Parameterization of basal hydrology near grounding lines in a one-dimensional ice sheet model

Gunter Leguy, Xylar Asay-Davis, William Lipscomb

Los Alamos National Laboratory

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Introduction

We are exploring how basal physics influences resolution in a 1-D vertically integrated flowline model. Goals:

- Present a new parametrization of the basal shear stress (in the case of a Marine ice sheet):
  - Physically motivated (ocean connection)
  - Transitions smoothly between finite basal friction in the ice sheet and zero basal friction in the ice shelf
- Hope for $\approx 1$km resolution near the grounding line.
Valid in both, the ice sheet and the ice shelf:

### conservation of mass

\[ H_t + (uH)_x = a, \]

### vertically integrated stress – balance equation

\[ \tau_l + \tau_b + \tau_d = 0. \]

\[
\tau_l = \left[ 2\bar{A}^{-\frac{1}{n}} H|u_x|^{\frac{1}{n}-1}|u_x| \right]_x, \]

\[ \tau_d = -\rho_i g H s_x, \]

\[ \tau_b = \text{basal shear stress}, \]

\[ s = \begin{cases} 
    H - b & x < x_g \\
    \left(1 - \frac{\rho_i}{\rho_w}\right) H & x \geq x_g
\end{cases} \]
Assumption and Boundary conditions

Symmetric ice sheet at the ice divide

\[ u = 0 \]
\[ s_x = (H - b)_x = 0 \] at \( x = 0 \),

At the grounding line, we have the flotation condition and the balance between \( \tau_I \) and \( \tau_d \).

\[ H = \frac{\rho_w}{\rho_i} b, \]
\[ 2\bar{A}^{-\frac{1}{n}} |u_x|^{\frac{1}{n}-1} u_x = \frac{1}{2}\rho_i g \left( 1 - \frac{\rho_i}{\rho_w} \right) H \] at \( x = x_g \).
Basal stress

We adopt the formulation from Schoof (2005):

$$
\tau_b = - C |u|^{\frac{1}{n}} - u \left( \frac{N^n}{\kappa u + N^n} \right)^{\frac{1}{n}}.
$$

- $\kappa = \frac{m_{\text{max}}}{\lambda_{\text{max}}A_b}$
- $\lambda_{\text{max}} =$ wavelength of bedrock bumps
- $m_{\text{max}} =$ maximum bed obstacle slope
- $A_b =$ average ice temperature at the bed
- Effective pressure: $N \equiv p_i - p_w$
  - $p_i \equiv \rho_i g H$
  - $p_w$?
Basal stress (continued)

\[ N(p) = \rho_i g H \left( 1 - \frac{H_f}{H} \right)^p , \]

where \( H_f = \max(0, \frac{\rho_w}{\rho_i} b) \), and \( p \in [0, 1] \).

\( N(p) \) satisfies the following limits:

- When \( p = 0 \), \( N(p) = \rho_i g H \) (no water-pressure support).
- When \( p = 1 \), \( N(p) = \rho_i g (H - H_f) \) (full water-pressure support from the ocean wherever the ice-sheet base is below sea-level).
- At the grounding line when \( p > 0 \), \( N(p) = 0 \) (\( \tau_b \) is continuous across the grounding line).
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Basal stress (continued): \( \tau_b = -C|u|^{1/n-1}u \left( \frac{N(p)^n}{\kappa u + N(p)^n} \right)^{1/n} \).

- 2 asymptotic limits:
  - if \( \kappa u \ll N(p)^n \) and \( \tau_b \approx -\frac{\gamma}{\kappa^{1/n}}|u|^{1/n-1}u \) (frozen ice at the bed)
  - if \( \kappa u \gg N(p)^n \) and \( \tau_b \approx -CN(p) \frac{u}{|u|} \). (hydrological connection)

Transitions smoothly from non-zero basal stress in the ice sheet to zero basal sliding in the ice shelf.

Basal stress term (kPa)
Numerics

Chebyshev polynomials:
- Spectrally accurate
- GL lies on a grid point
- Used as a benchmark solution for fixed-grid model
- Harder to implement in 3-D models

Fixed-grid
- Suitable for 3-D model
- Constant resolution
- Numerically less accurate
- GL usually falls between two grid points leading to interpolation error
  - Possibility to use GLP
Numerics: Fix grid GLP

In our code we make sure that the grounding line is located in the last grounded cell.

- sub-grid-scale interpolation of the grounding line
- use a GLP similar to PA_GB1 in Gladstone et al. (20120a):
  1. Determine GL using linear interpolation of the function \( f \equiv \frac{H_f}{H} \)
  2. Compute basal and driving stresses once each assuming that the cell is entirely grounded and then entirely floating.
  3. The stresses are linearly interpolated between their grounded and floating values.

\[
\begin{align*}
H_{f,N-1}, H_{N-1} & \quad f = 1 \\
H_{f,N}, H_N & \quad \text{grounded: } f < 1 \quad \text{floating: } f > 1
\end{align*}
\]
How to compare our results for all p-values?

- **Qualitative**
  - Schoof (2007) boundary layer solution (in the case of frozen bed type basal sliding)

- **Quantitative**
  - Schoof (2007) boundary layer solution (good approximation for $p=0$)
  - Chebyshev polynomial numerical scheme
Experimental set-up:

- Fixed-grid resolution comparison: 3.2 km, 1.6 km and 0.8 km.
- Fixed-grid with no GLP and with GLP.
- We will show results for 3 values of $p$.
- Constant accumulation rate: $a = 0.3$ m/yr.
- MISMIP experiment: neutral equilibrium experiments.
- Two different bed topographies (as in Schoof (2007a)).
Results linear bed

The fixed-grid solution is inaccurate in capturing the retreat.

Grounding-line position (10^3 km)
p = 0, Res = 0.8 km

1/A
Results linear bed (continued): MISMIP-type experiments 1 & 2

Difference in grounding line position

<table>
<thead>
<tr>
<th>Res</th>
<th>No GLP</th>
<th>With GLP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8 km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference (km)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^25</td>
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<td></td>
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<tr>
<td>10^26</td>
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</tbody>
</table>

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### Results poly bed (continued): MISMIP-type experiment 3

#### Difference in grounding line position

<table>
<thead>
<tr>
<th>Res</th>
<th>Difference inifestyles (km)</th>
<th>No GLP</th>
<th>With GLP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Res = 0.8 km</td>
<td>-750, -400, -30, -20, -10, 0</td>
<td>-750, -400, -30, -20, -10, 0</td>
<td>-750, -400, -30, -20, -10, 0</td>
</tr>
<tr>
<td>Res = 1.6 km</td>
<td>-750, -400, -30, -20, -10, 0</td>
<td>-750, -400, -30, -20, -10, 0</td>
<td>-750, -400, -30, -20, -10, 0</td>
</tr>
<tr>
<td>Res = 3.2 km</td>
<td>-750, -400, -30, -20, -10, 0</td>
<td>-750, -400, -30, -20, -10, 0</td>
<td>-750, -400, -30, -20, -10, 0</td>
</tr>
</tbody>
</table>

- **Res** = resolution
- **GLP** = grounding line position
- **Difference** = difference in grounding line position

![Graph showing difference in grounding line position](image_url)
Results poly bed (continued): MISMIP-type experiment 3

Error estimate in grounding line position

No GLP

With GLP

Chebyshev GL position (10^3 km)
conclusion

- Error estimate decreases substantially as the value of $p$ increases and therefore the required model resolution decreases as $p$ increases.
- Adding a GLP is always beneficial besides sometimes for high values of $p$.
- $p$ does not play any role in the bulk of the ice sheet. It impacts a relatively small distance (no more than 20 km from the grounding line in our experiments) which is enough to impact the solution.
- Resolution of $\approx 1$km or coarser is sufficient to capture the solution.

**Paper submitted in the cryosphere**
Ratio $\kappa u/N(p=1)^n$ for values between 0.01 and 1
THANK YOU