The MPAS-Ocean Vertical Coordinate

Mark Petersen
and the MPAS-Ocean development team

Los Alamos National Laboratory
The MPAS-Ocean Vertical Coordinate

- **Z-Level**: Fixed coordinate.  *POP, MOM, MIT-GCM, NEMO*
- **Z-star**: Layers expand with SSH.  *MOM, recently POP, others*
- **sigma**: terrain-following.  *ROMS, NEMO*
- **isopycnal**: *MyCOM, GOLD*
- **hybrid isopycnal**: *HyCOM*
- **partial bottom cells** (in addition to others)
- **z-tilde**: frequency-filtered coordinate (in addition to others)

What is the best vertical coordinate for MPAS-Ocean?
The MPAS-Ocean Vertical Coordinate

- **Z-Level**
- **Z-star**
- **sigma**: only tested in idealized cases so far
- **isopycnal**: idealized only, no zero thickness layers
- **hybrid isopycnal**: under development
- **partial bottom cells**
- **z-tilde**: frequency-filtered coordinate
**Arbitrary Lagrangian-Eulerian (ALE) Vertical Coordinate**

**Thickness equation:**

\[ \frac{\partial h_k}{\partial t} = -\nabla \cdot \left( h_k^{\text{edge}} \mathbf{u}_k \right) + w_{k+1}^{\text{top}} - w_k^{\text{top}} \]

\( w_{\text{top}} \) is transport through interface

\(-\nabla \cdot \left( h_k^{\text{edge}} \mathbf{u}_k \right) - w_{k+1}^{\text{top}} + w_k^{\text{top}} \)

**isopycnal** (for adiabatic, idealized studies) \( w = 0 \)

**z-level** \( \frac{\partial h_k}{\partial t} = 0 \), except layer 1

**z-star** Layer thickness changes in proportion to SSH

\[ \frac{\partial h_k}{\partial t} = \sum_k c_k \frac{\partial \eta}{\partial t} \]
Test Problems (Ilicak et al. 2012)

- Lock exchange
- Baroclinic eddies
- Overflow
- Internal gravity wave
- Sub Ice-Shelf
Lock Exchange Test Case

- Zero tracer diffusion
- Vary horizontal viscosity
- Linear equation of state
- Simplest test of mixing

Ilicak et al. (2012) compares ROMS, MITgcm, MOM, GOLD

Theoretical wave propagation speed is

\[ u_f = \frac{1}{2} \sqrt{gH \left( \delta \rho / \rho_0 \right)} \]

Ilicak et al. (2012)
Resting Potential Energy (RPE): a measure of mixing

- Definition (Ilicak et al. 2012): \[ RPE = g \iiint \rho^* z \, dV \]
- \( \rho^* \) is the sorted density state, with heaviest on the bottom.

\[ \rho(x, z) \quad \rightarrow \quad \rho^*(z) \]

\[ RPE = g \sum_i \rho^*_i z_i V_i \]

**Example 1: No mixing**

\[ RPE = 16360 \, gV_{cell} \]

**Example 2: some mixing**

\[ RPE = 16370 \, gV_{cell} \]

**Example 3: fully mixed**

\[ RPE = 16400 \, gV_{cell} \]
Resting Potential Energy (RPE): Lock Exchange

- RPE increases with time as fluid is mixed
- RPE depends on horizontal viscosity as follows:
  - high horizontal viscosity
  - low Reynolds number
  - low RPE, less mixing
  - low horizontal viscosity
  - high Reynolds number
  - high RPE, more mixing

\[
\frac{RPE(t) - RPE(0)}{RPE(0)}
\]

\[
\frac{dRPE(t)}{dt}
\]

Data from Ilicak et al. (2012)

Graphs showing RPE over time for different horizontal viscosities and the rate of change of RPE with respect to grid Reynolds number.
Baroclinic Eddies Test Case

- Idealized ACC: periodic channel, f-plane
- Compare to POP z-level and POP z-star

\[
\frac{dRPE(t)}{dt}, \text{ W/m}^2
\]

10km resolution

less mixing

\[
10^0 \quad 10^1 \quad 10^2 \quad 10^3
\]

grid Reynolds number
Overflow Test Case

- Zero tracer diffusion
- Vary hor. viscosity
- Test z-level, z-star, partial bottom cells, and sigma coordinate
Internal Wave Test Case

The figure shows a contour plot of potential temperature in a water column, with depth and y-axis km. The legend indicates various color levels from 11 to 19 degrees Celsius. The bottom graph plots \( \frac{dRPE(t)}{dt} \) versus grid Reynolds number, with different lines and markers representing MPAS-O z-level, MPAS-O z-star, MITGCM, and MOM models.
Frequency-filtered thickness: $z$-tilde (Leclair & Madec 2011)

- **Motivation:** We would like internal gravity waves to not cause mixing.
- Here lines show grid cells, for $z$-star vertical grid:

  What if we allow layer thickness to oscillate with internal waves?
- This can be done with a low-pass filter on the divergence
Frequency-filtered thickness: z-tilde (Leclair & Madec 2011)

- A low-pass filter on the baroclinic divergence:

\[
D_k = \bar{D} + D_k' = \bar{D} + D_k^{lf} + D_k^{hf}
\]

\[
D_k = \nabla \cdot (h_k u_k)
\]

Divergence:
- Horizontal divergence
- Barotropic divergence
- Baroclinic divergence
- Low frequency baroclinic divergence
- High frequency baroclinic divergence

Low-pass filter:
\[
\frac{\partial D_k^{lf}}{\partial t} = -\frac{2\pi}{\tau_{Dlf}} \left[ D_k^{lf} - D_k' \right]
\]

- \(\tau_{Dlf}\) is the filter time scale, typically five days.
- It controls the time scales included in the low frequency divergence.

\(\tau_{Dlf}\) is the filter time scale, typically five days.
- short time, high frequency oscillations change layer thickness
- long time, low frequency oscillations do not change layer thickness
Frequency-filtered thickness: $z$-tilde (Leclair & Madec 2011)

- A low-pass filter on the baroclinic divergence:

Divergence:

$$D_k = \bar{D} + D'_k = \bar{D} + D_{lf}^k + D_{hf}^k$$

hor. divergence
barotropic
baroclinic

high frequency baroclinic div.
low frequency baroclinic div.

Low-pass filter:

$$\frac{\partial D_{lf}^k}{\partial t} = -\frac{2\pi}{\tau_{Dlf}} \left[ D_{lf}^k - D'_k \right]$$

High-frequency thickness equation:

$$\frac{\partial h_{hf}^k}{\partial t} = -D_{hf}^k + \frac{2\pi}{\tau_{hhf}} h_{hf}^k + \nabla \cdot \left( \kappa_{hhf} \nabla h_{hf}^k \right)$$

forcing
restoring
diffusion

Revised thickness equation:

$$\frac{\partial h_k}{\partial t} = \frac{\partial h_{ext}^k}{\partial t} + \frac{\partial h_{hf}^k}{\partial t}$$

z-star
part
z-tilde
part

Two new prognostic equations
Frequency-filtered thickness: Internal Wave Test Case

- It works!
- Here lines show grid cells, for z-tilde vertical grid:
Frequency-filtered thickness: Internal Wave Test Case

- Similar results for global simulations

![Graph showing the effect of grid Reynolds number on vertical transport through layer interface.](image)

- Short time, high frequency oscillations change layer thickness
- Long time, low frequency oscillations do not change layer thickness

- Stronger $z$-tilde, less vertical transport, less mixing
For coupled ocean-ice shelf modeling, we need to depress the ocean surface with the weight of the ice shelf.

Observations: Pine Island Glacier

image from Joughin ea. Science, 2012

image from Jenkins ea. Science, 2010
MPAS-Ocean: Ice Shelf Above Ocean Surface

- For coupled ocean-ice shelf modeling, we need to depress the ocean surface with the weight of the ice shelf.

Test 2: Driven Cavity

- MPAS-Ocean model
  - 22 layers, 50 m each
  - surface wind stress of 0.1 N/m²

- cavity, $S=34.3$ throughout

- ice shelf, imposed by surface pressure

- varying slope

- fixed slope

- $A: 100$ m (varies)
- $B: 15$ km (varies)
- $S_{\text{top}}=34.5$
- $S_{\text{bot}}=34.7$

- linear stratification in salinity, constant temperature

- $250$ m

- $30$ km

- $140$ km
For coupled ocean-ice shelf modeling, we need to depress the ocean surface with the weight of the ice shelf.

Ocean layers were compressed to 5 cm thickness with no negative effects.

Sheer cliff face may be used at ice shelf edge.

Tests used linear EOS. For nonlinear EOS, must account for sigma-coordinate correction.
The MPAS-Ocean Vertical Coordinate

- **Z-Level**
- **Z-star**
- **sigma**: only tested in idealized cases so far
- **isopycnal**: idealized only, no zero thickness layers
- **hybrid isopycnal**: under development
- **partial bottom cells**
- **z-tilde**: frequency-filtered coordinate