MOM6 diagnostic consistency on non-native grids (work in progress)

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January 25, 2022
Motivating Application:
Diagnostics for Preconditioner in Newton-Krylov
Based Solver for Tracer Spin-up

- NK spin-up operates on tracers on a fixed \( z^* \) grid
  - to ease computation/interpretation of difference
    \( \text{tracer}^{\text{end}} - \text{tracer}^{\text{beg}} \)

- Krylov preconditioner based on terms of the form
  \( \partial (\text{tend}) / \partial (\text{tracer}) \) for different tracer tendency terms
  - \( \text{tend} \) and \( \text{tracer} \) on \( z^* \) grid

- Terms computed by computing tendency terms from impulses on \( z^* \) grid, generating tendency diagnostics on \( z^* \) grid

- Remapped diagnostics didn’t work quite as expected…
Features of thickness budget desired to hold on non-native grids

1. \( dhdt = \text{boundary}_\text{forcing}_h\_\text{tendency} + \text{vert}_\text{remap}_h\_\text{tendency} + \text{dynamics}_h\_\text{tendency} \)

2. \( \int_{t_0}^{t_1} dhdt \, dt = h(t_1) - h(t_0) \)
Reconstruct $dhdt$ from terms
native grid, daily mean, column in N Pac

MOM6 diagnostic consistency on non-native grids
Reconstruct dhdt from terms rho2 grid, daily mean, column in N Pac
Reconstruct dhdt from terms $z^*$ grid, daily mean, column in N Pac
Reconstruct mean $\frac{dh}{dt}$ from snapshots $z^*$ grid, daily mean, column in N Pac
Remapping Tendency Terms in Mathematical Terms

Consider the tendency of some field $F$:

$$\frac{\partial F^n}{\partial t} = \frac{F^{n+1} - F^n}{\Delta t}$$

Current approach in MOM6 to remapping this:

$$L \left( \frac{\partial F^n}{\partial t} \right) := L \left( \frac{F^{n+1} - F^n}{\Delta t} \right)$$

$L$ remaps from native grid to diagnostic grid, depending on layer thicknesses on src and dst grids.

Difficulty arises from choosing thickness time-levels.
Remapping Tendency Terms in Mathematical Terms

Generalization:
\[
L\left( \frac{\partial F^n}{\partial t} \right) := \frac{L^{n,n+1}(F^{n+1}) - L^{n,n}(F^n)}{\Delta t}
\]

RHS of adding two successive time steps:
\[
\frac{L^{n+1,n+2}(F^{n+2}) - L^{n+1,n+1}(F^{n+1})}{\Delta t} + \frac{L^{n,n+1}(F^{n+1}) - L^{n,n}(F^n)}{\Delta t}
\]

For eqn 2 to hold, middle terms need to cancel:
\[
L^{n+1,n+1}(F^{n+1}) = L^{n,n+1}(F^{n+1})
\]
Remapping Tendency Terms in Mathematical Terms

Remapping operator needs to be independent of time-level of tendency, and instead depend on time-level of $F$ being remapped.

One approach to satisfy this constraint is for $L^n$ to use thicknesses just prior to process related to $F$ is applied, and $L^{n+1}$ use thicknesses just after process is applied.

\[
L \left( \frac{\partial F^n}{\partial t} \right) := \frac{L^{n+1}(F^{n+1}) - L^n(F^n)}{\Delta t}
\]
Reconstruct dhdt from terms rho2 grid, daily mean, column in N Pac
Reconstruct dhdt from terms z* grid, daily mean, column in N Pac
Reconstruct mean $dhdt$ from snapshots $z^*$ grid, daily mean, column in N Pac

![Diagram showing depth at cell center vs. $|dhdt|$ and $|dhdt$ Recon Err|](image-url)
Reconstruct mean opotemptend from snapshots native grid, daily mean, column in N Pac
Reconstruct mean \( \text{tripletrend} \) from snapshots \( z^* \) grid, daily mean, column in N Pac
Summary and Continuing Work

• Generalizing definition of remapped tendency terms enables reconstruction of thickness tendency by terms and finite differences from snapshots to hold on diagnostic grids.

• Reconstruction of tracer layer content tendency from snapshots works on diagnostic grids. Implementation of remapping tracer tendency terms is in progress.

• Can lateral fluxes be remapped to be consistent with divergence operator on diagnostic grid? not sure