u^2 & \text{for surface model grid boxes} \\
 0 & \text{for non-surface model grid boxes}
\end{cases}
\]
Approach to Equilibrium: $O_2$

**Graph:***

- **X-axis:** Number of iterations
- **Y-axis:** RMS drift (mmol/m$^3$/year)

- **Lines:**
  - Black: NK with preconditioner (L2)
  - Green: NK with preconditioner (L4)
  - Red: NK without preconditioner
  - Blue: Usual time-stepping

- **Convergence Threshold:**
  - The graph shows the convergence threshold and the approach to equilibrium.

**Notes:**

- 1 iteration = a 1 year model run

**Additional Information:**

- CESM-POP2
  - ~1x1 deg
  - 60 levels
Approach to Equilibrium: $\text{O}_2$

last 10 years of 300 year spin-up

10 year run initialized with NK solution
There is significant drift in surface and deep ocean nutrients after a 300 year spin-up. Newton-Krylov solver is effective for eliminating the drift.

We have working preconditioners for P-cycle model (DIP,DOP,POP), and O2-cycle, as well as untested code for N-cycle model (DIN,DON,PON).

We think we can improve the P-cycle preconditioner by reducing the amount of lumping used.

So far the offline biogeochemistry for NK solver is a simplified version of CESM biogeochemistry model. Near term goal is to couple offline tracer transport matrix with the CESM biogeochemistry (MARBL).