Wave Modeling and Langmuir Mixing

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The inverse turbulent Langmuir mixing number accounts for nonaligned wind and wave fields.

It is defined as

\[
La_i = \begin{cases} 
\left( \frac{U_{stokes} \cdot u^*}{|u^*|^2} \right)^{1/2}, & |\theta| < \pi/2; \\
0, & |\theta| \geq \pi/2.
\end{cases}
\]

where \( \theta \) is the difference in wind and wave directions.
Previous Work: A Simple Climatology

- Used output from NWW3 to estimate areas of Langmuir mixing and derive a simple climatology
A Simple Scaling for Langmuir Depth/Entrainment: (Li & Garrett, 1997)

Use $Fr$ to determine $H$

Large came up with clever choices for $N$, $H$ that lead to a robust implementation in KPP

If $H$ is deeper than KPP Boundary Layer depth, use $H$

With these choices, $H$ and BLD converge over time.

\[
Fr = \frac{\omega}{NH} \approx 0.6 \quad \omega \approx \frac{V}{1.5} \approx \frac{\sqrt{u^*u_s}}{1.5}
\]

The Algorithm
Use $Fr$ to determine $H$

CAM related to CAM $u^*$ by WW3 Climatology
Previous Work: Shown Sensitivity to Inclusion

(a) CFC in CCSM 3.5 & P14S WOCE obs
(b) August mixed layer depths
Problem 1: Calculating the Surface Friction Velocity

- Installed WW3 on bluefire (details later)
- Obtained similar calculations of $La_i$ using WW3's $u^*$ with COREv2 forcings
Problem 2: Estimating Stokes Drift

For monochromatic waves, it can be shown that at the surface

\[ U_{\text{stokes}} = \frac{\pi^3 Hs^2}{gTm^3} \]

where \( Hs = 4\sqrt{m_0} \) and \( m_0 \) is the zeroth moment of the variance.

However, this is not true for anything other than monochromatic waves.
Problem 3: Different Definitions of Mean Wave Period

WaveWatch: \( Tm_0 = \overline{(f^{-1})} \)

ERA40: \( Tm_1 = \frac{1}{\overline{f}} \)

TOPEX: \( Tm_2 = \frac{1}{\sqrt{\overline{f^2}}} \)

\[ m_n = \int_0^{2\pi} \int_0^{\infty} f^n S(f, \theta) \, df \, d\theta \]

\( Tm_0 = \frac{m_{-1}}{m_0}, \quad Tm_1 = \frac{m_0}{m_1}, \quad Tm_2 = \left( \frac{m_0}{m_2} \right)^{1/2} \)
A Quick Example

Pierson-Moskowitz Spectrum

\[ S(f, \theta) = S(f) = \frac{\alpha g^2}{(2\pi)^4} f^{-5} \exp \left[ -\frac{5}{4} \left( \frac{f_p}{f} \right)^4 \right] \]

where \( \alpha \) is the Phillips constant and \( f_p \) the peak frequency

\[ \left( \frac{T_{m0}}{T_{m1}} \right)^3 = 1.37, \quad \left( \frac{T_{m0}}{T_{m2}} \right)^3 = 1.76 \]

\[ \left( \frac{T_{m1}}{T_{m2}} \right)^3 = 1.28 \]
From previous work by Kenyon (1969) and McWilliams & Restrepo (1999), we can reformulate Stokes drift using the 2-D spectrum as

\[ U_{stokes} = \frac{16\pi^3}{g} \int_0^{2\pi} \int_0^\infty f^3 S(f, \theta) \, df \, d\theta \, \mathbf{\hat{e}}_d \]

\[ = \frac{16\pi^3}{g} m_3 \, \mathbf{\hat{e}}_d \]

where \( \mathbf{\hat{e}}_d \) is the dominant direction of wave propagation.

As a result, we no longer need the previous \( U_{stokes} \) approximation!
Refining our Stokes Drift Approximation

• Would still like to be able to estimate Stokes drift using satellite and buoy data for comparison

• Currently examining if there is an empirical or mathematical relationship that we can use such as

\[ U_{\text{stokes}} \approx a(f) \frac{\pi^3 Hs^2}{gTm^3} \hat{e}_d \]
Current Estimate of $L_i^2$
Problem 4: Numerical Cost

- 3rd generation wave model
- Solves the spectral action density balance equation
- 15-20 sec per time step (1 hr) for one processor \( \approx 35-50 \text{ hr/yr} \)
- Plan on scaling back the number of bins significantly and turning off some interactions
- Alternative 2nd generation model developed by George Mellor (Princeton) worth exploring
Applications of Coupling a Wave Model

• Calculate Langmuir Mixing forcing prognostically
  • A coupled wave model will allow use of more sophisticated and validated parameterizations (e.g., Smyth et al, 04; Harcourt & D’Asaro, 08; Grant & Belcher, 09)

• Improve the air-sea momentum flux

• Improve the air-sea tracer flux

• Conduct climate change studies like erosion

• Others?
Some Properties of $a(f)$