Towards a potential-vorticity based mesoscale closure scheme

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The ocean interior is stratified and quasi-adiabatic, so much so that we infer the global ocean circulation from tracer distribution along isopycnals.

Figures taken from the World Ocean Circulation Experiment (WOCE)
It is essential to get the isopycnal placement correctly in global ocean simulations for estimating global heat and tracer transport. The thickness-weighted averaged (TWA) framework provides us with a path forward.
The Gent-McWilliams and Redi diffusivity

- The Gent-McWilliams (GM) skew diffusivity diffuses the isopycnal thickness in a similar manner to how baroclinic instability would if resolved.

- The Redi diffusivity represents the enhanced tracer stirring along isopycnals due to eddies.

- GM and Redi should be related to one another.

- Can we capture the full eddy feedback and not just the release of available potential energy?
Employing a coarse-graining method, Aluie et al. (2018) examined the direction of kinetic energy cascade from a model simulation.

Blue shadings indicate the eddies fluxing kinetic energy back into the mean flow.

A general conclusion we can deduce from Figs. 8–10 is that an upscale energy transfer does not take place everywhere in the ocean, even at the higher latitudes. On the other hand, if we average over large enough regions (of order $10^3$ km in size or larger) in the ocean, away from the equator, we find from Figs. 4–7 that the energy fluxes transfer in the Gulf Stream core east of Florida and the Carolinas. This persists well beyond the separation point (Cape Hatteras), indicating that energy is transferred from mesoscale eddies into the Gulf Stream, accelerating and focusing the current. Flanking both sides of this (dark blue) core, we see downscale transfer (red) most probably associated with barotropic instabilities resulting from strong shear. Overall, an upscale transfer dominates in the Gulf Stream, in accordance with QG. A similar pattern, though not as pronounced, exists in the North Brazil Current. The (shallow) North Equatorial Current, which in our simulation is around $5^\circ$N, exhibits an upscale energy transfer.

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Mesoscale eddies energize the Gulf Stream.
Can we say more on how and where?
\[
\begin{align*}
\hat{u}_{t} + \hat{u}\hat{u}_{x} + \hat{v}\hat{u}_{y} + \hat{\omega}\hat{u}_{b} - f\hat{v} + \bar{m}_{x} &= -\vec{e}_1 \cdot (\vec{\nabla} \cdot \vec{E}) + \hat{\chi} \\
\hat{v}_{t} + \hat{u}\hat{v}_{x} + \hat{v}\hat{v}_{y} + \hat{\omega}\hat{v}_{b} + f\hat{u} + \bar{m}_{y} &= -\vec{e}_2 \cdot (\vec{\nabla} \cdot \vec{E}) + \hat{\gamma}
\end{align*}
\]

- \(\hat{u} \left( = \frac{\overline{\sigma u}}{\sigma} \right)\): the thickness-weighted averaged (TWA) velocity.

- \((\cdot)\): the ensemble mean.

- \(\sigma \left( = \zeta_{b} \right)\): the isopycnal thickness.

- \(\omega\): the diapycnal velocity.

- \(m \left( = \phi - b\zeta \right)\): the Montgomery potential.

Young (2012); Ringler et al. (2017)
A 24-member ensemble simulation

- No. of ensemble members: 24.

- Resolution: 1/12°; Duration: 50 years (1963-2012).

- Model: MITgcm; Basin: North Atlantic.

- Surface boundary condition: partially air-sea coupled.

- Lateral boundary condition: relaxation and radiation conditions.
• Focus on an isopycnal whose ensemble-mean depth ($\zeta$) is below the ensemble-mean mixed-layer depth (MLD).

• The isopycnal shoals drastically across the separated Gulf Stream.
The eddy feedback onto the mean flow

\[
\hat{u}_t + \hat{u}\hat{u}_x + \hat{v}\hat{u}_y + \hat{\omega}\hat{u}_b - f\hat{v} + \overline{m}_x = -\bar{e}_1 \cdot (\tilde{\nabla} \cdot \mathbf{E}) + \hat{\chi}
\]

\[
\hat{v}_t + \hat{u}\hat{v}_x + \hat{v}\hat{v}_y + \hat{\omega}\hat{v}_b + f\hat{u} + \overline{m}_y = -\bar{e}_2 \cdot (\tilde{\nabla} \cdot \mathbf{E}) + \hat{\gamma}
\]

- The net eddy feedback onto the (TWA) mean fields are encapsulated in the Eliassen-Palm flux \((\mathbf{E})\) divergence.
- For an eddy closure, it shifts the focus from the buoyancy equation (GM) to the momentum equations.

If we can parametrize \(\bar{e} \cdot (\nabla \cdot \mathbf{E})\), we have a physically consistent eddy closure scheme which represents the eddy buoyancy & momentum fluxes.
Can we parametrize the Eliassen-Palm flux divergence?

- The Eliassen-Palm flux divergence is directly related to the eddy Ertel potential vorticity (PV) flux ($F^{\Pi}$).
- This primes us to connect the Eliassen-Palm flux divergence to the large-scale Ertel PV.
- We relate the eddy Ertel PV flux to the local-gradient flux of the mean Ertel PV ($\Pi^#$) via an anisotropic eddy diffusivity tensor ($K$).

\[
\begin{bmatrix}
\langle u'' \theta'' \rangle & \langle v'' \theta'' \rangle \\
\langle u'' s'' \rangle & \langle v'' s'' \rangle
\end{bmatrix} = - \begin{bmatrix}
\hat{\theta}_x & \hat{\theta}_y \\
\hat{s}_x & \hat{s}_y
\end{bmatrix} \cdot \begin{bmatrix}
\Pi^#_x & \Pi^#_y
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
F^{\Pi1} & F^{\Pi2}
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
\kappa_{uu} & \kappa_{vu} \\
\kappa_{uv} & \kappa_{vv}
\end{bmatrix}
\]
Can we parametrize the Eliassen-Palm flux divergence?

$F^1_\Pi \quad (= \bar{\sigma}^{-1} \hat{e}_2 \cdot (\hat{\nabla} \cdot \mathbf{E}))$

$F^2_\Pi \quad (= -\bar{\sigma}^{-1} \hat{e}_1 \cdot (\hat{\nabla} \cdot \mathbf{E}))$

$-(\kappa^{uu} \Pi^u + \kappa^{uv} \Pi^v)$

$-(\kappa^{uv} \Pi^u + \kappa^{vv} \Pi^v)$

Residual ($F^1_\Pi$)

Residual ($F^2_\Pi$)
• The Eliassen-Palm flux divergence, which is directly related to the eddy PV flux, encapsulates the net eddy feedback onto the mean flow.

• The eddy PV flux can be related to the TWA field as an active tracer.

• The $2 \times 2$ diffusivity tensor $K$, which provides a closure for the eddy PV flux, single-handedly includes the information of eddy momentum fluxes in addition to bringing the GM and Redi diffusivities together.

• For a prognostic closure scheme, we would need to inform the “$\kappa$”s via the (resolved?) TWA field.
With a realistic equation of state (EOS) the vertical coordinate cannot “naively” be defined by potential density ($\rho_\theta$) as the pressure anomaly ($\phi = \int -g\rho_0^{-1}(\rho_\theta - \rho_0)d\zeta$) does not translate to a body force in buoyancy coordinates, i.e. $\nabla_h \phi \neq \tilde{\nabla}_h m$. We, therefore, argue for the use of in-situ density anomaly $\delta \ (= \rho - \rho(\zeta))$ where $\rho$ is the in-situ density and $\rho$ is a function of only depth. The buoyancy coordinate can then be defined as $\tilde{b} = -g\rho_0^{-1}\delta$ which removes a large portion of compressibility; the iso-surfaces of $\tilde{b}$ become close to neutral surfaces. The formulation of $\tilde{b}$ is analogous to where the buoyancy reduces to $\tilde{b} = -g\rho_0^{-1}(\rho - \rho_0)$ for a linear EOS.
\( \hat{u}_t + \hat{u}\hat{u}_x + \hat{v}\hat{u}_y + \hat{\omega}\hat{u}_b - f\hat{v} + \bar{m}_x = -\vec{e}_1 \cdot (\hat{\nabla} \cdot \mathbf{E}) + \hat{\lambda} \)

\( \hat{v}_t + \hat{u}\hat{v}_x + \hat{v}\hat{v}_y + \hat{\omega}\hat{v}_b + f\hat{u} + \bar{m}_y = -\vec{e}_2 \cdot (\hat{\nabla} \cdot \mathbf{E}) + \hat{\gamma} \)

\[
\mathbf{E} = \begin{pmatrix}
\hat{u}''\hat{u}'' + \frac{1}{2\sigma} \hat{\zeta}'^2 & \hat{u}''\hat{v}'' & 0 \\
\hat{v}''\hat{u}'' & \hat{v}''\hat{v}'' + \frac{1}{2\sigma} \hat{\zeta}'^2 & 0 \\
\hat{\omega}''\hat{u}'' + \frac{1}{\sigma} \hat{\zeta}'\hat{m}'_x & \hat{\omega}''\hat{v}'' + \frac{1}{\sigma} \hat{\zeta}'\hat{m}'_y & 0 \\
\end{pmatrix}
\]

- \( \mathbf{u}'' (= \mathbf{u} - \hat{\mathbf{u}}) \): the eddy velocity.

- \( (\cdot)' (= (\cdot) - \overline{(\cdot)}) \): the residual from the ensemble mean.
Each column is laid out so that the sum of the first three rows sum up to the E-P flux divergence shown in the bottom row.

The terms associated with eddy momentum flux and baroclinic instability tend to cancel each other out.
**Diffusivity tensor**

\[
\varphi_C = \arccos \left( \frac{\mathbf{F}_C \cdot \mathbf{G}_C}{|\mathbf{F}_C||\mathbf{G}_C|} \right)
\]
Temperature parametrization
(Extra slide) Salinity parametrization

\[ u''s'' \]

\[-(\kappa^{u\mu}s_\chi + \kappa^{u\nu}s_\gamma) \]

Residual \( u''s'' \)

\[-(\kappa^{v\mu}s_\chi + \kappa^{v\nu}s_\gamma) \]

Residual \( v''s'' \)

\[(m/(kg\cdot s))\]
Correlation and error of parametrization

\[
\sum [(F_C - \langle F_C \rangle)(F_{\text{param}} - \langle F_{\text{param}} \rangle)] \nonumber
\]

\[
\sum (F_C - \langle F_C \rangle)^2 \sum (F_{\text{param}} - \langle F_{\text{param}} \rangle)^2
\]

\[
\frac{|F_C - F_{\text{param}}|}{|F_C|}; \quad F_{\text{param}} = G_C \cdot K_C
\]