Modeling tensile and shear fracture in ice using damage mechanics

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Tensile fractures in Ross Ice Shelf

Shear fracture (cliff failure)

• Surface meltwater driven hydro-mechanical (tensile) fracture could accelerate calving of ice shelf fronts [Scambos et al., 2003, 2009; MacAyeal et al., 2003]

• Ice cliff failure (shear fracture) of grounded marine-terminating glaciers could cause rapid retreat of the ice terminus and contribute to sea level rise [DeConto and Pollard, 2016].

• Can hydrofracture and cliff failure cause a meter or more of sea level rise within the 21st century?
NSF CAREER: Fracture Mechanics of Antarctic Ice Shelves

**Research Aim 1**
Better understand crevasse propagation

- Tensile fracture occurs away from the terminus
- Shear fracture occurs near the ice cliff

**Educational Aim 1**
Enhance computational modeling skills

- Cross-college course in computational modeling
- High performance computing with CISM and LIVVkit

**Research Aim 2**
Physics-based calving laws for ice sheet models

- Observed velocity field
- CISM simulation result

**Educational Aim 2**
Increasing participation and diversity

- Cyberlearning tools for outreach to K-12 students
- Undergraduate and high school student research experiences
A Continuum Damage Mechanics (CDM) Model for Hydrofracture

- Strain rate or rate of deformation tensor
  \[ \dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \dot{\varepsilon}^{eq}_{ij} = \sqrt{\frac{1}{2} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}} \]

- Nonlinear viscous rheology of ice [Glen, 1955]
  \[ \tau_{ij} = 2(1 - D)\eta \dot{\varepsilon}_{ij} \quad \eta = \frac{1}{2} B (\dot{\varepsilon}^{eq}) \frac{1-N}{N} \]

- Poro-Damage approximation [Biot, 1955]
  \[ \sigma_{ij} = \tau_{ij} - p\delta_{ij} + Dp_w \delta_{ij} \]

- Incompressible Full-Stokes equations
  \[ \frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0 \quad \frac{\partial v_j}{\partial x_j} = 0 \]

- Gradient nonlocal damage evolution [Bazant, 1988; Peelings et al., 2001]
  \[ \dot{D} - \frac{l_c^2}{4k} \frac{\partial^2}{\partial x_i^2} \dot{D} = \dot{D}^{loc} \]

- Power-law creep damage rate [Murakami, 1983]
  \[ \dot{D}^{loc} = \hat{B} \frac{\langle \chi \rangle^r}{(1 - D)^{k_r}} \text{ if } p \geq 0 \]

- Brittle-ductile failure criterion [Hayhurst, 1972]
  \[ \chi = \alpha \tilde{\sigma}^{(I)} + \beta \tilde{\sigma}^{y} + (1 - \alpha - \beta) \tilde{\sigma}_{kk} \]

- Tensile fracture
- Shear fracture
- Pressure dependence

Hydrofracture of Surface Crevasses Filled with Meltwater

Water filled crevasses in a rectangular marine terminating glacier with free basal slip

- Self weight of ice causes compressive stress in the vertical direction.
- Seawater pressure causes compressive stress in the horizontal direction.
- Ice flow causes horizontal tensile stress at the top and horizontal compressive stress at the bottom.
- Water pressure in crevasses causes tensile stress due to wedging-effect.
Damage field in the domain is resolved with a 2 m mesh size

Jimenez et al., Journal of Glaciology (in prep.)
Can a water-filled crevasse propagate the entire ice thickness?

**Continuum damage mechanics deviates from linear elastic fracture mechanics (LEFM) theory for a near-floating grounded glacier**

Jimenez et al., Journal of Glaciology (in prep.)
Implementation of Damage Mechanics Model in SSA in Elmer Ice

Shallow Shelf Approximation [Winkelmann et al., 2011]

Velocity

Stress

Hydraulic pressure

Damage

Crevasse depth

Creep damage evolution [Murakami, 1983]
Benchmark Simulation: MISMIP+ with Damage Evolution in Elmer Ice

Domain is 640000 m long and 80000 m wide.

- Floating
- Grounded

Thickness (m)
- 27 days
- 54 days
- 219 days
- 356 days

Velocity Magnitude (m/yr)

Damage

Huth et al., Journal of Glaciology (in prep.)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>2015 Landsat-8 mosaic (Pope, 2016)/NASA Measures (Rignot et al., 2017)</td>
</tr>
<tr>
<td>Ice shelf thickness</td>
<td>Cryosat-2, swath processed (2015)</td>
</tr>
<tr>
<td>Bed elevation, grounded ice thickness</td>
<td>Bedmap2 (Fretwell et al., 2013)</td>
</tr>
<tr>
<td>Surface temperature</td>
<td>RACMO2.3 (Van Wessam et al., 2015)</td>
</tr>
<tr>
<td>Surface mass balance</td>
<td>RACMO2.3 (Van Wessam et al., 2015)</td>
</tr>
<tr>
<td>Geothermal heat flow</td>
<td>Satellite magnetic data (Fox-Maule et al., 2005)</td>
</tr>
</tbody>
</table>

Huth et al., *Journal of Glaciology* (in prep.)
Phase Field Damage Approach for Tension/Shear Failure

- Small strain tensor is the gradient of displacement
  \[ \varepsilon = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \]

- Constitutive law for linear elastic compressible ice
  \[ \sigma = 2\mu(1-D)\varepsilon + \lambda(1-D)\text{tr}(\varepsilon)I \]

- Mechanical equilibrium
  \[ \nabla \cdot \sigma + b = 0 \]

- Phase field damage evolution
  \[ D - l_c^2 \nabla^2 D + (1-D)l_c \left( \frac{\mathcal{H}_V(\varepsilon)}{G_V} + \frac{\mathcal{H}_D(\varepsilon)}{G_D} \right) = 0 \]

- Tensile component of crack driving force
  \[ \mathcal{H}_V = \kappa \langle \text{tr}(\varepsilon) \rangle_+^2 \]

- Shear component of crack driving force
  \[ \mathcal{H}_D = \mu \text{dev}(\varepsilon^2) \]
Wing Crack Growth in Brittle Rock-like Materials

Compression test experimental crack morphology (Gypsum plaster)

Phase field damage model predicted crack morphology

Devendiran and Duddu, Computational Mechanics, in prep.
Mixed-mode Fracture of Brittle Rock-like Materials

Nooru-Mohammed test  
experimental crack morphology

Phase field damage model  
predicted crack morphology

The two cracks emanating from the notches move away from each other and do not intersect

Devendiran and Duddu, Computational Mechanics, in prep.
Cliff Failure Modeling of Rock Slopes Using Damage Mechanics

Damage model features:

1. Strain-based brittle failure for tension

2. Mohr-Coulomb failure criterion for shear

Centrifuge experiments show that tensile fracture occurs away from the terminus and shear failure occurs near the rock cliff.

Li et al., Computers and Geotechnics, 36: 1246–1258, 2009
1. The CDM model predicts calving can occur due to hydrofracture of crevasses in grounded glaciers except for near floating condition

2. The CDM model can be implemented within the shallow shelf or other higher order Stokes approximations, which reduces computational cost

3. The new phase-field CDM model is based on fracture energy criterion and unifies CDM and LEFM approaches

4. The phase field CDM model can capture both tensile (opening mode I) and shear (sliding mode II) fracture, but cliff failure is more complicated.
Wing Crack Growth in Brittle Rock-like Materials