Relating inverse-derived basal sliding coefficients beneath ice sheets to other large-scale variables

David Pollard
Pennsylvania State University

Robert DeConto
University of Massachusetts

Land Ice Working Group/CESM meeting
NCAR, February 14-15 2013
1. Deduce basal sliding coefficients $C(x,y)$ by simple model inversion
   - (like last year)

2. Don’t impose any constraints due to basal temperature or hydrology
   - (unlike last year)

3. Then compare $C(x,y)$ patterns with basal temperature, melt, topography
   - new parameterization for $C(x,y)$?

4. Fails…Why?
Common basal sliding laws in Antarctic-wide models

- Sliding velocity depends on basal shear stress, intrinsic bed conditions, and basal hydrology or temperature:

\[ u_b = C(x,y) N^q \tau_b^n \quad \text{or} \quad u_b = C(x,y) f(T_b) \tau_b^n \]

where

- \( u_b \) = basal ice velocity,
- \( \tau_b \) = basal shear stress,
- \( N \) = effective pressure,
- \( T_b \) = basal temperature,
- \( f(T_b) = 0 \) if bed is frozen, 1 if bed is at melt point

Blue: \( C = 10^{-10} \text{ m a}^{-1} \text{ Pa}^{-2} \)
Orange: \( C = 10^{-5} \text{ m a}^{-1} \text{ Pa}^{-2} \)

Crude \( C(x,y) \) map:
sediment if rebounded bed is below sea level, hard bedrock if above
Typical surface elevation (or thickness) errors

model minus observed:

- **Axiom of talk:**
  
  \[ u_b = C(x, y) f(T_b) \tau_b^n \]

- Golledge et al., PNAS, 2012
- Ritz et al., JGR, 2001
- Martin et al., The Cryo., 2011
- Pollard and DeConto, The Cryo., 2012
- Whitehouse et al., QSR, 2012
**Simple Inversion Method**

Very simple procedure to deduce basal sliding coefficients $C(x,y)$, fitting to observed ice surface elevations.

- Run model forward
- Every 2000 years, decrease (stiffen) $C(x,y)$ if the *local* ice surface is too low, or increase (soften) $C(x,y)$ if *local* surface is too high:
  - $C_{\text{new}} = C \times 10^{\Delta z / 2000}$
    - where $\Delta z$ = model – observed surface elevation (m)
  - Constrain $C$ to remain in range $10^{-15}$ to $10^{-4}$ m a$^{-1}$ Pa$^{-2}$
- Run model for ~100,000 years until convergence

*Ignores $\partial/\partial x$, $\partial/\partial y$’s….as if effects are local!*

*Ignores all other potentially canceling model errors!*
2 strategies in using the inverse method

We can write the sliding law either as

\[ u_b = C(x,y) \cdot f(T_b, \text{hydrol., topog., } \ldots) \cdot \tau_b \]

or as

\[ u_b = C'(x,y) \cdot \tau_b \]

Imagine that we know \( f(\ldots) \), and apply it during the inversion procedure, to deduce \( C(x,y) \) representing intrinsic bedrock properties.

Don’t apply \( f(\ldots) \) during inversion. Invert for \( C'(x,y) \). Then try to find a function \( f \) so that \( C' = C \cdot f \), i.e., \( f(\ldots) \approx 0 \) in regions with \( C' \approx 0 \), and \( f = 1 \) outside.

(last year’s talk, and The Cryo, 2012)
Results of inverse method, \textit{no} basal temperature constraint

- Purple regions are where sliding $\approx 0$
- Ideally, they correspond to frozen beds, or no basal water supply
- But they don’t correspond to $T_b < 0$
- Can we find a function $f(T_b, \text{topog., melt})$ that does?

Final elevation error $\Delta h_s$

Deduced sliding coefficients $C'$

Basal temperature $T_b$

$r = 0.109$
Attempt at $f(\ldots)$ using basal temperature and sub-grid bed roughness

$$C' = C(x,y) \, f(T_b, s)$$

"Sub-grid valley bottoms may still be unfrozen even if $T_b < 0$"

- But resulting "$f(T_b,s) \approx 0$" pattern does not resemble purple regions $C' \approx 0$.
- Main problem is that $T_b$ and $s$ both resemble large-scale bed topography.

$r = -0.006$
Attempt at $f(\ldots)$ using basal liquid supply (m/yr)

$$C' = C(x,y) f(B)$$

$B$ (m/yr) = basal liquid supply due to:
- melt (GHF+friction+conduction)
- percolation from surface

$log_{10}(f) = 3 + log B$

$\log_{10}(f) + \log_{10}(B) = \log_{10}(fB) = \log_{10}(f) + \log_{10}(B)$

Percolation from surface (m/yr)
Basal melt (m/yr)
Bed topography (Bedmap2)

- Again, resulting $f(B) \approx 0$ pattern does not resemble purple regions $C' \approx 0$
- Again, main problem is that $B$ resembles large-scale bed topography

$r = 0.018$
Why have these attempts at $f(\ldots)$ failed?

1) Incorrect internal deformation (mostly SIA) - incorrect enhancement factor $E$?
2) Incorrect longitudinal stress dynamics (hybrid model)
3) Basal hydrologic flow system (re-arranges $B$)
4) Geothermal Heat Flux distribution
**Why # 1: Results of inverse method, different enhancement factors $E$**

*Inverse with no basal temperature constraint on sliding*

- **Final elevation error $\Delta h_s$**
- **Deduced sliding coefficients $C'$**
- **Basal temperature $T_b$**

![Maps showing the results for different $E$ values](image)

- Increasing $E$ requires less sliding, more areas with $C' \approx 0$ (around EAIS flanks)
- None of the purple $C' \approx 0$ patterns resemble frozen-bed patterns $T_b < 0$
- Fabric? $E(x,y,z)$? Anisotropic models?
  
  e.g., Wang and Warner, Ann. Glac, 1999; Seddick et al., TC 2011
Why # 2: Incorrect longitudinal-stress dynamics (hybrid model)

- Many areas with $C' \approx 0$ (purple) are close to ice sheet margins
- Could be compensating for dynamical errors in hybrid model – too much internal shear flow near margins?
- Test with Full Stokes models
Why # 3: Basal hydrologic flow system (re-arranges B)

- Current model lacks basal hydrology
- Basal flow could transport water supply B, and produce patterns like purple \( C' > 0 \) regions (?)

A subglacial water-flow model for West Antarctica
A.M. Le Brocq, A.J. Payne, M.J. Siegert, R.B. Alley
J. Glaciol., 2009

Fig. 4. Relationship between expected basal sliding parameter and subglacial water depth: (a) subglacial water depth; (b) expected basal sliding parameter; (c, d) expected basal sliding parameter plotted against subglacial water depth, all cells where \( \Delta t > 15 \) mm.
Why # 4: Geothermal Heat Flux distribution

Geothermal heat flux (mW m\(^{-2}\)):

- Perhaps real GHF distribution has more structure, influencing basal melt
- Nb: Modern Siple coast is streaming, Wilkes basin outlet is not – due to high GHF and volcanism upstream of Siple? *

* Behrendt, GPC 2004; Blankenship et al., ARS 2001; Parizek et al., GRL 2002
But...regardless of basal physics...the only input to the model with fine structure are Bedmap2 elevation maps

Surface: \( \nabla^2 h_s = \partial^2 h_s / \partial x^2 + \partial^2 h_s / \partial y^2 \)

Bed: \( \nabla^2 h_b = \partial^2 h_b / \partial x^2 + \partial^2 h_b / \partial y^2 \)

Bed |slope|: \( \sqrt{ \left( \partial h_b / \partial x \right)^2 + \left( \partial h_b / \partial y \right)^2 } \)

cf. Plan curvature (Le Brocq et al., GRL 2008)

- Still no clear connection with \( C' \approx 0 \) (purple) patterns
- So where do the \( C' \approx 0 \) patterns come from in the model?
- Do they indicate any real physical process?
End
Results of inverse method, *no* basal temperature constraint

- Purple regions are where sliding $\approx 0$
- Ideally, they correspond to frozen beds, or no basal water supply
- But they don’t correspond to $T_b < 0$
- Can we find a function $f(T_b, \text{topog.}, \text{melt})$ that does?

Final elevation error $\Delta h_s$

Deduced sliding coefficients $C'$

Basal temperature $T_b$

$\text{Basal temperature, Pattyn, EPSL, 2010}$

$r = 0.109$
Constraining the internal-flow enhancement factor $E$

- If $E$ is too small, nearly all motion has to be basal sliding $\Rightarrow$ positive surface errors where base is frozen
- If $E$ is too large, too much internal flow. If it exceeds the balance velocity, $C$ inversion can’t help $\Rightarrow$ large ubiquitous negative surface errors
- Best results for $E \approx 1$
Results of inverse method, no $T_b$ effect, $E = 0.75 \times f(\text{distance to dome})$

Fabric, anisotropy, variable enhancement coefficients:

- Ren et al., 2011, JGR.
- $E = f(z)$: Mangeney and Califano, 1998, JGR
- Anisotropic models: Gillet-Chaulet et al., 2005, J. Glac. (GOLF law)
- Ma et al., 2010, J. Glac. $\rightarrow E$ (sheet vs. shelf).
- Seddick et al., 2011, The Cryo (CAFFE model)

$\Delta h_s$, $C(x,y)$, $T_b$, $r = 0.132$
Results of inverse method, with basal temperature constraint

$$u_b = C(x,y) f(T_b, s) \tau^n_b$$

where \( f(T_b) = 0 \) for frozen bed, ramps to 1 for bed at melt point, and width of ramp increases with sub-grid bed roughness \( s \)

- \( \Delta h_s \) over mountain ranges is improved by dependence on \( s \)
- But not completely – \( h_s \) still too high over mountains

Final elevation error \( \Delta h_s \)

Deduced sliding coefficients \( C(x,y) \)

Basal temperature \( T_b \)

Basal fraction unfrozen (0 to 1)
Plan curvature
Le Brocq et al., GRL, 2008
Previous basal inversions for Antarctica

• Previous work has deduced basal-stress or sliding-coefficient maps using control theory (Lagrangian multiplier/adjoint) methods, fitting modeled vs. observed velocities, with ice geometry (thickness, elevation) fixed from observations.


Ice Stream E (MacAyeal, 1992):

Pine Island and Thwaites Glaciers (Joughin et al., 2009; Morlinghem et al., 2010):

PISM (U. Alaska):

ISSM (JPL):
Relating inverse-derived basal sliding coefficients beneath ice sheets to other large-scale variables

David Pollard
Pennsylvania State University

Robert DeConto
University of Massachusetts

Land Ice Working Group/CESM meeting
NCAR, February 14-15 2013
Summary

• Simple inverse method “works”:
  (a) converges, (b) reduces surface elevation errors, (c) deduces reasonable $C(x,y)$ patterns.

• Independent of ice model. Just needs:
  (a) run for \(~200,000\) years, (b) bedrock parameter(s) that make $u_b$ increase or decrease.

• **BUT** some of the deduced $C(x,y)$ *must* be due to other model errors, not real bed conditions.
  Lesser of two evils: cancelling errors vs. $O(500\text{m})$ biases in surface elevation

• Next steps:
  - Combine with large-ensemble techniques?  (Stone et al., The Cryo. 2010; Tarasov et al., EPSL, 2011)
  - Apply to last deglaciation  (Briggs et al., ISAES abs., 2011.; Whitehouse et al., QSR, 2012)