Spinning Up Tracers in the Ocean

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Statement of Problem

• Generate tracer distributions that are in balance with respect to (non-stationary) ocean model circulation (advection and mixing).
  – More precise statement later

• Applications:
  – Initializing transient experiments
  – Compare modeled tracers to observations
  – Optimize parameters to reduce model bias
    • Requires ability to spinup repeatedly
Target Model Configurations

• x1 grid: 320x384x60 (~4.2x10^6 grid points)
• Tracer modules
  – Abiotic radiocarbon: 2 tracers
  – Dye tracers: arbitrary # of independent tracers
  – CESM 1.2 ecosystem: 27 tracers
  – Active tracers: temperature & salinity
• Daily surface forcing w/ interannual variability
  – CORE or coupled model forcing
• Parameterized diurnal cycle for shortwave
  – Precludes taking large time steps for ecosystem
Newton-Krylov Solvers
Li & Primeau (2008), Khatiwala (2008)

• Let \( u(t) \) denote tracer state.
• Model Map: \( u(t) = \Phi(u(0), t) \)
• \( \Phi \) incorporates advection, mixing, surface fluxes, interior BGC, etc.

• Solve \( \Phi(u_0, T) = u_0 \) for \( u_0 \).
  – \( T \) is period of forcing and circulation.

• Rewrite as \( F(u) = \Phi(u, T) - u = 0 \)
• Newton’s Method:
  \[
  u_{k+1} = u_k - (\partial F / \partial u)^{-1} \ast F(u_k)
  \]
Newton-Krylov Solvers
Li & Primeau (2008), Khatiwala (2008)

• Use Krylov iterative method (GMRES) to solve:
  \[(\frac{\partial F}{\partial u})(\delta u_k) = -F(u_k)\]

• Construct Krylov basis
  \[y_0, (\frac{\partial F}{\partial u})y_0, (\frac{\partial F}{\partial u})^2y_0, (\frac{\partial F}{\partial u})^3y_0, \ldots\]

• Find linear combination of basis that minimizes
  \[|(\frac{\partial F}{\partial u}) x + F(u_k)|^2\]

• Each GMRES iteration evaluates \((\frac{\partial F}{\partial u})(\delta u)\)
• Finite Difference Approximation
  – \((\frac{\partial F}{\partial u})(\delta u) \approx (F(u+\sigma\delta u)-F(u))/\sigma\)
  – Note this is a forward model run of length T.
Newton-Krylov Solvers
Li & Primeau (2008), Khatiwala (2008)

- Preconditioner is a MUST for GMRES
  \[(P^{-1}(\partial F/\partial u))(\delta u_k)=(P^{-1})(-F(u_k)), \quad P\approx(\partial F/\partial u)\]

- We use P based on time mean advection and mixing operators, extracted from POP with impulse response function tracer module.
Results for Abiotic Radiocarbon

- Normal Year Forcing, x1 grid
- Active Ice Model
- Physics spun up for 150 years
- OCMIP2 protocols (implementation from A. Jahn)
- Spin up C and $^{14}$C wrt model year 0151
  - Natural $^{14}$C, no bomb signal
- 4 Krylov Iterations per Newton Iteration

<table>
<thead>
<tr>
<th>Newton Iteration</th>
<th>Residual RMS change</th>
<th>CO$_2$ Gas Flux (PgC/yr)</th>
<th>% ocean where drift in $\Delta^{14}$C &gt; 10$^{-3}$ %o/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.5370</td>
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<td>100% (IC for $\Delta^{14}$C = 0)</td>
</tr>
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<td>1</td>
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<td>-0.228</td>
<td>79.2%</td>
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</table>
Comparing Results to GLODAP (physics spun up for 150 years)

Model $\Delta^{14}C$ Age

GLODAP $\Delta^{14}C$ Age

14C Age, Model, 190<LON<210

14C Age, GLODAP, 190<LON<210
How long does the physics spinup need to be? N. Pac. O(1000 years)
How long does the physics spinup need to be? S. Atl. O(300 years)
Interannual Variability

• Spinup tracers with respect to an N-year segment of a run with interannual variability
• Each evaluation of $F(u) \equiv \Phi(u,T)-u$ now requires a forward model run of N years
  – More years is useful to avoid anomalous segments, but increases spinup cost
  – Q: How many years should you use? TBD
Interannual Variability

• Model physics (circulation and mixing operators, temperature, salinity) are no longer cyclostationary

• Results using 5 year segment of CORE hindcast indicate that this is not a problem, solver still finds cyclostationary tracer solution

<table>
<thead>
<tr>
<th>Newton Iteration</th>
<th>Residual RMS change</th>
<th>CO₂ Gas+Virtual Flux (PgC/yr)</th>
<th>% ocean where drift in Δ¹⁴C &gt; 10⁻³ ‰/yr</th>
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</thead>
<tbody>
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</tbody>
</table>
Summary & Ongoing Work

• NK spinup for $^{14}$C and dye tracers (not shown) works on x1 grid
  – Useful diagnostic of deep ocean ventilation
  – Time need for physics spinup varies regionally, can be very long
• Technique works with interannual variability
  – Work is underway to study how spunup solution depends on segment of forcing
• Application to CESM 1.2 ecosystem is ongoing
  – Technique that worked in CCSM 3.5 not working
  – Alternatives are being explored
• Q: Can technique be extended to T & S?