A simultaneous heat and water transfer model in frozen and thawed soil

Zhenghui Xie, Liye Song, Aiwen Wang
ICCES/LASG, Institute of Atmospheric Physics, Chinese Academy of Sciences
(zxie@lasg.iap.ac.cn)

Xiaobing Feng (University of Tennessee at Knoxville)

LMWG and BGCWG Meetings, NCAR, Mesa Lab, 8-10 February, 2010
Outline

- Introduction
- A moving boundary problem
- A heat and water transfer model
- Model validation
- Summary and discussion
Soil freeze/thaw processes including change of frost/thaw depths significantly influence energy and water exchanges between land surface and sub-surface, as well as vegetation growth and organic matter decomposition through thermal and hydrological processes.
Frozen soil: all kinds of ice-containing frozen soil at 0°C or below 0°C.

- Permafrost and seasonally frozen soil account for 24% and 30% of the land area in the north Hemisphere, respectively;
- Russia and Canada are the countries where the frozen soil are most widely distributed.
Distribution of frozen soil types in China
Earth System Responses and Feedbacks To Soil Freeze/Thaw Processes

Soil Freeze/Thaw Processes

Cold Land/Atmosphere Energy Exchanges
Boundary Layer Turbulence and Stability
Effects of Clouds on Radiation Energy Fluxes
Precipitation Characteristics
Liquid Water Movement through Soil
Water Vapor Movement Through Soil

Energy Sink

Process-Oriented State Variables

Soil Frozen/Thaw Depths
Frozen Soil Internal Energy (relative to melting point)
Soil Moisture
Soil Temperature
Liquid Water Content
The soil freezing-thawing processes including change of frost/thaw depth, significantly influence energy and water exchanges between land surface and sub-surface;

Accurate representation of frost and thaw depths and their climate feedback is significant for improving simulations of the hydrological and greenhouse gas exchange processes in cold regions;

Current land surface models used for climate studies do not represent suitably the dynamics of frost/thaw depths and their feedback to the climate system, which give delayed or rapid freezing/thaw due to the frozen soil parameterization in the models.
The purpose of this work

- A heat and water transfer model with representation of the dynamics of frost/thaw depths and their feedback to the climate system, was developed;
- It treats the soil frost/thaw depths as moving interfaces governed by some Stefan-type moving boundary conditions, and describes the liquid water and solid ice states as well as the positions of the frost/thaw depths;
- An adaptive mesh method for the moving boundary problem is adopted to solve the relevant equations and to determine frost/thaw depths, water content and temperature distribution.
Outline

- Introduction
- A moving boundary problem
- A heat and water transfer model
- Model validation
- Summary and discussion
Two moving boundaries: Phase-transition interfaces
Frost and thaw depths

- A thawed zone from ground surface to the first phase-transition interface (namely thaw depth);
- A frozen zone from the first phase-transition interface to the second phase-transition interface (namely frost depth);
- An unfrozen zone from frost depth to the bottom of the calculation depth.
Control equations

Energy balance equations

\[
\frac{\partial (c_u T)}{\partial t} = \frac{\partial}{\partial z} \left( \lambda_u \frac{\partial T}{\partial z} \right), 0 < z < \xi,
\]

\[
\frac{\partial (c_f T)}{\partial t} = \frac{\partial}{\partial z} \left( \lambda_f \frac{\partial T}{\partial z} \right), \xi < z < \zeta,
\]

\[
\frac{\partial (c_u T)}{\partial t} = \frac{\partial}{\partial z} \left( \lambda_u \frac{\partial T}{\partial z} \right), \zeta < z < L,
\]

Mass balance equation

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left( D \frac{\partial \theta}{\partial z} \right) - \frac{\partial K}{\partial z} + \frac{\partial q_v}{\partial z} + S, 0 < z < \xi, \zeta < z < L,
\]
A moving boundary problem for heat and water transfer processes

\[
\frac{\partial (c_u T)}{\partial t} = \frac{\partial}{\partial z} \left( \lambda_u \frac{\partial T}{\partial z} \right), \quad 0 < z < \xi, \xi < z < L, \quad \frac{\partial (c_f T)}{\partial t} = \frac{\partial}{\partial z} \left( \lambda_f \frac{\partial T}{\partial z} \right), \quad \xi < z < \zeta,
\]

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left( D \frac{\partial \theta}{\partial z} \right) - \frac{\partial K}{\partial z} + \frac{\partial q_v}{\partial z} + S, \quad 0 < z < \xi, \xi < z < L,
\]

Initial and boundary conditions

\[
T(0, t) = f_1(t), t > 0, \quad \theta(0, t) = f_2(t), t > 0,
\]

\[
T(z, 0) = g_1(z), 0 < z < L, \quad \frac{\partial T}{\partial z}|_{z=L} = G_g,
\]

\[
\theta(z, 0) = g_2(t), 0 < z < L, \quad \theta(L, t) = \theta_r, t > 0,
\]

Moving boundary conditions

\[
T|_{z=\xi^+} = T|_{z=\xi^-} = T_f, \quad T|_{z=\xi^+} = T|_{z=\xi^-} = T_f,
\]

\[
\lambda_f \frac{\partial T}{\partial z}|_{z=\xi} - \lambda_u \frac{\partial T}{\partial z}|_{z=\xi} = Q \frac{d\xi}{dt}, \quad \lambda_f \frac{\partial T}{\partial z}|_{z=\zeta} - \lambda_u \frac{\partial T}{\partial z}|_{z=\zeta} = Q \frac{d\zeta}{dt}.
\]

Xie, Song, Feng, Science in China(A), 2008
Outline

● Introduction

● A moving boundary problem

● A heat and water transfer model

● Model validation

● Summary and discussion
\[ D^b \theta^{k+1}_j - \frac{\omega}{h_{j+\frac{1}{2}}} \left( D^{k+1} E^f \theta^{k+1}_j - D^{k+1} E^b \theta^{k+1}_j \right) \]

\[- \left(1 - \omega\right) \frac{h_{j+\frac{1}{2}}}{h} \left( D^k E^f \theta^k_j - D^k E^b \theta^k_j \right) - \omega E^c K^{k+1}_j - \left(1 - \omega\right) E^c K^k_j \]

\[- S_j - \omega \frac{q^{k+1}_{v,j+1} - q^{k+1}_{v,j-1}}{h_j + h_{j+1}} - (1 - \omega) \frac{q^1_{v,j+1} - q^k_{v,j-1}}{h_j + h_{j+1}} = 0, \]

\[ D^b \left( c^{k+1}_u T^{k+1}_j \right) - \frac{\omega}{h_{j+\frac{1}{2}}} \left( \lambda^{k+1}_{u,j+\frac{1}{2}} E^f T^{k+1}_j - \lambda^{k+1}_{u,j-\frac{1}{2}} E^b T^{k+1}_j \right) \]

\[- \left(1 - \omega\right) \frac{h_{j+\frac{1}{2}}}{h} \left( \lambda^k_{u,j+\frac{1}{2}} E^f T^k_j - \lambda^k_{u,j-\frac{1}{2}} E^b T^k_j \right) = 0, \]

\[ D^b \left( c^{k+1}_f T^{k+1}_j \right) - \frac{\omega}{h_{j+\frac{1}{2}}} \left( \lambda^{k+1}_{f,j+\frac{1}{2}} E^f T^{k+1}_j - \lambda^{k+1}_{f,j-\frac{1}{2}} E^b T^{k+1}_j \right) \]

\[- \left(1 - \omega\right) \frac{h_{j+\frac{1}{2}}}{h} \left( \lambda^k_{f,j+\frac{1}{2}} E^f T^k_j - \lambda^k_{f,j-\frac{1}{2}} E^b T^k_j \right) = 0, \]

\[ \omega = 1 \]

the backward Euler scheme

\[ \omega = 0 \]

the explicit Euler scheme
Algorithm

- **Step 1.** Choose the initial datum functions;
- **Step 2.** Compute soil temperature and soil moisture, and Soil frozen/thaw depths \( \theta_j^{k+1}, T_j^{k+1}, \xi^{k+1}, \zeta^{k+1} \) by the Finite difference scheme;
- **Step 3.** Check whether the stopping criterion is met. If the relative error of two iterates is less than a prescribed tolerance, stop the iteration and set \( k := k + 1 \); go back to Step 2; Otherwise, let the iteration run until the maximum number of iterations is reached, then either stop or set \( k := k + 1 \) and go back to Step 2.
- **Step 4** Continue the time integration until \( t_{k+1} = t_{final} \).
Outline

- Introduction
- A moving boundary problem
- A heat and water transfer model
- Model validation
- Summary and discussion
Numerical Experiment 1

- Purpose: To test the effects of the surface temperature on the simulated frost and thaw depths;
- With the simulated surface temperature

\[ f(t) = T_0 + G_t t + A_0 \sin(\omega t) \]

- \( T_0 \): mean annual ground surface temperature (GST)
- \( G_t \): the rising rate of GST
- \( A_0 \): annual amplitude of GST

\[ A_0 = 13^\circ C; \quad \omega = \frac{2\pi}{8760} \]
The ground surface temperature and the simulated frost/thaw depths

(a) Soil surface temperature;

(b) frost and thaw depths for $G_t=0$;

(c) those for $G_t=0.02$;

(d) those for $G_t=-0.02$. 
Numerical Experiment 2

- With the observed surface temperature and soil moisture as the upper boundary conditions
- D66, D110
The Simulated frost and thaw depths at D66 station from 1997.8 to 1998.8

(a) Surface temperature

(b) Frost and thaw depths

- Observation
- Thaw depth
- Frost depth
The Simulated soil moisture and temperature at D66 station from 1997.8 to 1998.8.
The Simulated frost and thaw depths at D110 station from 1997.8 to 1998.8

- Begin to freeze in late September
- Begin to melt in Early April
- Finish a freeze-thaw cycle in July
Outline

● Introduction
● A moving boundary problem
● A heat and water transfer model
● Model validation
● Summary and discussion
A new simultaneous heat and water transfer model for simulating the active layer and frost/thaw depths is developed;

The new model explicitly tracks the freezing-thawing interface and frost/thaw depths;

An adaptive mesh method for the moving boundary problem is adopted to solve the relevant equations and to determine frost/thaw depths, water content and temperature distribution.
Future works and discussion

- The dynamical representation of frost/thaw depth was coupled to CLM 3.0, coupling with updated version of the CLM Models and its validation should be done;
- Soil temperature was calculated by using frost and thaw depths as mesh points, then soil liquid and ice content were adjusted by the variety of phase change;
- Land surface models with dynamical representation of frost/thaw depth can improve the ability to model the soil moisture and temperature.
Thanks for your Attentions!

http://web.lasg.ac.cn/staff/xie/xie.htm

Reference