Data assimilation and prognostic whole ice sheet modelling with the variationally derived, higher order, open source, and fully parallel ice sheet model VarGlaS

Douglas J. Brinkerhoff and Jesse V. Johnson

Department of Computer Science
University of Montana, Missoula

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Capabilities.

- Stokes’ and first-order.
- Enthalpy balance.
- Advanced time stepping.
- Data assimilation.

Features.

- Parallel.
- Built on open-source FEM platform FEniCS.
- Mesh refinement.
Better living through the calculus of variations.

\[ \dot{V} = -\rho g \cdot u \] Potential energy of ice.

\[ \dot{D} = \frac{2n}{n+1} \eta (\dot{\epsilon}^2) \dot{\epsilon}^2 \] Conversion to heat through internal friction.

\[ \dot{F} = \beta^2 h^r \mathbf{u} \cdot \mathbf{u} \] Conversion to heat through basal friction.

\[ \mathbf{I}_c = P(\nabla \cdot \mathbf{u}) \] Incompressibility.

\[ \mathbf{I}_p = P(\mathbf{u} \cdot \mathbf{n}) \] Impenetrability.

The Stokes’ problem

\[
\min \int_\Omega \dot{V} + \dot{D} + \mathbf{I}_{c} \, d\Omega + \int_{\Gamma_B} \dot{F} + \mathbf{I}_{p} \, d\Gamma \tag{1}
\]

The FO problem

\[
\min \int_\Omega \dot{V}_1 + \dot{D}_1 + \mathbf{I}_{c} \, d\Omega + \int_{\Gamma_B} \dot{F}_1 + \mathbf{I}_{p} \, d\Gamma \tag{2}
\]
Better living through the calculus of variations.

\[ \dot{V} = -\rho g \cdot u \] Potential energy of ice.

\[ \dot{D} = \frac{2n}{n+1} \eta (\dot{\epsilon}^2) \dot{\epsilon}^2 \] Conversion to heat through internal friction.

\[ \dot{F} = \beta^2 h^r u \cdot u \] Conversion to heat through basal friction.

\[ I_c = P(\nabla \cdot u) \] Incompressibility.

\[ I_p = P(u \cdot n) \] Impenetrability.

The Stokes’ problem

\[ \min \int_{\Omega} \dot{V} + \dot{D} + I_c \, d\Omega + \int_{\Gamma_B} \dot{F} + I_p \, d\Gamma \] (1)

The FO problem

\[ \min \int_{\Omega} \dot{V}_1 + \dot{D}_1 + I_c \, d\Omega + \int_{\Gamma_B} \dot{F}_1 + I_p \, d\Gamma \] (2)
Newton’s method

\[ \delta A(u) = 0 \]  \hspace{1cm} (3)

\[ \delta^2 A \Delta u = -\delta A \]  \hspace{1cm} (4)

solvers and efficiency.

- Currently using gmres and hypre-euclid (ILU) for Stokes’.
- Currently using gmres and hypre-amg (AMG) for FO.
- Ongoing research into what makes an efficient solver here.
ISMIP-HOM A

ISMIP-HOM Experiment A

Ice Speed (m/a)

5 km

10 km

20 km

40 km

80 km

160 km

Normalized X coordinate

FOA Output
Stokes Output
Full Stokes Mean
Full Stokes Std. Dev.
First Order Mean
First Order Std. Dev.

The University of Montana
Enthalpy versus temperature.

\[ \rho(\partial_t + \mathbf{u} \cdot \nabla)H = \rho \nabla \cdot \kappa(H) \nabla H + Q \quad (5) \]

\[ \kappa(H) = \begin{cases} \frac{k}{\rho C_p} & \text{if cold} \\ \frac{\nu}{\rho} & \text{if temperate,} \end{cases} \quad (6) \]
▶ Easy to implement.
▶ Yields water content.
▶ Avoids contact problem.
▶ Self consistent boundary conditions.

\[ \rho \kappa (H) \nabla H \cdot n = q_{geo} + \dot{F} - \frac{\rho m_b}{L} \]  

(7)
Dynamic boundaries

\[ \frac{\partial S}{\partial t} = \dot{a} + \mathbf{u} \cdot \mathbf{n} \quad (8) \]

Implementation.

- Total Variation Diminishing Runge-Kutta (TVD-RK).
- Shock capturing.
- Streamline upwind Petrov-Galerkin (SUPG).
ISMIP-HOM Experiment F

No Slip

Slip

Surface elevation (m)

Surface speed (m/a)

Normalized X coordinate

- FOA Output
- Stokes Output
- Full Stokes Mean
- Full Stokes Std. Dev.
- First Order Mean
- First Order Std. Dev.
EISMINT-II

T w.r.t. PMP (K)

Thickness (m)

EISMINT II A

EISMINT II F
Data assimilation.

\[ u \text{ Model variable.} \]
\[ d \text{ Data.} \]
\[ p \text{ Unknown parameter.} \]
\[ \mathcal{I} \text{ Objective (Cost) functional.} \]
\[ \delta \mathcal{A} \text{ Forward model.} \]
\[ \lambda \text{ Lagrange multiplier (Adjoint variable).} \]

The problem

\[ \min \mathcal{F} = \min \int_{\Omega} \mathcal{I}(u, d, p) \, d\Omega + \int_{\Omega} \lambda \delta \mathcal{A} \, d\Omega \]
Inverting ISMIP-HOM C.
Mesh refinement.

\[ e(c) \propto \max_{i \in E} x_i^T M x_i \]  

(9)

\[ M = V^T |\Lambda| V. \]  

(10)
Parallelism.
Inverting  *Joughin 2007-2008 two year average velocity*

**Observed velocity 2007-2008**

**Modeled velocity**
Greenland steady state, fixed geometry.

Surface Velocity (m/a)

$\beta^2$ (Pa s/m$^2$)

T w.r.t. PMP (°C)
500 years with constant climate.
Mass through time.
Next steps.

- Marine margin treatment (The equality constrained variational principle become *inequality* constrained).
- Adaptive mesh refinement.
- Mass continuity initialization.
How to try your own copy.

- Install FEniCS 1.1.0 stable
- bzr branch lp:up-feism
- Comment on The Cryosphere Discuss.