Unified Formula for Land Biogeochemical Models

Yiqi Luo and many contributors

Center for Ecosystem Science and Society, Northern Arizona University, AZ, USA

Yiqi.Luo@nau.edu

http://www2.nau.edu/luo-lab/?home

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Matrix equation of CLM4.5

\[
\frac{dX(t)}{dt} = B(t)I(t) - A(t)KX(t) - V(t)X(t)
\]

\[
X(t) = (X_1(t), X_2(t), X_3(t), ..., X_{70}(t))^T
\]

\[
A = \begin{pmatrix}
A_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & A_{22} & 0 & 0 & 0 & 0 & 0 \\
A_{31} & 0 & A_{33} & 0 & 0 & 0 & 0 \\
A_{41} & 0 & 0 & A_{44} & 0 & 0 & 0 \\
0 & A_{52} & A_{53} & 0 & A_{55} & A_{56} & A_{57} \\
0 & 0 & 0 & A_{64} & A_{65} & A_{66} & 0 \\
0 & 0 & 0 & 0 & A_{75} & A_{76} & A_{77}
\end{pmatrix}
\]

\[
A_{31} = \text{diag}(-f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31})
\]

\[
V(t) = \begin{pmatrix}
V_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & V_{22}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & V_{33}(t) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & V_{44}(t) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & V_{55}(t) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & V_{66}(t) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & V_{77}(t) & 0
\end{pmatrix}
\]

\[
V_{22} = \text{diag}(z_1, z_2, ..., z_{10})^{-1}
\]

\[
\begin{pmatrix}
g_1 & -g_1 & 0 & 0 & ... & 0 & 0 & 0 \\
-h_2 & h_2 + g_2 & -g_2 & 0 & ... & 0 & 0 & 0 \\
0 & -h_3 & h_3 + g_3 & -g_3 & ... & 0 & 0 & 0 \\
0 & 0 & -h_4 & h_4 + g_4 & ... & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & ... & h_8 + g_8 & -g_8 & 0 \\
0 & 0 & 0 & 0 & ... & -h_9 & h_9 + g_9 & -g_9 \\
0 & 0 & 0 & 0 & ... & 0 & -h_{10} & h_{10}
\end{pmatrix}
\]
What have we done in the past year?
General equation for C and N model

\[
\begin{align*}
\frac{d}{dt}X(t) &= A_C \xi(t)K_C X(t) + u(N, t)B \\
\frac{d}{dt}N(t) &= A_N \xi(t)K_N N(t) + k_u F \Pi
\end{align*}
\]

\begin{align*}
X(t=0) &= X_0 \\
N(t=0) &= N_0
\end{align*}
CLM vegetation C&N: phenology, fire etc.

- L: leaf
- FR: fine root
- LS: live stem
- DS: dead stem
- LR: live coarse root
- DR: dead coarse root
- L_X: leaf transfer
- FR_X: fine root transfer
- LS_X: live stem transfer
- DS_X: dead stem transfer
- LR_X: live coarse root transfer
- DR_X: dead coarse root transfer
- L_S: leaf storage
- FR_S: fine root storage
- LS_S: live stem storage
- DS_S: dead stem storage
- LR_S: live coarse root storage
- DR_S: dead coarse root storage

Allocation, Phenology
Phenology offset, Background turnover, Gap mortality, Fire
Phenology, Fire
Phenology Procedure
Controlling Procedure
Matrix equation of vegetation C&N dynamics

\[
\frac{d}{dt} X(t) = (A_{ph}(t)K_{ph}(t) + A_{gm}(t)K_{gm}(t) + A_{fi}(t)K_{fi}(t))X(t) + B(t)F(t)
\]
Matrix equation of soil C&N dynamics

\[
\frac{d}{dt} X(t) = \left( A\xi(t)K - V(t) - V_f(t) \right) X(t) + B(t)I(t)
\]
Matrix Vs default RCP 8.5 (global-level)
Matrix Vs default bias in 2099 RCP8.5 (Global pattern)

(Matrix – Default)/Default * 100%
Diagnostic variables related to C storage capacity ($X_C$) and C storage potential ($X_P$)

\[
X_C = -\left( A\xi K \right)^{-1} BI \\
X_P = X_C - X
\]

Luo et al. 2017

$\xi$: Environmental scalar  
$A$: Carbon transfer coefficient  
$K$: Turnover rate  
$B$: Partitioning coefficients for influx  
$I$: Influx  
$X$: state variable of C storage

Add 100 variables: 36 Vegetation C output variables, 36 Vegetation N output variables (18 vegetation pools), 14 Soil C variables and 14 Soil N variables (7 soil pools) for both capacity and potential.
ORCHIDEE matrix model

Huang et al. under review

JAMES

32 Layers Vertical Mix

CO₂ Transfers to 32 SOC pools

Active SOC x5-36 Slow SOC x37-68 Passive SOC x69-100

Transfers among 3 SOC types in each soil layer

Huang et al. under review JAMES
What is the use of it?
1. Semi-analytical spinup

400 years semi-analytical matrix result vs. 200,000 years ORCHIDEE-MICT

Acceleration of spin-up by 500 times
2. Comprehensive sensitivity analyses

Huang et al. under review

JAMES
3. Data assimilation for both flux- and pool-related data to constrain global SOM

MCMC

Harmonized soil C content by Wieder et al., (2015)

From Zheng Shi
4. One formula to unify all land carbon cycle models

\[
d X(t) / dt = B I(t) - A_\xi(t) K X(t)
\]

Luo et al. 2001
Luo et al. 2003 GBC
Luo and Weng 2011 TREE
Luo et al. 2012
Luo et al. 2015
Luo et al. 2017
5. Hierarchical models

Vertical profile

\[
\frac{dX(t)}{dt} = (A\xi(t)K + V(t))X(t) + B(t)u(t)
\]

\[
\frac{dX(t)}{dt} = A\xi(t)KX(t) + Bu(t)
\]

C transfer of phenology
C transfer of gap mortality
C transfer of fire

Pool state
Input
Allocation

Vegetation dynamics

\[
\frac{d}{dt}X(t) = (A_{ph}(t)K_{ph}(t) + A_{gm}(t)K_{gm}(t) + A_{fi}(t)K_{fi}(t))X(t) + B(t)F(t)
\]
6. Unified Diagnostic System
Or 1-3-5 scheme

• One (1) formulae unifies all land C cycle models

• One 3-D space to evaluate all model outputs

• Five (5) Traceable components to pinpoint uncertainty sources

Luo et al. 2017 AGU
7. Analytic attribution of CO$_2$ impacts

**Matrix Simulations** (matlab)

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<th>S0</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
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</table>

CLM Run 1
CO$_2$(280ppm)

CLM Run 2
eCO$_2$(560ppm)

Huang et al. 2018 GCB
8. Modeling conundrum

- Models behave so differently;
- Uncertainty has been documented in almost all model intercomparison projects (MIPs);
- Uncertainty becomes larger instead of smaller as we incorporate more processes into models;
- We become more confused with uncertainty as we invest more time to address this issue.

Friedlingstein et al. 2006
8. Modeling conundrum

✓ High degree of complexity and sophistication of model implementations hinders our understanding of holistic system behavior

✓ Matrix approach provides a general framework for the qualitative understanding of models without compromising detail in process representation
What is the future?
CLM5.0 matrix CN model

- Done all tests
- Ready to be incorporated into trunk of CLM
- Will make the offline version of CLM5.0-CN matrix model publically available
- Welcome more people to use matrix models
- Willing to help the community for the transformation
- Summer training course at Flagstaff in May
Challenges and opportunities for you

Challenges
• Luring you out of your comfort zone for modeling
• Basic training in matrix algebra
• Other obstacles?

Benefits and opportunities
• Most likely to make your life easier
  – Simplicity in coding
  – Cleaner and more efficient code
  – Faster for spin-up
• Enabling you do analysis (e.g., Sobol) you usually can not do
• Understanding your model results much easier
General equation for biogeochemical models

<table>
<thead>
<tr>
<th>Matrix models</th>
<th>In progress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CLM 3.5</td>
<td>1. JULES</td>
</tr>
<tr>
<td>2. CLM4.0</td>
<td>2. LM3V-N</td>
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<td>3. CLM4.5</td>
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<td>4. CLM5.0</td>
<td>10 more models to participate in the summer training course</td>
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<td>5. CABLE</td>
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<td>6. LPJ-GUESS</td>
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<td>7. ORCHIDEE</td>
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<tr>
<td>8. BEPS</td>
<td>10 nonlinear Microbial models by Carlos Sierra</td>
</tr>
</tbody>
</table>
General dynamical equation of terrestrial carbon cycle

1. Transforming land carbon cycle modeling into more theoretically based science
2. Asking a whole set of new questions on land carbon cycle dynamics (e.g., dynamic disequilibrium, predictability)
3. Implications on empirical research are yet to be uncovered