GCM - Ice Model Coupling:
Adventures in Energy Conservation

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Synchronous Two-Way Coupling

Important to resolve transients (human timescales).

**Challenge:**
- Balance mass and energy budget for (potentially) non-conservative ice model.
- Compute non-conservation; dump extra in ocean.
Energy Budget

Account for energy flux in each 2D ice grid cell:

\[
\psi(x, y, t) = \psi_0 + \int_{t_0}^{t} (e_s + e_b + e_c + h_s + h_b + h_i + \nabla \cdot \psi \mathbf{u} + \epsilon) \, dt
\]
Energy Budget

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| \( \psi_0 \) | Initial enthalpy state of ice sheet | J/m² |
| \( \psi \) | Enthalpy of ice sheet | m/s |
| \( \mathbf{u} \) | Ice velocity field | |
| \( e_s \) | Enthalpy flux of SMB (from snow/firn) | W/m² |
| \( e_b \) | Enthalpy flux of runoff | |
| \( e_c \) | Enthalpy flux of calving | |
| \( h_s \) | Conductive heat flux through top surface | W/m² |
| \( h_b \) | Conductive basal heat flux | |
| \( h_i \) | Strain heating rate | |
| \( \epsilon \) | Unaccounted energy flux | |

- GCMs do not track gravitational potential.
- GCM must dispose of \( h_i + \epsilon \) in non-physical way.
Coupling Fields

**Initialization:**

**Ice Model → GCM**

1. $T$, top of ice sheet
2. Depth of top layer
3. Elevation on ice grid

**GCM Computes:**

1. Conductive Heat Flux

**GCM → Ice Model**

1. Surface Mass Balance
2. Enthalpy of SMB
3. $T$ at bottom of ice surface model

**Ice Model → GCM**

**Mass and Enthalpy:**

1. SMB
2. Internal Advection
3. Basal Runoff
4. Vertically-Integrated State
5. $\epsilon$ non-conservation (mass, energy)

**Energy:**

1. Strain Heating
2. Geothermal Flux

**Other:**

1. $T$, top of ice sheet
2. Depth of top layer
3. Elevation on ice grid
Step 1: Initialization

Elevation (m)  
Surface T (°C)  
Depth of Top Grid Point (m)
Step 2: GCM $\longleftrightarrow$ Ice Heat Flow

**Goal:**
Compute $q_n$, heat flux between models

**Challenges:**

1. **Differing Parameterizations**
   - Solving heat equation between FD and non-FD model.
   - This FD ice model has no gridpoint at surface.

2. **Differing scales**
   - Large $\Delta z$ yields large $\Delta T$, inappropriate for small scale of $z_1 \ldots z_n$.
   - $T_{n+1}$ doesn’t change over multiple timesteps for $T_n$
Step 3: GCM Outputs

Surface Mass Balance
\((\text{kg m}^{-2} \text{ s}^{-1})\)

Surface T (°C)
PISM Mass Budget (kg m$^{-2}$ s$^{-1}$)

Surface Mass Balance

Internal Advection

Basal Runoff
PISM Mass Budget (kg m$^{-2}$ s$^{-1}$)

Total Mass Flux

$\epsilon$: mass

- $-3.5 \times 10^{-16}$
- $0$
- $3.5 \times 10^{-16}$
PISM Energy Budget: Enthalpy Flux (W/m²)

- Surface Mass Balance
- Internal Advection
- Basal Runoff
PISM Energy Budget: Heat Flux (W/m\textsuperscript{2})

Strain Heating

Geothermal Flux
PISM Energy Budget: Results ($W\,m^{-2}$)

Total Enthalpy Flux

$\epsilon$: enthalpy
Discussion

Why the enthalpy problem? Possibilities:

- No grid point at top of ice model? (Uncontrolled forcing when setting Dirichlet BC)
- Disparate time and space scales? (with explicit timestepping at model interface)
- Would Neumann BC for ice model help?
- Problematic parameterization in ice surface?
- Just a spin-up problem?
- We will find out with 1-D prototype.

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