BIOMASS PARTITIONING USING AN OPTIMIZATION APPROACH FROM ECONOMIC THEORY

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MOTIVATION AND BACKGROUND

- Optimal partitioning theory: plants allocate biomass to most limiting resource
- Most LSMs used fixed ratios for biomass partitioning
PARALLELs BETWEEN ECOLOGY AND ECONOMICS

- Bloom et al., 1985:
  - Plants acquire resources when they are cheap and store them for later use
  - Plants produce roots and leaves until they cannot benefit from further growth of that component
  - Plants adjust allocation such that growth limitation is equal for all resources
  - Plants adjust phenology to changes in resources
COBB-DOUGLAS PRODUCTION FUNCTION

\[ Y = K^\alpha L^\beta \]

- Currency for plants can be carbon, nitrogen, water, etc.
- The Caveat:
  - This is a first order highly simplified approach – a proof-of-concept
  - Two resources: carbon and nitrogen
  - Two plant components: leaves and fine roots
  - Solve for fine root:leaf ratio

CARBON PARTITIONING WITH COBB-DOUGLAS EQUATIONS

\[ P(u_s, u_r) = \pi_C^\alpha \pi_N^\beta \ldots \pi_X^\lambda \]

\( \alpha \) and \( \beta \) are fixed based on CN ratios in the model \( \alpha + \beta = 1 \)

Goal: Optimize NPP.
Inputs: carbon and nitrogen.

\[ \pi_C = H_C(u_s) - c_C(u_s) - c_C(u_r) \]

\( \pi \) is the harvest of carbon or nitrogen

\[ \pi_N = \sum_{j=1}^{m} H_{jN}(u_r) - c_N(u_s) - c_N(u_r) \]

Must follow the law of diminishing returns

\[ \frac{\partial P(u_l, u_r)}{\partial u_l} = \frac{\partial P(u_l, u_r)}{\partial u_r} = 0 \]

Solver uses Newton-Raphson with a finite difference approximation for the derivative
EXAMPLE OF SINGLE RESOURCE

\[ H(x), \text{ gross resource harvest (g m}^{-2} \text{ yr}^{-1}) \]

\[ u_x \text{ biomass strategy (g m}^{-2} \text{ yr}^{-1}) \]

Lynch, 2015
EXAMPLE OF MULTIPLE RESOURCES

McNickel et al., 2016
DYNAMIC ALLOCATION, UPDATED ANNUALLY (GRASS EXAMPLE)

- $H_c(l) = GPPp_{ot} \times (1 - e^{-l})$
- $cc(l) = l \times (mr + gr)$
- $cc(fr) = fr \times (mr + gr)$
- $H_n(fr) = Nallocp_{ot} \times (1 - e^{-fr})$
- $cn(l) = (mr + gr) \times (l / leafcn)$
- $cn(fr) = (mr + gr) \times (fr / frootcn)$
RAN THE SIMULATION IN POINT MODE AT 30 FLUXNET2015 SITES IN ELM

30 sites
14 countries
11 PFTs (6 mixes sites)

DCA: dynamic carbon allocation model

CONTROL: default fixed allocation model

http://fluxnet.fluxdata.org/data/fluxnet2015-dataset/
FINE ROOT:LEAF RATIO ARE DEPENDENT ON PHENOLOGY TYPE

<table>
<thead>
<tr>
<th>Phenology</th>
<th>Fine Root:Leaf Ratio</th>
<th></th>
<th>Fine Root:Leaf Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>Range</td>
</tr>
<tr>
<td>Evergreen</td>
<td>0.96</td>
<td></td>
<td>0.51 – 2.18</td>
</tr>
<tr>
<td>Seasonal</td>
<td>0.37</td>
<td></td>
<td>0.16 – 0.55</td>
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<tr>
<td>Deciduous</td>
<td></td>
<td></td>
<td>Stress Deciduous</td>
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<tr>
<td>Stress</td>
<td>0.41</td>
<td></td>
<td>0.34 - 0.51</td>
</tr>
<tr>
<td>Deciduous</td>
<td></td>
<td></td>
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</tbody>
</table>
GENERAL IMPACTS ON GPP

![Bar chart showing general impacts on GPP with categories ENFT, ENFB, EBFT, EBFT1, DBFT, BDSB, BDST, C3CA, C3G, and C4G. The chart compares GPP (kg C m⁻² yr⁻¹) across different treatments: DCA, CONTROL, and Observations.]
EXAMPLE 1: DCA MODELED GPP INCREASE

DE-Tha Site:
Evergreen Needleleaf Temperate

Fine Root:Leaf 0.51
EXAMPLE 2: DCA MODELED GPP DECREASE

IT-Lav Site: Evergreen Needleleaf Boreal

Fine Root:Leaf 2.18
EXAMPLE 3: DCA MODELED GPP INCREASE

US-UMB Site:
Seasonal Deciduous Temperate
Fine Root:Leaf 0.16
TAYLOR DIAGRAM OF GPP SHOWS IMPROVEMENT IN SD BUT NOT CORRELATION
SUMMARY

Fine root:leaf ratios vary with PFT:
   Evergreen phenology has highest fine root:leaf ratios (i.e., N limited).
   Deciduous phenology has lowest fine root:leaf ratios (i.e., C limited).

DCA simulated *increases* in GPP at all sites with fine root:leaf ratio < 1
DCA simulated *decreases* in GPP at all sites with fine root:leaf ratio > 1
Changes in fine root:leaf ratio have stronger impact on evergreen than deciduous PFTs.

Standard deviation of the DCA model is closer to observations than CONTROL for most ecosystems
DCA model correlation with observations is unchanged compared with the CONTROL
CONCLUSIONS

The Cobb-Douglas dynamic carbon allocation model shows promise for including a dynamic approach to carbon partitioning in ESMs.

In the future

- The Cobb-Douglas algorithm should consider woody tissue (stem and coarse roots)
- Water, phosphorus, and distinguishing the nitrogen species of nitrate and ammonium should be included the Cobb-Douglas equation
- Test alternate solvers
- Test alternate equations of harvest and cost
- Bring in competition – game theory
THANK YOU

QUESTIONS?