Nonlinear Scale Interactions and Energy Pathways in the Ocean

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Ocean Circulation

- Multiscale Flow: $O(10^4) \text{ km} -- O(1) \text{ mm}$
Ocean Circulation

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- Inhomogeneous and Anisotropic

Source: NASA
Ocean Circulation

• Multiscale Flow: $O(10^4)$ km -- $O(1)$ mm
• Inhomogeneous and Anisotropic
• Mean Kinetic energy (KE) is concentrated in narrow intense currents.

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The Problem

• What is the flow of energy between different spatial scales, different forms?
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• What is the flow of energy between different spatial scales, different forms (how big are these arrows)?
Traditional Approach

Source: NASA
Our Approach

Coarse-graining (Filtering)

\[ \bar{u}_\ell(x) = \int d\mathbf{r} \ G_\ell(\mathbf{r}) u(x + \mathbf{r}) \]

Leonard (1974),
Germano (1992), Eyink (1994),
Piomelli et al. (1991), Liu et al (1994),
Chen, Ecke, Eyink (2003),…
Cascade of Energy

Large-scale energy budget

\[ \partial_t \frac{|\mathbf{u}|^2}{2} + \nabla \cdot [\cdots] = -\Pi_{\ell}^E - \nu |\nabla \mathbf{u}|^2 \]

Energy flux
Cascade of Energy

Large-scale energy budget

\[ \partial_t \frac{|\mathbf{u}|^2}{2} + \nabla \cdot [\cdots] = -\Pi^E_k - \nu |\nabla \mathbf{u}|^2 \]

- Every point \( \mathbf{x} \) and every instant \( t \)
- Variable scale \( \ell \)

Energy flux

\( 0 \quad \quad \quad \quad \quad \quad \quad \quad K = 1/\ell \quad \quad \quad \quad \quad \quad \quad \quad k \)
Large-scale energy budget

\[ \partial_t \frac{|\mathbf{u}|^2}{2} + \nabla \cdot [\cdots] = -\Pi_E - \nu |\nabla \mathbf{u}|^2 \]
Measuring Energy Transfer

Large-scale energy budget

\[
\partial_t \frac{|\mathbf{u}|^2}{2} + \nabla \cdot [\cdots] = -\Pi^E_{\ell} - \nu |\nabla \mathbf{u}|^2
\]

Subgrid scale (SGS) flux

\[
\Pi^E_{\ell}(\mathbf{x}) = -\partial_j \bar{u}_i \left[\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j\right]
\]
Measuring Energy Transfer

Large-scale energy budget

\[ \partial_t \frac{|\bar{u}|^2}{2} + \nabla \cdot [\cdots] = -\Pi_{\ell}^E - \nu |\nabla u|^2 \]

Subgrid scale (SGS) flux

\[ \Pi_{\ell}^E (x) = -\partial_j \bar{u}_i \left[ \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j \right] \]


\[ \Pi_{\ell} (x) = \bar{u}_i \ u_j \ \partial_j (u_i - \bar{u}_i) \]

sweeping energy in space

between structures
Measuring Energy Transfer

Large-scale energy budget

\[
\partial_t \frac{|\mathbf{u}|^2}{2} + \nabla \cdot [\cdots] = -\Pi^E_\ell - \nu |\nabla \mathbf{u}|^2
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Subgrid scale (SGS) flux

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\Pi^E_\ell (\mathbf{x}) = -\partial_j \bar{u}_i \left[ \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j \right]
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\Pi_\ell (\mathbf{x}) = \bar{u}_i \ \partial_j (u_j \bar{u}_i)
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Measuring Energy Transfer

Any measure of the energy exchange must satisfy:

1. Galilean Invariance
2. Vanish in the absence of subscale fluctuations
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Simulation Domain

SGS definition

Frisch (1995) definition
Measuring Energy Transfer

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Measuring Energy Transfer

Any measure of the energy exchange must satisfy:
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South America

North America

Europe

Africa

South America

SGS definition $\ell = 200 \text{ km}$

Frisch definition $\ell = 200 \text{ km}$
South America  
North America  
Europe  
Africa  
South America

SGS definition  \( \ell = 200 \text{ km} \)  
\[ \langle \Pi^\text{SGS}_\ell \rangle = +0.9 \text{ W/km}^2/\text{m} \]

Frisch definition  \( \ell = 200 \text{ km} \)  
\[ \langle \Pi^\text{Frisch}_\ell \rangle = -22.9 \text{ W/km}^2/\text{m} \]
SGS definition \( \ell = 200 \text{ km} \)

\[ \langle \Pi^\text{SGS}_\ell \rangle = +0.9 \text{ W/km}^2/\text{m} \]

\[ = -197 \text{ W/km}^2/\text{m} \]

Frisch definition \( \ell = 200 \text{ km} \)

\[ \langle \Pi^\text{Frisch}_\ell \rangle = -22.9 \text{ W/km}^2/\text{m} \]

\[ = +319 \text{ W/km}^2/\text{m} \]
In summary

- Traditional homogeneous/isotropic turbulence tools break down in the Ocean.

- Spectral Transfer and Flux can be qualitatively wrong.

- Guided by basic physical principles (*Galilean invariance, ....*), the SGS approach to coarse-graining provides a robust means to measure energy transfer at different locations in the Ocean.
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  • Since then, have worked through issues of calculating energy transfer on the sphere (scale-dependent commutation of vector operations and filter).