Progress in modeling glacier hydrology

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\[ \approx 1 \text{ m} \]

\[ \approx 1 \text{ m s}^{-1} \]
Introduction & background

Hydrology and dynamics are linked in alpine glaciers ...

Change in subglacially stored water, Kennicott Glacier, Alaska, 29 Jun–3 Jul, 2006 (Bartholomäus et al., 2008)
Introduction & background

... and in the continental ice sheets

Subglacial lakes and active drainage systems in Antarctica (Bell, 2008)
Introduction & background

Basal effective pressure, and hence basal water pressure (over the relevant length scales), is a key link between hydrology and dynamics.

\[ N = P_i - P_w \]

Left: glacier surface speed vs. borehole water level at Findelengletscher, 1980-82 (Iken and Bindschadler, 1986)
Introduction & background

Subglacial water pressure, Trapridge Glacier

"Slow" systems: $P_w \uparrow \mu_b \uparrow$

Linked cavities

Pore flow

"Fast" systems: $P_w \downarrow \mu_b \downarrow$

R-channel

N-channel

Diagrams courtesy of T. Creyts
Comprehensive modeling efforts (Arnold et al. 1998)

- spatially fixed, temporally evolving conduit network
- slow system approximated as small or wide conduits
- slow-to-fast transition prescribed as snowline passes moulins
- surface melt (calculated from energy balance) routed to moulins
- simulations performed with EPA storm water management model
Previous work: 2.5-D multicomponent modeling

Glacier drainage systems
1. Supraglacial
2. Englacial
3. Subglacial
4. Subsurface

For each system (in 2-D plan view):

\[ h = \text{fluid volume} \ [\text{L}] \]

\[ K(h) = \text{system conductivity} \ [\text{L/T}] \]

\[ \psi(h) = \text{fluid potential} \ [\text{M/LT}^2] \]

\[ Q(K, h, \nabla \psi) = \text{fluid flux} \ [\text{L}^2/\text{T}] \]
Previous work: 2.5-D multicomponent modeling

Example: subglacial drainage (3)

\[ h = \text{fluid volume \ [L]} \]

\( h \) is an areally-averaged water volume and may depend on the effective porosity or configuration of the subglacial drainage system

Diagram (left) courtesy of T. Creyts
Previous work: 2.5-D multicomponent modeling

Example: subglacial drainage (3)

\[ K(h) = \text{system conductivity} \ [L/T] \]

Hydraulic conductivity, \( K(h) \), is a measure of subglacial hydraulic “connectivity”, and can be used to emulate a transition between fast and slow drainage systems.
Example: subglacial drainage (3)

\[ \psi(h) = \text{fluid potential} \quad [\text{M/LT}^2] \]

\[ \psi = p + \rho_w g \ z_b \]

Fluid potential depends on subglacial water pressure, \( p(h) \), which depends on character of the glacier bed.

Diagrams (lower left) courtesy of T. Creyts
Example: subglacial drainage (3)

\[ Q(K, h, \nabla \psi) = \text{fluid flux} \ [L^2/T] \]

\[ Q = - \frac{K(h) h}{\rho_w g} \nabla \psi, \]

Fluid flux is described by a non-linear form of Darcy’s Law
Mass conservation in each drainage system:

1. Supraglacial

\[ \frac{\partial h^r}{\partial t} + \frac{\partial Q_j^r}{\partial x_j} = M + R - \phi^{r:e} - \phi^{r:a} \]

2. Englacial

\[ \frac{\partial h^e}{\partial t} + \frac{\partial Q_j^e}{\partial x_j} = \phi^{r:e} - \phi^{e:s} \]

3. Subglacial

\[ \frac{\partial h^s}{\partial t} + \frac{\partial Q_j^s}{\partial x_j} = b^s + \phi^{e:s} - \phi^{s:a} \]

4. Subsurface

\[ \left( \frac{h^a}{\rho^a} \right) \frac{\partial \rho^a}{\partial t} + \frac{\partial h^a}{\partial t} + \frac{\partial Q_j^a}{\partial x_j} = \phi^{s:a} + \phi^{r:a} \]

Flowers and Clarke, 2002
Previous work: 2.5-D multicomponent modeling

This simple model can reproduce various qualitative features of borehole water pressure records.

Subglacial water pressure data from Trapridge Glacier, Yukon Territory, 9-23 July 1997

Flowers and Clarke, 2002
Previous work: coupling hydrology and dynamics

Parameterization of basal sliding including hydrology

$$u_b = C \tau_b \frac{P_w}{P_i}$$

This implementation of hydrology can enhance or reduce sliding, as opposed to a parameterization based on surface melt volume.

Marshall et al., 2005
## Previous work: 2.5-D multicomponent modeling

### Pros:
- Harmonized treatment of each drainage system (model layer)
- Description of each system tied loosely to system morphology
- Parameterized vertical coupling replaces prescribed vertical fluxes or full 3-D model
- Explicit description of each system potentially allows more objective simulation of observed behavior
- Fast and slow subglacial drainage systems emulated with extreme simplicity at grid scale

### Cons:
- Description of each system tied loosely to system morphology
- Physics of subgrid channelized drainage missing
- Simple treatment of subglacial drainage system requires prescribed relationship between basal water volume & pressure
- Explicit description of each system introduces more parameters, necessitating more data for model calibration
- Ice dynamics absent from description of subglacial system
Subglacial drainage morphology

“Fast” system

“Slow” system

\[ K(h) = \text{system conductivity [L/T]} \]

\[ \log(K) \]

\[ h \]

Diagrams courtesy of T. Creyts
Subglacial drainage morphology

Conduit in Kötlujökull, Iceland (Näslund and Hassinen, 1996)
Two-component flowband model of basal hydrology

Flowers, 2008
Flowband model description: hydrology

Water balance (continuity):

$$\frac{\partial h_s}{\partial t} + \frac{\partial Q_{sx}}{\partial x} = b_s - \phi^{s:c}$$

Water flux:

$$Q_{sx} = -\frac{K_s}{\rho_w g} h_s \frac{\partial \psi_s}{\partial x}$$

Fluid potential:

$$\psi_s = p_s + \rho_w g z$$

Basal water pressure:

$$p_s = p_s(h_s)$$

function of bed character, geometry

$h_s$ = effective water-sheet thickness
$Q_{sx}$ = water flux
$b_s$ = source term
$\phi_{s:c}$ = water exchange term
$K_s$ = hydraulic conductivity
$\psi_s$ = fluid potential
$p_s$ = basal water pressure
$t$ = time
$x$ = horizontal position
$\rho_w$ = density of water
$g$ = gravitational acceleration
Flowband model description: hydrology

Conservation of mass:

\[
\frac{\partial S}{\partial t} = - \frac{Q_{cx}}{\rho_i L} \left( \frac{\partial \psi_c}{\partial x} - c_t \rho_w c_w \frac{\partial p_c}{\partial x} \right) - 2S \left( \frac{p_i - p_c}{nB} \right)^n
\]

Conduit discharge:

\[
Q_{cx} = - \left( \frac{8S^3}{P_w \rho_w f_R} \right)^{1/2} \frac{\partial_x \psi_c}{|\partial_x \psi_c|^{1/2}}
\]

Sheet-conduit water exchange:

\[
\phi_{s:c} = \chi_{s:c} \frac{K_{s:c} h_{s:c}}{\rho_w g d_c^2} (p_s - p_c)
\]

- \(S\) = conduit cross-sectional area
- \(Q_{sc}\) = conduit discharge
- \(f_R\) = friction coefficient
- \(P_w\) = conduit wetted perimeter
- \(n\) = flow law exponent
- \(B\) = flow law coefficient
- \(\psi_c\) = conduit fluid potential
- \(p_s\) = basal water pressure
- \(p_c\) = conduit water pressure
- \(L\) = latent heat of fusion
- \(c_t\) = pressure melting coefficient
- \(c_w\) = heat capacity of water
Flowband model description: hydrology

Conservation of mass:

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Sheet-conduit water exchange:

\[ \phi_{s:c} = \chi_{s:c} \frac{K_{s:c}}{\rho_w g d_c^2} h_{s:c} (p_s - p_c) \]

Allows representation of parallel, non-interacting conduits, given a conduit density per unit width \( d_c \)
Simulated seasonal evolution of glacier hydrology

Prescribed: annual & diurnal sinusoidal variations in water input for an idealized glacier geometry

Coupling to ice dynamics described tomorrow by Sam Pimentel

Flowers, 2008
Final comments and outlook

• Details of the subgrid physics are important in glacier hydrology and have significant implications for ice dynamics: they (or their effects) must be parameterized or described in a fashion that can be implemented in current continuum models.

• May be worth investigating statistical descriptions of subgrid conduit networks for large-scale modeling.

• Neglecting short-term transient events in the drainage system probably leads to an underestimation of the influence of hydrology, thus asynchronous coupling with steady-state hydrology may not be the best method of coupling with ice dynamics.

• Oversimplified parameterizations of the effects of basal hydrology (e.g. sliding proportional to degree-days) can produce behavior inconsistent with well-established physics and should probably be avoided.

• How can we effectively use data to increase the validity of these models? What data would be most appropriate?