Quantifying the Uncertainty in Ice Sheet Model Parameters via Model Calibration

Calibration of the Community Ice Sheet Model

M. T. Pratola\textsuperscript{1}, S. Price\textsuperscript{2}, J. Gattiker\textsuperscript{1} and D. Higdon\textsuperscript{1}

\textsuperscript{1}Statistical Sciences Group, Los Alamos National Laboratory

\textsuperscript{2}COSIM, Fluid Dynamics and Solid Mechanics Group, Los Alamos National Laboratory

June 22, 2011
Outline

- Introduction: Community Ice Sheet Model (CISM)
- Some idealized experiments using CISM
- A Statistical Framework for Combining CISM with field data
- Early results, future directions
Historically, ice sheets thought to respond slowly to short-term climate change.

However, recent observations indicate significant ice sheet volume changes as a result of decadal-scale climate forcing.

Potential changes in discharge from Greenland and/or Antarctic ice sheets are the largest unknown w.r.t. future sea-level rise.

CISM describes ice sheet evolution (velocities, thickness, temperature, etc.) assuming appropriate boundary and initial conditions and atmospheric and oceanic forcing (e.g., from CESM).
Goals

- Our goal is to leverage a statistical model calibration framework to better understand and quantify uncertainties in ice-sheet evolution as simulated by CISM.
- Here, we investigate idealized scenarios with uncertainties in:
  - Flow law exponent, $n$
  - Flow law rate factor activation energy, $Q$
  - Constant (in $x,t$) ice-shelf basal melt rate, $m$
Experimental Setup

- Modified version of the standard “confined shelf” test case:
  - isothermal, rectangular shelf of uniform thickness
  - confined at upstream and lateral margins (zero flux bc)
  - open to the ocean at downstream margin (specified stress bc)

- Constant and steady surface mass balance applied for experiments where $n$ and $Q$ vary

- Constant $n$, $Q$, and surface accumulation for experiments where $m$ varied

- All experiments evolve to approximate SS from $t=0$ to $t=1000$ yrs
CISM Confined Shelf Example
Represent CISM as $\eta(\theta)$ where $\theta$ is some parameter vector of interest

Ensemble of CISM runs \( \{ Y_i^c = \eta(\theta_i) \} \) at different $\theta_i$'s to get ensemble of outputs

Choose a “true” value $\theta_0$ to simulate a field observation. The observation, $Y^f$, is constructed as $\eta(\theta_0) +$ error.

- Experiment 1: $\theta = n$, select 7 settings for $n \in [1.5, 4.0]$ for model runs. $n_0 = ?$
- Experiment 2: $\theta = (n, Q)$, select 8 settings for $n, Q \in ([1.5, 4.0], [4e4, 8e4])$. $(n_0, Q_0) = ?$
- Experiment 3: $\theta = m$, select 7 settings for $m \in [1.0, 5.0]$. $m_0 = ?$
A Statistical Framework for Uncertainty Quantification

- Statistical computer model calibration experiments - e.g. Kennedy & O’Hagan (2001), Higdon et al. (2004), amongst others.

- Useful in situations where
  - model costly to run
  - combine field observations and model output
  - quantify uncertainty in parameters and model predictions
Statistical Calibration Model

Model: \( y^f(x) = \eta(x, \theta_0) + \epsilon(x) \); \( y^c_i(x, \theta_i) = \eta(x, \theta_i) \)

- We have computer model outputs
  \[ Y^c = (Y^c_1^T, \ldots, Y^c_N^T)^T \]

  and field observations
  \[ Y^f = (y^f(x_1), \ldots, y^f(x_M))^T. \]

- Then a joint model for all the data is:
  \[
  \begin{pmatrix}
  Y^f \\
  Y^c
  \end{pmatrix}
  \sim
  \mathcal{N}
  \left(
  \begin{pmatrix}
  \mu \\
  \mu
  \end{pmatrix},
  \sigma^2
  \begin{bmatrix}
  R^f & R^{fc} \\
  R^{fc} & R^c
  \end{bmatrix}
  +
  \begin{bmatrix}
  \sigma^2 \epsilon I_n & 0 \\
  0 & 0
  \end{bmatrix}
  \right),
  \]

  \[ \text{corr}(y^c_k(x_i, \theta_k), y^f(x_j, \theta_0)) = e^{-\sum_{p=1}^{P} \gamma_p (x_{ip} - x_{jp})^2 - \sum_{q=1}^{Q} \phi_q (\theta_{kq} - \theta_{0q})^2} \]
Posterior of parameters is
$$\theta_0, \mu, \sigma^2, \sigma^2_\epsilon, \gamma, \phi | \mathbf{Y}^f, \mathbf{Y}^c \propto L(\mathbf{Y}^f, \mathbf{Y}^c | \cdot) \pi(\theta_0, \mu, \sigma^2, \sigma^2_\epsilon, \gamma, \phi)$$
which we can sample using MCMC to get our parameter estimates.

Can also sample the posterior predictive distribution,
$$y^c(x, \theta_0) | \mathbf{Y}^f, \mathbf{Y}^c$$ for making predictive inference.

In particular,
$$E[y^c(x, \theta_0) | \mathbf{Y}^f, \mathbf{Y}^c, \cdot] = \mathbf{w}^T \left( \begin{pmatrix} \mathbf{Y}^f \\ \mathbf{Y}^c \end{pmatrix} - \mu \right)$$

where $$\mathbf{w}^T = \mathbf{c}^T \Sigma^{-1}$$, with $$\Sigma$$ as before, and
$$\mathbf{c}^T = \left[ \text{cov} \left( y^c(x, \theta_0), y^f(x_1, \theta_0) \right), \ldots \right]$$.
The Idea (in pictures)
The Idea (in pictures)
The Idea (in pictures)
The Idea (in pictures)
The Idea (in pictures)
The Idea (in pictures)
The Idea (in pictures)
Results: Experiment 1 ($n_0 = 3$)
Results: Experiment 2 \((n_0 = 3, Q_0 = 60 \times 10^3)\)
Results: Experiment 3 \((m_0 = 2.72)\)
Calibrated Prediction (eg: melt experiment)
Calibrated Prediction (eg: melt experiment)
Calibrated Prediction (eg: melt experiment)
Calibrated Prediction (eg: melt experiment)
Conclusions & Future Directions

- Outlined a typical approach to statistical uncertainty quantification in model calibration
- Method gave reasonable results, but improvements needed to analyze simulated examples that are closer to the real-world problem
- Scientific goal is to perform uncertainty quantification for an ice-shelf where, for example, $n$, $Q$ and $m$ (or other parameters of interest) are unknown (we’re not there yet).