Towards a 2D, Computationally Light, Single-Head Ice Sheet Hydrology Model

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CESM Land Ice Working Group Winter Meeting
Study Site and Data: Sermeq Avannarleq Flowline

1. Thomsen and Olsen (1991)
2. Iken et al. (1993)
3. Lüthi et al. (2002)

(Joughin et al., 2010)
Empirical Motivation: InSAR 2005/2006 Velocities

Pursuing a hydrology model which can be used to explain velocity variations…

(Joughin et al., 2010)
Empirical Motivation: GPS 1996/2008 Velocities

(Jay Zwally, per. comm.)
Theoretical Motivation: “Alpine” Sliding Model

Variations in sliding velocity are due to changes in water storage over time (i.e. \( \frac{dS}{dt} \))…

\[
\frac{dS}{dt} = \text{inputs} - \text{outputs}
\]

\[+VE \frac{dS}{dt} = \text{sliding}\]

\[-VE \frac{dS}{dt} = \text{no sliding}\]

…where output rate is dominated by conduit efficiency.

\[(Bartholomaus \ et \ al.,\ 2007)\]
Single-Head Hydrology Model (1D): Overview

**Water Storage:**
1. Ice aquifer \((S_i)\)
2. Conduits \((S_c)\)

\[ S = S_i + S_c \]

**Water Mass Conservation:**
1. External input \((Iw)\)
2. Internal melt \((m/\rho_w)\)
3. Horizontal divergence \((dQ/dx)\)
4. Change in conduit storage \((dS_c/dt)\)

\[
\frac{dS}{dt} = \varphi W \frac{\partial h_e}{\partial t} = Iw + \frac{m}{\rho_w} - \frac{\partial Q}{\partial x} \frac{\partial S_c}{\partial t}
\]
Single-Head Hydrology Model (1D): Conduits

Conduit geometry:

\[ S_c = \frac{\pi r^2}{2} (n_c w) \]

Conduit Mass Conservation:
1. Internal Melt \((m/\rho_w)\) +VE
2. Deformation (…) –/+ VE

\[
\frac{dS_c}{dt} = \frac{m_i}{\rho_i} - 2A \left( \frac{\pi r^2}{2} \right) \cdot (n_c w) \cdot \left| \frac{P_i - P_w}{n} \right|^n \cdot \text{sign}(P_i - P_w)
\]
1D Results: Animation

Water input (surface & bed)
Englacial water table elevation
Subglacial conduit radius
Rate of change in englacial water elevation
1D Results: Animation

VIEW ANIMATION
1D Results: Stability and Residence Time

Spin-up to stable state in < 10 years…

Mean residence time ($t_{res}$) varies between 1.1 and 3.3 years (depending on bulk ice porosity; $\phi$)

$$t_{res} = \frac{\sum S}{\sum (Iw)}$$
1D Results: Flotation Fraction ($P_w/P_i$)

Entire flowline annually oscillates close to flotation ($P_w/P_i = 1$)… consistent with *in situ* observations.

Annual mean and minimum are not sensitive to choice of bulk ice porosity ($\phi$)… but annual maximum is.

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CIRES
1D Results: Changes in Water Storage Over Time \( (dS/\text{dt}) \)

Support for summer speedup during \(+\text{VE} dS/\text{dt}\) and fall slowdown during \(-\text{VE} dS/\text{dt}\)…

\[ \frac{dS}{dt} = w \phi \frac{dh_e}{dt} \]

(…with strong 1D artifacts).
Aside: “Perennial” Conduits

Portions of the conduit system may "overwinter" to be "reactivated" the following melt season. “New" system may not have to migrate upglacier from the terminus each melt season.
Single-Head Hydrology Model (2D): Overview

Water Mass Conservation:

\[ \varphi \cdot \partial x \cdot \partial y \cdot \frac{\partial h_e}{\partial t} = I \cdot \partial x \cdot \partial y + \frac{\dot{m}}{\rho_w} - \frac{\partial Q}{\partial x} \frac{\partial Q}{\partial y} - \frac{\partial \bar{b}}{\partial t} \]

Sheet-type flow: \( \bar{b} \) becomes the mean “gap” width at the ice-bed interface… higher order conduit geometry not required.
Plan: Implement transient equations that describe flow in the center of a *wide* conduit to approximate “gap” flow…

\[ \frac{\partial h}{\partial t} = \frac{m}{\rho_i} - \hat{w}, \quad \hat{w} = \frac{N_c}{2\eta_i} \sqrt{l^2 - x^2}. \]

\[ (Ng, 1999) \]
For now $\frac{dB}{dt} = 0$...
2D Sample Region Output: Animation

Model Time [a]: 0.02

Water input (surface only) [cm/d]

Margine

Inland extent of temperate bed

Englacial water (or bedrock) elevation [m]
2D Sample Region Output: Animation

VIEW
ANIMATION
## Computational Efficiency (Single Processor)

<table>
<thead>
<tr>
<th></th>
<th>1D (Flowline)</th>
<th>2D (Test Region)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical Method</td>
<td>ode15s</td>
<td>1\textsuperscript{st} iteration Piccard</td>
</tr>
<tr>
<td>Transients (/node)</td>
<td>$h_E$ and $S_c$</td>
<td>$h_E$ and $\bar{b}$</td>
</tr>
<tr>
<td>Unknowns (/node)</td>
<td>47</td>
<td>46 (est.)</td>
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<tr>
<td>Constants</td>
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<td>~ 50</td>
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<td>Nodes</td>
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<td>5000</td>
</tr>
<tr>
<td>$dt$</td>
<td>1 d</td>
<td>6 hr</td>
</tr>
<tr>
<td>Processor Time (/dt)</td>
<td>0.2 s ($\approx$ 1.1 min / a)</td>
<td>10 s ($\approx$ 4.1 hr / a)</td>
</tr>
<tr>
<td>Processor Time (/dt/node)</td>
<td>0.0017 s</td>
<td>0.0020 s</td>
</tr>
</tbody>
</table>

A 1 by 1 km application to the entire Greenland Ice Sheet would require ~ 2,000,000 nodes… in the realm of feasibility with a quad- or eight core unit.
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