A Simple Inverse Method for Deducing the Large-Scale Distribution of Basal Sliding Coefficients beneath the Antarctic Ice Sheet

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Outline

1. Motivation, previous work
2. Simple inverse method # 1
3. Simple inverse method # 2 (with basal temperature)
4. Comparison with previous work
5. Summary
**Motivation**

- Ice sheet geometry is sensitive to basal boundary conditions, mainly deformable sediment ($C=10^{-5}$) vs. hard bedrock ($C=10^{-10}$)

  \[ u_b = C(x,y) f(T_b) \tau_b^2 \]

- Primary cause of $O(500 \text{ m})$ elevation errors in Antarctic continental paleo ice-sheet models?

where

- $u_b = \text{basal ice velocity}$,
- $\tau_b = \text{basal shear stress}$,
- $T_b = \text{basal temperature}$,
- $f(T_b) = 0$ if bed is frozen,
- $1$ if bed is at melt point
Typical surface elevation or thickness errors
In continental (paleo) Antarctic models

Modern surface elevations, model minus observed
Previous basal inversions for Antarctica

- Previous work has deduced basal-stress or sliding-coefficient maps using control theory (Lagrangian multiplier/adjoint) methods, fitting modeled vs. observed velocities, with ice geometry (thickness, elevation) fixed from observations.

- **Regional:** MacAyeal, 1992; Vieli and Payne, 2003; Joughin et al. 2009; Morlighem et al., 2010.
- **Continental:** ISSM, Larour et al., ISSM, issm.jpl.nasa.gov; Bueler et al., PISM, www.pism-docs.org.
  Also Price et al. (PNAS, 2011), Greenland, local method.

Ice Stream E (MacAyeal, 1992):

Pine Island and Thwaites Glaciers (Joughin et al., 2009; Morlinghem et al., 2010):

PISM (U. Alaska):

ISSM (JPL):

**Basal stress, Pine Island and Thwaites Glaciers.** Joughin et al., J. Glac., 2009

**Basal stress, Pine Island Gl:** Morlighem et al., GRL, 2010

**PISM basal drag coefficient (Pa s m⁻¹).** Lingle et al., JPL PARCA meeting, 2007

**ISSM basal drag coefficient (ms⁻¹/²).** Larour et al., JPL PARCA meeting, 2009

The Inversion Method

- Very simple procedure to deduce basal sliding coefficients $C(x,y)$, fitting to observed ice geometry (surface elevation).
- Run model forward, and every 5 kyrs adjust $C$ locally depending on current ice surface elevation mismatch with observed.

Details:
- Every 5000 years, decrease (stiffen) $C(x,y)$ if the local ice surface is too low, or increase (soften) $C(x,y)$ if local surface is too high:
  - $C_{\text{new}} = C \times 10^{-\Delta z / 500}$
    where $\Delta z =$ model – observed surface elevation (m)
  - Constrain $C$ to remain in range $10^{-20}$ to $10^{-5}$ m a$^{-1}$ Pa$^{-2}$
- Run model forward for ~200,000 years until convergence

Ignore $\partial / \partial x, \partial / \partial y$'s....as if effects are local

Ignore all other potentially canceling model errors! (e.g. internal deformation $\partial u / \partial z$)

Ignore GIA - assume modern ice sheet is in equilibrium
Spinup in a 400,000 year run. The method converges!

Model surface elevations minus observed
Results of Method # 1 (no basal temperature effect)

- Turn off effect of basal temperature on sliding
- Allow minimum \( C = 10^{-20} \), so inverse procedure can find "frozen" (stuck) areas

- But when run full model with \( C(x,y) \) prescribed, frozen areas differ from inverse-deduced stuck areas.
- Large surface elevation errors re-occur.
Results of Method # 2 (with basal temperature effect)

- $T_b$ affects sliding during inversion procedure
- Minimum $C = \log_{10} (m \text{ a}^{-1} \text{ Pa}^{-2}) = 10^{-10}$ (hard bedrock)
- Guarantees same results when full model is run with resulting $C(x,y)$ prescribed

- Remaining errors over mountain ranges
- Reduce further by modifying $f(T_b)$ in sliding law, using sub-grid slope amplitude $s.a.$

$u_b = C(x,y) f(T_b, s.a) \tau_b^2$
Different grid resolutions: results are ~unchanged

<table>
<thead>
<tr>
<th>Grid Resolution</th>
<th>Maps</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 km (2nd + s.a.)</td>
<td><img src="image1" alt="Map" /> <img src="image2" alt="Map" /> <img src="image3" alt="Map" /></td>
</tr>
<tr>
<td>20 km (2nd + s.a.)</td>
<td><img src="image4" alt="Map" /> <img src="image5" alt="Map" /> <img src="image6" alt="Map" /></td>
</tr>
<tr>
<td>10 km nested (2nd + s.a.)</td>
<td><img src="image7" alt="Map" /> <img src="image8" alt="Map" /> <img src="image9" alt="Map" /></td>
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</tbody>
</table>

- **Δh_s**
- **C(x,y)**
- **T_b**
Comparison with previous basal inversions

- Previous studies have fitted model to observed velocities, with ice geometry fixed from observations.
- The method here fits model to observed ice geometry, with no constraints on velocities.
- In principle, this should yield ~same results for $C(x,y)$, due to unique relationship between surface mass balance, ice thickness and balance velocity.
Summary

• Simple inverse method “works”:
  (a) converges, (b) reduces surface elevation errors, (c) deduces reasonable $C(x,y)$ patterns.

• Independent of ice model. Just needs:
  (a) run for ~200,000 years, (b) bedrock parameter(s) that make $u_b$ increase or decrease.

• **BUT** some of the deduced $C(x,y)$ must be due to other model errors, not real bed conditions.
  Lesser of two evils: cancelling errors vs. $O(500m)$ biases in surface elevation

• Next steps:
  - Combine with large-ensemble techniques? (Stone et al., The Cryo. 2010; Tarasov et al., EPSL, 2011)
  - Apply to last deglaciation (Briggs et al., ISAES abs., 2011.; Whitehouse et al., QSR, 2012)