Matrix approaches to modeling land carbon and nitrogen cycles

Yiqi Luo
University of Oklahoma, USA

yluo@ou.edu
http://ecolab.ou.edu
CESM WG meeting, March 2, 2017
Observation-theory-model-process integrated approach

Bonan 2014 New Phytologist
Terrestrial carbon cycle

Observation & experimentation
Benchmark analysis

Systemic understanding
Data-theory-model fusion

Modelling & prediction
Structural analysis

Targeted data collection
Update understanding
Targeted model refinements

Model-theory-data fusion

Luo, Keenan, Smith 2015 GCB
Matrix approach to carbon cycle modeling
A: Basic processes

B: Shared model structure

C: Similar algorithm

D: General model

\[
\frac{dX(t)}{dt} = A \xi(t) CX(t) + BU(t)
\]

\[
X(t = 0) = X_0
\]

Luo et al. 2003 GBC
Luo and Weng 2011 TREE
Luo et al. 2015 GCB
Luo et al. 2017 BG
Yuanyuan’s work

a. Matrix solution
b. Fast spin-up to steady-state solution
c. Traceability analysis to evaluate model components
d. Data assimilation to improve models

\[
\begin{align*}
\frac{dX(t)}{dt} &= \xi ACX(t) + BU(t) \\
X(t = 0) &= X_0
\end{align*}
\]

Tractability

Luo et al. 2003 GBC
Luo and Weng 2011 TREE
Luo et al. 2015 GCB
Luo et al. 2017 BG
Matrix representation

LPJ-GUESS

CLM 3.5, 4.0, 4.5

TECO

CABLE
**Applications**

1. Diagnostics (Zhou et al. *J Climate*, Jiang et al. *GCB,*
   - a. 3D parameter space
   - b. Traceability analysis
   - c. Variance decomposition

2. Faster spin-up (Huang et al. *In prep.*, Xia et al. 2012)

3. Data assimilation (Hararuk et al. 2014, 2015, Shi et al. *In prep.*)

4. Theoretical understanding
   - a. Dynamic disequilibrium (Luo and Weng 2011)
   - b. Predictability of the terrestrial carbon cycle (Luo et al. 2015)
   - c. Transient dynamics (Luo et al. 2017)

\[
\begin{align*}
\frac{dX(t)}{dt} &= \xi ACX(t) + BU(t) \\
X(t = 0) &= X_0
\end{align*}
\]

Luo et al. 2003 GBC
Luo and Weng 2011 TREE
Luo et al. 2015 GCB
Luo et al. 2017 BG
3D parameter space

\[
\frac{dX}{dt} = I + A \varepsilon k X - \frac{\text{Tri}}{dz} X
\]

\(X, C\) pools  \(A,\) transfer matrix  \(\text{Tri},\) tridiagonal matrix

\(I,\) external input  \(\varepsilon,\) scalar  \(k,\) decomposition rate

Transient dynamics = Capacity - Potential

\[X(t) = \tau_E(t) NPP(t) - X_p(t)\]

Residence time  Production  3rd dimension

Luo et al. 2017
3D parameter space

$X(t) = \tau_E(t)NPP(t) - X_p(t)$

C storage dynamics = Capacity — Potential

Residence time  C input

Zhou et al. *J Climate*
Traceable components

\[ X(t) = (A_n(t)K)^{-1}Bu(t) - (A_n(t)K)^{-1}X'(t) \]
Variance decomposition I

Zhou et al. J Climate
Variance decomposition II

Zhou et al. *J Climate*
Matrix approach to nitrogen Modeling
Carbon cycle

- LAI
- Ta
- PAR
- RH

- $J_c$
- $A$
- $J_e$
- $Gs$

- $An$
- $Ac$

- $CO_2$

- Foliage
- Woody
- Fine Roots

- Metabolic Litter
- Structural Litter

- Fast SOM
- Slow SOM
- Passive SOM

Nitrogen cycle

- Deposition
- Fixation
- Uptake

- Soil Mineral N

Mineralization
Leaching Denitrification

Du et al. 2017 JAMES
Coupled carbon-nitrogen matrices

\[
\begin{align*}
\frac{d}{dt} X(t) &= A \xi(t) K(N) X(t) + u(N, t) B \\
\frac{d}{dt} N(t) &= A \xi(t) K(N) R^{-1} X(t) + \kappa_u N_{\min} \Pi \\
\frac{d}{dt} N_{\min} &= -(\kappa_u + \kappa_L) N_{\min} + A \xi(t) \phi_1^* K(N) R^{-1} X(t) + F(t)
\end{align*}
\]

\[
\begin{align*}
X(t = 0) &= X_0 \\
N(t = 0) &= N_0 \\
N_{\min}(t = 0) &= N_{\min,0} \quad \text{Shi et al. 2016 JPE} \\
&\quad \text{Du et al. 2017 JAMES}
\end{align*}
\]
\[ R = \begin{pmatrix} 
\rho_1 \\
\rho_2 \\
\rho_3 \\
\rho_4 \\
\rho_5 \\
\rho_6 \\
\rho_7 \\
\rho_8 
\end{pmatrix} = \text{diag}(\rho) \]
LM3V-N

CO₂, N₂, reactive N

Fire

Respiration

Photosynthesis (+)

Deposition

Fixation

Litterfall

Uptake

Mineralization

Stabilization (+)

Inorganic C

Mineral N

Organic C/N

Litter

Soil organic matter

Ammonium

Nitrate

Uptake

Nitrification

Immobilization

NH₃ volatilization

Leaching

Denitrification

Gerber et al., 2010; Huang and Gerber, 2015
Matrix Representation of LM3V-N

\[
\frac{dX(t)}{dt} = B(t)V(t) - A(t)\xi(t)K(t)X(t)
\]

- N input
- Scalar
- N allocation
- N transfer
- N turnover

\[
X = \begin{pmatrix}
\text{leaf} \\
\text{root} \\
\text{sapwood} \\
\text{wood} \\
\text{fast litter} \\
\text{slow litter} \\
\text{slow SOM} \\
\text{passive SOM} \\
\text{amm} \\
\text{nitr}
\end{pmatrix}
\]

\[
B = \begin{pmatrix}
b_{n1} & 0 \\
b_{n2} & 0 \\
b_{n3} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & i_1 \\
0 & 1 - i_1
\end{pmatrix}
\]

\[
V = \begin{pmatrix}
\text{BNF}(t) \\
\text{Dep}(t) + \text{Fert}(t)
\end{pmatrix}
\]

Yuanyuan Huang, unpub
Matrix representation of nitrogen model of CLM5
Matrix representation of demographic model
The future

• A library of carbon matrix equations from most of the ESMs
• A library of nitrogen matrix equations from most of the ESMs
• Ensemble modeling with multiple carbon and nitrogen matrices
• Traceable components and their contributions to uncertainty
• Benchmarking traceable components with observations
• Data assimilation to train model components with observations
To constrain structures and parameters of those well-understood model components

To allow structural variations for those poorly understood components

Luo, Keenan, Smith 2015 GCB