Parameter estimation for grounding-line transition

Gunter Leguy, Xylar Asay-Davis, William Lipscomb

Los Alamos National Laboratory

February 15th, 2013
Introduction

We are exploring how basal physics influences resolution in a 1-D vertically integrated flowline model.

Several models of the form $\tau_b = \beta^2 u$ have been investigated:

- Schoof (2007): $\beta^2 = - C |u|^\frac{1}{n-1}$
- Pattyn (2006): $\beta^2 = - \exp[\beta_0 (x_g - x)]$

Goals:

- Present a new parametrization of the basal shear stress (in the case of a Marine ice sheet):
  - Physically motivated (ocean connection)
  - Transitions smoothly between finite basal friction in the ice sheet and zero basal friction in the ice shelf
- Hope for $\approx 1$km resolution near the grounding line.
Model equations

Valid in both, the ice sheet and the ice shelf:

conservation of mass: \( H_t + (uH)_x = a \),
stress – balance equation: \( \tau_I + \tau_b + \tau_I = 0 \).

\[
\tau_I = \left[ 2\bar{A}^{-\frac{1}{n}} H |u_x|^{\frac{1}{n}-1} |u_x| \right]_x,
\]
\[
\tau_d = - \rho_i g H s_x,
\]
\[
\tau_b = \text{basal shear stress}.
\]
Assumption and Boundary conditions

Symmetric ice sheet at the ice divide

\[
\begin{align*}
  u &= 0 \\
  s_x = (H - b)_x &= 0
\end{align*}
\] at \( x = 0 \),

At the grounding line, we have the flotation condition and the balance between \( \tau_I \) and \( \tau_d \).

\[
\begin{align*}
  H &= \frac{\rho_w}{\rho_i} b, \\
  2A^{-\frac{1}{n}} |u_x|^\frac{1}{n-1} u_x &= \frac{1}{2} \rho_i g \left( 1 - \frac{\rho_i}{\rho_w} \right) H
\end{align*}
\] at \( x = x_g \).
Basal stress

We adopt the formulation from Schoof (2005):

$$
\tau_b = -\gamma N \left( \frac{u}{u + \frac{\lambda_{\text{max}}}{m_{\text{max}}} A N^n} \right)^{\frac{1}{n}}.
$$

- $\lambda_{\text{max}}$ = wavelength of bedrock bumps
- $m_{\text{max}}$ = maximum bed obstacle slope
- Effective pressure: $N \equiv p_i - p_w$
  - $p_i \equiv \rho_i g H$
  - $p_w$?
Basal stress (continued)

\[ N(p) = \eta(p; H, b) \rho_i g H, \quad \eta \text{ non-dimensional function} \]

- \( \eta \in [0, 1] \)
- \( p \in [0, 1] \)

We want the following limit for \( \eta \):

- When \( p = 0 \), \( \eta = 1 \): no water-pressure support.
- When \( p = 1 \), \( \eta = 0 \): subglacial water has a free connection to the ocean, Paterson(2010)
- At the grounding line, \( \eta = 0 \) (effective pressure goes to zero to allow continuity).

One expression with the right limit:

\[ N(p) = \rho_i g H \left( 1 - \frac{H_f}{H} \right)^p, \quad \text{where} \]

\[ H_f = \max(0, \frac{\rho_w}{\rho_i} b) \]
Basal stress (continued)

- Right limits:
  - if $u \ll \kappa N(p)^n$ and $\tau_b \approx -\frac{\gamma}{\kappa^{1/n}} |u|^{1/n-1} u$ (frozen ice at the bed)
  - if $u \gg \kappa N(p)^n$ and $\tau_b \approx -CN(p)\frac{u}{|u|}$. (hydrological connection)

- Physically motivated
- Transitions smoothly from non-zero basal stress in the ice sheet to zero basal sliding in the ice shelf

![Graph showing basal stress term (kPa) vs. Distance to grounding line (km)]

- a) $p = 0$
- b) $p = 1$
Numerics

Moving-grid:
- Numerically more accurate
- GL must lie on a grid point
- Could be used as a benchmark solution for fixed-grid model
- Harder to implement in 3-D models

Fixed-grid
- Suitable for 3-D model
- Constant resolution
- Numerically less accurate
- GL usually falls between two grid points leading to interpolation error
How to compare our results for all p-values?

**Qualitative**
- Schoof (2007) boundary layer solution (in the case of frozen bed type basal sliding)

**Quantitative**
- Schoof (2007) boundary layer solution (good approximation for $p=0$)
- Chebyshev polynomial numerical scheme
- Moving grid with high enough resolution used as a benchmark for fixed-grid model
Numerics (end)

Experimental set-up:
- Moving grid resolution benchmark: $156 \text{m} \leq \text{Res} \leq 316 \text{m}$.
- Fixed-grid resolution comparison: 1.6 km and 0.8 km.
- We will show results for 3 values of $p$.
- Constant accumulation rate: $a = 0.3 \text{ m/yr}$.
- MISMIP experiment: neutral equilibrium experiments.
- Two different bed topographies (as in Schoof (2007a)).
Results linear bed

The fixed-grid solution is inaccurate in capturing the retreat.

Figure:
LANL (Los Alamos National Laboratory) Parameter estimation for grounding-line trans
February 15th, 2013 11 / 20
Results linear bed (continued)

MISMIP-type experiments 1 & 2

Ice sheet domain (10³ km)

Grounding line position (10³ km)

p = 0
p = 0.5
p = 1

Ice sheet surface elevation (km)

Res = 0.8 km

fix
moving (advance)
moving (retreat)
fix (advance)
fix (retreat)
Results linear bed (continued)

MISMIP-type experiments 1 & 2 (note the different scales for the different $p$-values)

Grounding line position error estimate

- $p=0$
  - $Res = 1.6 \text{ km}$
  - $Res = 0.8 \text{ km}$

- $p=0.5$

- $p=1$

Error estimate (km)

1/A

LANL (Los Alamos National Laboratory)  Parameter estimation for grounding-line trans...
Results linear bed (end)

MISMIP-type experiments 1 & 2 (note the different scales for the different $p$-values)

SLR volume error estimate

<table>
<thead>
<tr>
<th>$p$</th>
<th>1/A</th>
<th>Res (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>1.6 km</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>0.8 km</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Element of comparison: 2850 km$^3 \approx 7.2$ mm.
Results poly bed
MISMIP-type experiment 3

Ice sheet domain (10^3 km)

Grounding line position (10^3 km)

p=0
p=0.5
p=1

Ice sheet surface elevation (km)

Res = 0.8 km

fixed
moving
moving advance
moving retreat
fixed advance
fixed retreat

LANL (Los Alamos National Laboratory)  Parameter estimation for grounding-line trans...
Results poly bed (continued)

MISMIP-type experiment 3 (note the different scales for the different $p$-values)

Grounding line position error estimate

- $p=0$: Res = 1.6 km
- $p=0.5$: Res = 0.8 km
- $p=1$

LANL (Los Alamos National Laboratory)
Parameter estimation for grounding-line transition
February 15th, 2013
Results poly bed (continued)

MISMIP-type experiment 3 (note the different scales for the different $p$-values)

SLR volume error estimate

- $p=0$
  - Res = 1.6 km
  - Res = 0.8 km

- $p=0.5$

- $p=1$

LANL (Los Alamos National Laboratory)
For low p-values the need of high resolution in the vicinity of the GL remains.

Error estimate decreases substantially as the value of p increases and therefore the required model resolution decreases as p increases.

p does not play any role in the bulk of the ice sheet. It impacts a relatively small distance (no more than 25 km from the grounding line in our experiments) which is enough to impact the solution.

With $p \approx 1$, a resolution of $\approx 1$km is sufficient to capture the solution.
Future work

- 3-D implementation
- Add the missing physics (lateral drag, buttressing)
- Data comparison for admissible values of $p$ (any value besides 0 and 1 valid?)
"The winters are nothing like the old days!"