Inferences and Implications for Parameterizations from a Global Diagnosis of Mesoscale Tracer Stirring

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OMWG Meeting, 12/11 9:25–9:45
The Character of the Mesoscale

(Capet et al., 2008)

- Boundary Currents
- Eddies
- $Ro = O(0.1)$
- $Ri = O(1000)$
- Full Depth
- Projects on Fronts
- 100km, months

Eddy processes mainly **baroclinic & barotropic instability**. Parameterizations of baroclinic instability (GM, Visbeck...).
Tracer Flux-Gradient Relationship

\[ u' \tau' = -M \nabla \tau \]

- Virtually all extant subgridscale eddy closures may be written as above, e.g.: GM, Redi, FFH
- Relates the eddy flux to the coarse-grain gradients
- May have a flow/property dependent \( M \): (FFH, Visbeck, Green, Held & Larichev, Stone, Canuto & Dubovikov, Griffies et al ‘05)
- May consider gridscale (FFH, Hallberg & Adcroft)
- Isopycnal & lagrangian coordinate versions possible/known
\[
\begin{bmatrix}
\frac{\partial u'}{\partial \tau'} \\
\frac{\partial v'}{\partial \tau'} \\
\frac{\partial w'}{\partial \tau'}
\end{bmatrix} = - \begin{bmatrix}
M_{xx} & M_{xy} & M_{xz} \\
M_{yx} & M_{yy} & M_{yz} \\
M_{zx} & M_{zy} & M_{zz}
\end{bmatrix} \begin{bmatrix}
\overline{\tau}_x \\
\overline{\tau}_y \\
\overline{\tau}_z
\end{bmatrix}
\]

Diagnostically: 9 elements requires at least 3 similar-transport tracers to specify uniquely.

Could vary tracer by tracer, or active tracer vs. passive, etc. In practice we don’t do this.
\[ u' \tau' = -M \nabla \tau \]

**Anistropic** Redi Form

\[
\begin{bmatrix}
    u' \tau' \\
    v' \tau' \\
    w' \tau'
\end{bmatrix} = -
\begin{bmatrix}
    K_{xx} & K_{xy} & \hat{x} \cdot K \cdot \hat{\nabla} z \\
    K_{yx} & K_{yy} & \hat{y} \cdot K \cdot \hat{\nabla} z \\
    \hat{x} \cdot K \cdot \hat{\nabla} z & \hat{y} \cdot K \cdot \hat{\nabla} z & \hat{\nabla} z \cdot K \cdot \hat{\nabla} z
\end{bmatrix}
\begin{bmatrix}
    \tau_x \\
    \tau_y \\
    \tau_z
\end{bmatrix}
\]

**Yellow** Elements are horizontal stirring

**Blue** Elements in Redi (1982) are symmetric and scaled to make

eddy mixing along neutral surfaces

*Anistropic form due to Smith & Gent 04
Anisotropic* Gent-McWilliams

\[
\begin{pmatrix}
\overline{u'u'} \\
\overline{v'v'} \\
\overline{w'w'}
\end{pmatrix}
= -
\begin{pmatrix}
0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \nabla z \\
0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \nabla z \\
\hat{x} \cdot \mathbf{K} \cdot \nabla z & \hat{y} \cdot \mathbf{K} \cdot \nabla z & 0
\end{pmatrix}
\begin{pmatrix}
\overline{T_x} \\
\overline{T_y} \\
\overline{T_z}
\end{pmatrix}
\]

Antisymmetric Elements in GM (1990)
are scaled to overturn fronts, make vertical fluxes
extract PE, and restratify the fluid
equivalent to eddy-induced advection

Q: Same K as Redi?

*Anisotropic form due to Smith & Gent 04  *Tensor Form (Griffies, 98)
Fox-Kemper, Ferrari, & Hallberg (2008) form (a mixed layer (submeso) eddy param.):

\[
\begin{bmatrix}
\overline{u'\tau'} \\
\overline{v'\tau'} \\
\overline{w'\tau'}
\end{bmatrix} = -
\begin{bmatrix}
0 & 0 & -\Psi_y \\
0 & 0 & \Psi_x \\
\Psi_y & -\Psi_x & 0
\end{bmatrix}
\begin{bmatrix}
\overline{\tau}_x \\
\overline{\tau}_y \\
\overline{\tau}_z
\end{bmatrix}
\]

Antisymmetric Elements in Fox-Kemper, Ferrari, & Hallberg (2008) are scaled to overturn fronts, make vertical fluxes extract PE, and restratify the fluid, At a rate validated against eddying simulations!
Need a Natural, Mesoscale Eddy Environment to Test Out:

\[
[u'\tau'] = - M \nabla \bar{\tau}
\]

\[
\begin{bmatrix}
\frac{u'\tau'}{u'\tau'} \\
\frac{v'\tau'}{v'\tau'} \\
\frac{w'\tau'}{w'\tau'}
\end{bmatrix} = - \begin{bmatrix}
M_{xx} & M_{xy} & M_{xz} \\
M_{yx} & M_{yy} & M_{yz} \\
M_{zx} & M_{zy} & M_{zz}
\end{bmatrix} \begin{bmatrix}
\bar{\tau}_x \\
\bar{\tau}_y \\
\bar{\tau}_z
\end{bmatrix}
\]

3 equations/tracer
9 unknowns (M components)

BY USING 3 or MORE TRACERS, can determine M!!!
(a la Plumb & Mahlman '87, Bratseth '98)

Thursday, December 10, 2009
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[ u'\tau' = -M \nabla \tau \]

We Use:

Years 16-20 of a Global 0.1 Degree Model (sim to Maltrud & McClean '06)

9 Passive Tracers To Overdetermine \( M \)
Use a Natural, Mesoscale Eddy Environment to Test Out:

Testing the Diagnosis:

Note: T not used for diagnosis, active tracers are apparently transported as passive ones are!
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
    \overline{u'\tau'} \\
    \overline{v'\tau'} \\
    \overline{w'\tau'}
\end{bmatrix} = - \begin{bmatrix}
    K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\
    K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\
    \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z & \tilde{\nabla} z \cdot \mathbf{K} \cdot \tilde{\nabla} z
\end{bmatrix} \begin{bmatrix}
    \overline{\tau}_x \\
    \overline{\tau}_y \\
    \overline{\tau}_z
\end{bmatrix}
\]

Correct shape/scale at 150m depth:

Hor. Diffusivity is roughly Trace(\mathbf{M})

Peak Near 500 \text{ m}^2/\text{s}
Median: 2000m^2/s
<6% negative
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
u'\tau'
\nu'\tau'
\nu'\tau'
\end{bmatrix} = -
\begin{bmatrix}
0 & 0 & -\hat{x}\cdot K \cdot \tilde{\nabla} z \\
0 & 0 & -\hat{y}\cdot K \cdot \tilde{\nabla} z \\
\hat{x}\cdot K \cdot \tilde{\nabla} z & \hat{y}\cdot K \cdot \tilde{\nabla} z & 0
\end{bmatrix}
\begin{bmatrix}
\bar{T}_x \\
\bar{T}_y \\
\bar{T}_z
\end{bmatrix}
\]

Result 1: Antisymmetric (GM) Elements scale with corresponding Symmetric (Redi) elements in extratropics.

Thus, GM/Redi basic shape of M is roughly correct (some detailed validation remains)
Use a Natural, Mesoscale Eddy Environment to Test Out:

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\begin{bmatrix}
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  w'\tau'
\end{bmatrix} = -
\begin{bmatrix}
  0 & 0 & -\hat{x}\cdot K \cdot \nabla z \\
  0 & 0 & -\hat{y}\cdot K \cdot \nabla z \\
  \hat{x}\cdot K \cdot \nabla z & \hat{y}\cdot K \cdot \nabla z & 0
\end{bmatrix}
\begin{bmatrix}
  \overline{T}_x \\
  \overline{T}_y \\
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\hat{x} \cdot K \cdot \hat{\nabla} z & \hat{y} \cdot K \cdot \hat{\nabla} z & \hat{\nabla} z \cdot K \cdot \hat{\nabla} z
\end{bmatrix} \begin{bmatrix}
\bar{\tau}_x \\
\bar{\tau}_y \\
\bar{\tau}_z
\end{bmatrix}
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\end{bmatrix} = - \begin{bmatrix}
    0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\
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    \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z & 0
\end{bmatrix} \begin{bmatrix}
    \overline{T}_x \\
    \overline{T}_y \\
    \overline{T}_z
\end{bmatrix}
\]

Atlantic Section

Asym 3.1: GM@lon=345E

Asym 3.2: GM@lon=345E

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  v'\tau' \\
  w'\tau'
\end{bmatrix} = -
\begin{bmatrix}
  K_{xx} & K_{xy} & \hat{x}\cdot K\cdot \nabla z \\
  K_{yx} & K_{yy} & \hat{y}\cdot K\cdot \nabla z \\
  \hat{x}\cdot K\cdot \nabla z & \hat{y}\cdot K\cdot \nabla z & \nabla z\cdot K\cdot \nabla z
\end{bmatrix}
\begin{bmatrix}
  \overline{\tau}_x \\
  \overline{\tau}_y \\
  \overline{\tau}_z
\end{bmatrix}
\]
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\begin{bmatrix}
\overline{u' \tau'} \\
\overline{v' \tau'} \\
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\end{bmatrix}
= - \begin{bmatrix}
0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\
0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z & 0
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K_{yx} & K_{yy} & \hat{y} \cdot K \cdot \hat{\nabla} z \\
\hat{x} \cdot K \cdot \hat{\nabla} z & \hat{y} \cdot K \cdot \hat{\nabla} z & \hat{\nabla} z \cdot K \cdot \hat{\nabla} z
\end{bmatrix}
\begin{bmatrix}
\overline{\tau}_x \\
\overline{\tau}_y \\
\overline{\tau}_z
\end{bmatrix}
\]
These are clearly peaked near 0, but the distribution has big tails!
NSEF & Diabatic/Transition Layer

Danabasoglu & Marshall

Danabasoglu, Ferrari & McWilliams

Ferrari, McWilliams, Canuto, Dubovikov

Surface-intensified GM, no boundary condition issues, no over-restratification of Mixed Layer by Eddies

Fig. 2. A conceptual model of eddy fluxes in the upper ocean. Mesoscale eddy fluxes (blue arrows) act to both move isopycnal surfaces and stir materials along them in the oceanic interior, but the fluxes become parallel to the boundary and cross density surfaces within the BL. Microscale turbulent fluxes (red arrows) may contribute to mixing of surface water and the interior.
Near-surface eddy flux scheme (Ferrari, McWilliams, Canuto, Dubovikov)

EDDY-INDUCED MERIDIONAL OVERTURNING (GLOBAL)

Vertical profile of zonally-integrated total advective heat flux at 49°S
A new eddy parameterization (Ferrari, Griffies, Nurser & Vallis)

- The eddy streamfunction is given by the elliptic problem

\[
\left( c^2 \frac{d^2}{dz^2} - N^2 \right) \tilde{\Psi} = -\kappa \nabla \tilde{b}
\]

\[
\tilde{\Psi} = 0, \quad z = 0, -H
\]

Properties of the new parameterization
- releases mean available potential energy
- the eddy transport vanishes at the ocean boundaries
- the eddy transport is dominated by the first baroclinic mode (if \( c \) is set to speed of first baroclinic mode)
- does not require any tapering function
- reduces to GM for \( c=0 \)
Fig. 1. Annual mean thickness diffusivity ($K$) in m$^2$/s at 300 m depth in experiment CONST (a), VMHS (b), NSQR (c) and EG (d) after 500 years integration. Values of $K$ are known for the interior region only, i.e. values of $K$ in the (seasonal maximum) diabatic surface and transition layer are not shown and shaded black. Note the non-linear color scale for the thickness diffusivity. Note also that the data have been interpolated from the model grid to a regular rectangular grid of similar resolution prior to plotting. The bathymetric mask in the figure (taken from Smith and Sandwell (1997)) differs therefore slightly from the model’s land mask.
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Conclusions

- Passive Tracers are used in a global 0.1 model to diagnose Mesoscale Flux-Gradient Relationship

- Resembles $GM \approx \text{Redi}$ with $O(500 \text{ to } 2000 \text{m}^2/\text{s}$ at 150m depth, but long tails...

- Strongly anisotropic (mostly zonal, strong flow)

- Depth-dependent Streamfunction: MLB intensified changes behavior in diabatic/mixed layer

- $K$ does not scale well with Mean KE or $N^2$

- Active vs. Passive tracers apparently not an issue

- $M_{zx}$ & $M_{xz}$ are $O$(along-iso), detailed contrast later
Comparisons with Marshall et al.

Fig. 12. Inferred horizontal eddy diffusivity $\kappa$ (m$^2$ s$^{-1}$): (top) zonal mean and (bottom) vertical mean over the thermocline (0–1200 m). The contour intervals are (top) 500 and (bottom) 1000 m$^2$ s$^{-1}$. The thick line indicates the zero contour. Also indicated in the bottom panel are the 10-, 70-, and 130-Sv contours of the barotropic streamfunction.
Comparisons with Marshall et al.

Ferreira, Marshall, Heimbach 05

Zonal mean (scalar) diffusivity vs.
Eigenvalues of the symmetric tensor

Same shape--no negatives!

Thursday, December 10, 2009
Comparisons with Marshall et al.

Abernathy et al 09

Thursday, December 10, 2009
Comparisons with Marshall et al.

Abernathy et al 09

Critical Layer?
The Character of the Submesoscale

(Capet et al., 2008)

- Fronts & ageostrophic wind
- Eddies
- $R_o = O(1)$
- $R_i = O(1)$
- Near-surface
- 10km, days
- Parameterizations of eddies (FFH)
The Character of the Finescale
(Capet et al., 2008)

- 3d
- turbulent
- $Ro >> 1$
- $Ri < 1$ to $<< 1$
- near-surface, bottom
- surface wave (Langmuir, breaking)
- internal waves/loss of balance/nonhydrostatic
- $< 100 m$, minutes–hrs.