Enlargement of englacial conduits in cold ice –
basic theory and some simple experiments

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Basic processes enabling establishment of englacial hydrologic network in cold ice –
Fracture propagation, melt enlargement of conduits/passage
(need to work against creep closure and refreezing)
This research focuses only on melt enlargement and refreezing
Simplified model of water-carrying conduit in cold ice - neglecting advective transport in water (along conduit length), assuming water discharge rate is constant, water temperature = 0 degrees)

Heat conduction equation in cold ice (radial)

\[ \rho_i C_i \frac{\partial \theta}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left( r k_i \frac{\partial \theta}{\partial r} \right) = 0, \quad r > R(t) \]

Initial condition (cold ice)

\[ \theta(r, t = 0) = \theta_0, \quad r > R(t = 0) \]

Boundary conditions at ice-water interface \( R(t) \)...a moving boundary

\[ \theta(R(t), t) = \theta_{PM} = 0 \quad \text{at the phase change interface} \]

Energy Balance condition at interface

\[ \rho_w g Q \left| \frac{dh}{dx} \right| = \rho_w L \times 2\pi R \frac{dR}{dt} - 2\pi R k_i \frac{\partial \theta}{\partial r} \bigg|_{r=R(t)} \]

Energy supplied by viscous and turbulent dissipation in flowing water

Energy available to grow conduit

Energy conducted into cold ice (heat loss from water)

....all per unit length of conduit
Energy supplied by viscous and turbulent dissipation in flowing water

\[ \rho_w g Q \frac{dh}{dx} = \frac{\rho_w f Q^3}{4 \pi^2 R^5} \]

Discharge (flow rate)

Head loss per unit length

\[ \left| \frac{dh}{dx} \right| = \frac{f}{2R} \frac{V^2}{2g} = \frac{f}{2R} \frac{Q^2}{2g(\pi R^2)^2} \]

\( f = \text{Darcy-Weisbach friction factor} \)

(head loss formula for pipe flow)
Moody Diagram

Friction Factor

Reynolds Number, $Re = \frac{\rho V d}{\mu}$

Material

- Concrete, coarse
- Concrete, new smooth
- Drawn tubing
- Glass, Plastic, Perspex
- Iron, cast
- Sewers, old
- Steel, mortar lined
- Steel, clad
- Steel, structural or forged
- Water mains, old

Material E (mm)

- 0.25
- 0.025
- 0.0025
- 0.0025
- 0.15
- 3.0
- 0.1
- 0.5
- 0.025
- 1.0

Friction Factor = $\frac{2d}{\rho V^2} \Delta P$

Laminar Flow

Transition Region

Complete Turbulence

Smooth Pipe

Relative Pipe Roughness $\varepsilon$

Blasius Resistance Law

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Radius evolution equation

\[ \frac{dR}{dt} = \left( \frac{\rho_w f Q^3}{4 \pi^2 R^5} + 2 \pi R k_i \frac{\partial \theta}{\partial r} \bigg|_{r=R(t)} \right) / \left( 2 \pi R \rho_w L \right) \]

Stagnant water (no flow, no internal energy generation by dissipation)

\[ \frac{dR}{dt} = \left( 2 \pi R k_i \frac{\partial \theta}{\partial r} \bigg|_{r=R(t)} \right) / \left( 2 \pi R \rho_w L \right) \]

...obviously always get refreezing (energy in water is transferred to ice, causing water to refreeze, conduit radius shrinks)

Negative (warmer inside)

To get conduit to grow, need

\[ \frac{\rho_w f Q^3}{4 \pi^2 R^5} \] TO BEAT OUT \[ 2 \pi R k_i \frac{\partial \theta}{\partial r} \bigg|_{r=R(t)} \]

i.e. supply more energy to conduit wall by viscous/turbulent dissipation than is extracted by heat loss from water to ice by conduction......in an integrated sense over some time interval (the first term’s integral \( \sim \Delta t \), the conductive flux term’s integral \( \sim \sqrt{\Delta t} \) at early time) – not entirely satisfying analytical criterion for “critical flow rate” \( Q \) to produce growth.
Numerical Model

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Numerical Model

Ice

Water

Insulated Boundary

0°C Celsius Interface

Thermocouples

Numerical Model

Water

Ice

0°C Celsius Interface

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Insulated Boundary
Experimental Schematics

End Cross-Section View

Side Cross-Section View
Ice Sample Container Comparison

Insulated Box

PVC
Stagnant Water Tests

- Performed at CU
  - Conduit re-augered after each successful test
    - Multiple tests per ice sample

- Ice Dimensions
  - 6” Tall
  - 6” Wide
  - 12” Long
  - 3/8” Initial conduit radius

- 0 Degrees Celsius Water
  - Poured into each end

- 6 thermocouples
  - Recording temperature
Stagnant Comparison

- Model matches temperature values well
  - Locations furthest from center best
- Differences attributed to not perfectly radial heat transfer (square box)
Low Water Flow Rate Tests
Low Water Flow Rate Comparison

• Ice input as initial constant temperature
• Radial numerical code used

• Blasius smooth pipe approximation fits well
  – Darcy-Weisbach friction factor
• Model best at higher ice temperatures
Low Water Flow Rate Comparison

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Low Water Flow Rate Comparison

- Friction factor of 1 too high
  - Blasius smooth pipe approximation used (f=.02)
- Nearest temperature recording slightly underpredicts temperature
High Water Flow Rate Tests

• Performed at NASA GSFC
  – Conduit re-augered after each successful test
    – Multiple tests per ice sample

• Ice Dimensions
  – 6” Diameter
  – 26” Long
  – 3/8” Initial conduit radius

• 0 Degrees Celsius Water
  – Pumped through ice

• 12 thermocouples recording temperature
  – 2 profile locations

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Numerical Model
Experiments
Comparison
High Water Flow Rate Tests

Introduction  Experiments  Numerical Model  Comparison

[Diagram showing water flow process with nodes for Flow Box, Water Reservoir, Pump, Datalogger and Multiplexor, Ice Sample, Water Temperature Gauges, Thermocouple Wires, and Cold Room.]
High Water Flow Rate Tests

<table>
<thead>
<tr>
<th>High Flow</th>
<th>Initial Ice Temperature (Celsius)</th>
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<tbody>
<tr>
<td></td>
<td>-1.66</td>
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<td>-10.74</td>
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- Introduction
- Numerical Model
- Experiments
- Comparison

- Time (s from beginning)
- Distance from Conduit Center (in.)
High Water Flow Rate Tests

- Final conduit geometry
  - Radius not constant through ice sample length
  - Scalloping due to turbulent flow during conduit expansion
Scalloping

• Turbulent Flow
  – Eddies form and carve dimples
• Scallops turn walls from smooth to (very, very) rough
High Water Flow Rate Tests

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High Water Flow Rate Tests

- Friction factor of 1 too high, Blasius too low
  - $f=0.2$ best fit
- Nearest temperature recording slightly underpredicts temperature
  - Multidimensionality & scalloping
High Water Flow Rate Tests

- Ice input as initial constant temperature
- Radial numerical code used
  - Not significant time for latent heat to reach boundary to affect model
- Friction factor of 0.2 fits best
  - Darcy-Weisbach friction factor

- Model best at low time
  - Conduit expansion not uniform along length
  - Scalloping occurs

<table>
<thead>
<tr>
<th>Initial Ice Temperature (°C)</th>
<th>Critical Flow Rate (gpm)</th>
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<tr>
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Moody Diagram

Friction Factor

Material
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- Concrete, new smooth: 0.025
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- Glass, Plastic, Perspex: 0.0025
- Iron, cast: 0.15
- Sewers, old: 3.0
- Steel, mortar lined: 0.1
- Steel, lined: 0.5
- Steel, structural or forged: 0.025
- Water mains, old: 1.0

Transition Region

Complete turbulence

Friction Factor = \( \frac{2d}{\rho V^2 d \Delta P} \)

Smooth Pipe

Blasius Resistance Law

Reynolds Number, \( Re = \frac{\rho V d}{\mu} \)

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In Summary

Very good agreement between experiment and theory/numerical model

Very interesting insights on friction factor in englacial conduits even at this small laboratory scale

– under refreezing conditions, conduits behave like smooth conduits (Blasius Resistance law provides best fit)

– but under conditions of conduit growth, scalloping causes extremely high “friction factors” (f=0.2 > typical f used even for “rough” pipes)...confirms field observations reported in recent paper by Gulley et al. (2013) on roughness of englacial and subglacial conduits....scalloping is likely an inherent self-organized process resulting from interaction between turbulent eddies and melting (glacial karst) or dissolution (limestone karst)

Future Work: inclusion of creep closure effects