An anisotropic, elastic-decohesive constitutive relation for modeling Arctic sea ice

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Motivation

Explicitly represent lead formation

RGPS (Kwok, 1998) analysis of satellite images shows large ice deformation events occurring in long-lasting linear features that appear to correspond to displacement (or velocity) discontinuities in the deformation field due to leads.

Cracks in the ice (leads) occupy 1-2% of the ice cover in winter but account for half of the ocean-air heat flux. Heat flux through intact ice is 2-5 Wm$^2$ compared with 300-500 Wm$^2$ through leads.
Model

- Ice dynamics (horizontal momentum equation) is solved using the material point method (Peterson and Sulsky, 2012)
- Mass is conserved for each material point (continuity equation)
- Each material point solves column thermodynamics equations and tracks ice thickness distribution
- The sea ice code is coupled to the MITgcm (Marshall et al., 1997) ocean code through fluxes
- Atmospheric forcing is JRA-25 reanalysis data (Onogi et al., 2007)
- Use of an elastic-decohesive constitutive model for the ice
The Elastic-Decohesive Constitutive Model

- Intact ice is modeled as elastic
- Leads (cracks) are modeled as discontinuities
- Model predicts initiation, orientation and opening of leads
- Traction is reduced with lead opening until a complete fracture forms

The model introduces a jump in displacement as a crack is initiated in the simulation. Crack initiation is governed by a curve in stress space. What is that curve?
Laboratory data

Measurement by Schulson (2001) show the stress state when a crack forms and the orientation of the crack. The observed failure envelope in stress space that describes initiation of failure could be described mathematically by a function $F(\sigma) = 0$.

What is F?

(a) Loading is purely tensile.
(b) Biaxial loading - tension and compression.
(c) Axial loading - pure compression.
(d) Biaxial compression.
In (a-c) the crack has a normal in the direction of maximum principal stress. (d) transitions to shear failure with two possible crack orientations.
Corresponding Model

$F$ is a function of stress (Schreyer et al. (2005), Sulky et al. (2006)).

$$F = \max_n F_n(\sigma), \quad [\sigma] = \begin{pmatrix} \tau_n & \tau_t \\ \tau_t & \sigma_{tt} \end{pmatrix}$$

$$F_n = \left( \frac{\tau_t}{s_m \tau_{sf}} \right)^2 + e^{\kappa B_n} - 1$$

$$B_n = \frac{\tau_n}{\tau_{nf}} + \frac{(\sigma_{tt})^2}{f_c'^2} - 1$$

Model parameters:

- $\tau_{nf} = \text{tensile strength}$
- $\tau_{sf} = \text{shear strength}$
- $f_c' = \text{compressive strength}$
- $f_c = \text{shear magnification}$

Modeled failure envelope $F=0$. Arrows show the predicted direction of the normal to the crack surface. Directions match experiments at (a-d).
Sea ice concentration 2003

March | July | September | October

Grid resolution = 36km
Metrics
## Multi-category contingency table

<table>
<thead>
<tr>
<th>Forecast category</th>
<th>Observed category</th>
</tr>
</thead>
<tbody>
<tr>
<td>i j</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>(n(F_1, O_1))</td>
</tr>
<tr>
<td>2</td>
<td>(n(F_2, O_1))</td>
</tr>
<tr>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>K</td>
<td>(n(F_K, O_1))</td>
</tr>
<tr>
<td>Total</td>
<td>(N(O_1))</td>
</tr>
</tbody>
</table>

- \(\text{hits}[i] = n(F_i, O_i)\), event forecast to occur and did occur
- \(\text{false alarm}[i] = \sum_{j \neq i} n(F_i, O_j)\), event forecast to occur, but did not occur
- \(\text{misses}[i] = \sum_{j \neq i} n(F_j, O_i)\), event forecast not to occur, but did occur

<table>
<thead>
<tr>
<th>Name</th>
<th>Perfect</th>
<th>Definition</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias</td>
<td>1</td>
<td>(\frac{h + fa}{h + m})</td>
<td>How did the forecast frequency of ‘yes’ events compare to the observed frequency of ‘yes’ events?</td>
</tr>
<tr>
<td>POD</td>
<td>1</td>
<td>(\frac{h}{h + m})</td>
<td>What fraction of the observed ‘yes’ events were correctly forecast?</td>
</tr>
<tr>
<td>SR</td>
<td>1</td>
<td>(\frac{h}{h + m})</td>
<td>What fraction of the forecast ‘yes’ events were correctly observed?</td>
</tr>
<tr>
<td>TS</td>
<td>1</td>
<td>(\frac{h}{h + m + fa})</td>
<td>How well did the forecast ‘yes’ events correspond to the observed ‘yes’ events?</td>
</tr>
</tbody>
</table>
Performance diagram

- Roebber (2009)

- Use geometric relationship of 4 metrics:
  \[
  \text{bias} = \frac{\text{POD}}{\text{SR}},
  \]
  \[
  \text{TS} = \frac{1}{\text{SR}} + \frac{1}{\text{POD}} - 1.
  \]

- Easy to read and display
Model comparison
Sea ice concentration

Observations

- Nimbus-7 passive microwave data (Cavalieri et. al, 1996)
- Gridded resolution: 25 km * 25 km
- Sensitivity: ±5% in winter and ±15% in summer
- Late spring and summer months are the least accurate and have the highest RMSE
- Bias (mult) alone does not provide useful information
- Bad accuracy is driven by high concentration inaccuracy
- The bias shows the disparity of each bin (the mult Bias looks good because of compensation)
- SR and POD confirm the model’s inaccuracy in late spring and summer months
Conclusion from sea ice concentration comparison

- Sea-ice extent is well matched all year long
- Concentration is well matched year round besides in the summer during which forecast is weaker (larger error in observations as well though)
- Thermodynamics needs to be improved? (can’t wait for the column physics package release)
- A similar analysis with different bin size (e.g. equal bin size) provides similar conclusions
We developed a sea-ice model capable of representing sea-ice fractures and lead openings.

The model is running and simulates reasonable results.

Performance and frequency diagrams provide quantitative insight into the validation of multi-category variables. It has the advantage to be easy to read, interpret and implement.

We created a git repository with the code performing the comparison. It is easy to adapt for any model and is available for sea-ice concentration and thickness.

Work in progress for sea ice displacement validation and higher resolution runs.