Spectra related to the El Nino Southern Oscillation

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Spectra of 1PC of tropical SSTs, CESM2.0
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We would like to use a statistical model to get uncertainty from an single 100-year run.

Hypothesis: Observed spectra of interannual variability lie within the uncertainty of those produced by a LIM (Newmann et al 2008)
Spectrum of tropical PC₁, OBS

- Fit LIM to monthly means of observed SSTs in tropical band
- Generate 500 samples for 100 years each from the LIM
- **Light shading**: shows max/min power from all samples
- **Dark shading**: standard deviation
For most frequencies the OBS spectrum lies within one standard deviation of the LIM spectra. Nowhere does the observed spectrum lie outside the max/min values of the LIM.

The observed spectrum is not inconsistent with the LIM hypothesis.
Experiments/Data

Experiments:
- Large Ensemble CESM1.1 (LENS), 1850CNTL
- CESM2.0, 1850CNTL

ECMWF reanalysis of 20th-century (ERA-20C) = OBS
- 1900-2010 => 111 years

All results based on 100 year subsets

Caveat: ENSO in experiments with different forcing are compared to each other
Fit a LIM to the LENS 1850CNTL (1800 years)

The LENS spectrum is not inconsistent with the LIM hypothesis.
Uncertainty in LIM and LENS

- Left: Uncertainty given by 18 100yr-simulation from LENS
- Right: Uncertainty given as 500 100yr-simualations from LENS
Define ENSO band as spectrum between 3 and 7 years.

Histogram of power spectral density within ENSO band

LIM captures uncertainty of power spectra in ENSO band
Is the LENS consistent with observations?

- OBS within light blue shading:
  - We cannot reject the hypothesis that the power spectra density of OBS are within the uncertainty of LENS.

- Histogram of power spectral density of LIMs of OBS and LENS do not agree within ENSO band
  - LENS is unlikely consistent with OBS.
Is the LENS consistent with observations?

- Area under spectrum = Variance
- Variance has to be consistent
  - Variances of PC1 are unlikely consistent, but not impossible
Is CESM2.0 consistent with observations?

- CESM2.0 outside uncertainty obtained from OBS LIM
- Variance outside variance from OBS LIM
  - CESM2.0 is unlikely consistent with OBS
- CESM2.0 outside uncertainty obtained from LENS
  - CESM2.0 is not consistent with LENS
On the dynamical mechanisms governing ENSO regularity

SPPT reduces
- overly regular oscillation
- reduced amplitude of spectral peak

Berner et al. 2018
Damped harmonic oscillator forced by white noise

\[ \dot{\vec{x}} = L\vec{x} + S \vec{\varepsilon} \]

with

\[ L = \begin{pmatrix} -\nu & \omega \\ -\omega & -\nu \end{pmatrix} \]

Perturbing the frequency \( \omega \) results in decreased memory and no change in variance.

Perturbing the damping rate \( \nu \) results in increased memory and increased variance.

\[ \vec{x} \] State vector
\[ \vec{\varepsilon} \] White noise
\[ L \] Feedback matrix
\[ S \] Noise amplitude matrix
\[ \omega \] Frequency
\[ \nu \] Damping rate

**Autocovariance, damped oscillator**

- \( \tau_d = 21.0, C_0 = 10.5 \)
- \( \tau_d = 35.0, C_0 = 17.5 \)
- \( \tau_d = 15.0, C_0 = 10.5 \)
Thank you!


Berner, Sardeshmukh and Christensen, 2018:” Role of perturbed frequency and damping for El Nino Southern Oscillation irregularity, J.Clim,s 2018
Wiener–Khinchin theorem:
The autocorrelation function of a wide-sense-stationary random process has a spectral decomposition given by the power spectrum of that process.
Assume the evolution of a system can be described as:

\[ \dot{x} = Lx + S\tilde{\epsilon} \]

Then, the lagged covariance matrix is given by:

\[ C_\tau = e^{LT}C_0 \]

The fluctuation-dissipation relation (stationary Fokker-Planck equation)

\[ LC_0 + C_0L^T + Q = 0 \]

\[ Q = SS^T \]

Penland and Sardeshmukh, 1995