Ocean Modeling II
Parameterized Physics

Peter Gent
Oceanography Section
NCAR
Ocean Modelling Challenges

LO-RES (3°) O(100+ years/day)

WORKHORSE (1°) O(10-100 years/day)

HI-RES (0.1°) O(1 year/day)

Circumference of Earth ~4x10^5 km
MODEL EQUATIONS

Momentum equations:

\[
\frac{\partial}{\partial t} u + \mathcal{L}(u) - (uv \tan \phi)/a - f v = -\frac{1}{\rho_o a \cos \phi} \frac{\partial p}{\partial \lambda} + F_{Hx}(u,v) + F_V(u) \quad (2.1)
\]

\[
\frac{\partial}{\partial t} v + \mathcal{L}(v) + (u^2 \tan \phi)/a + fu = -\frac{1}{\rho_o a \cos \phi} \frac{\partial p}{\partial \phi} + F_{Hy}(u,v) - F_V(v) \quad (2.2)
\]

\[
\mathcal{L}(\alpha) = \frac{1}{a \cos \phi} \left[ \frac{\partial}{\partial \lambda}(u \alpha) + \frac{\partial}{\partial \phi} (\cos \phi \varphi \alpha) \right] + \frac{\partial}{\partial z}(w \alpha) \quad (2.3)
\]

\[
F_{Hx}(u,v) = A_M \left\{ \nabla^2 u + u(1 - \tan^2 \phi)/a^2 - \frac{2 \sin \phi}{a^2 \cos^2 \phi} \frac{\partial v}{\partial \lambda} \right\} \quad (2.4)
\]

\[
F_{Hy}(u,v) = A_M \left\{ \nabla^2 v + v(1 - \tan^2 \phi)/a^2 + \frac{2 \sin \phi}{a^2 \cos^2 \phi} \frac{\partial u}{\partial \phi} \right\} \quad (2.5)
\]

\[
\nabla^2 \alpha = \frac{1}{a^2 \cos^2 \phi} \frac{\partial^2 \alpha}{\partial \lambda^2} + \frac{1}{a^2 \cos^2 \phi} \frac{\partial}{\partial \phi} \left( \cos \phi \frac{\partial \alpha}{\partial \phi} \right) \quad (2.6)
\]

\[
F_V(\alpha) = \frac{\partial}{\partial z} \mu \frac{\partial}{\partial z} \alpha \quad (2.7)
\]

Continuity equation:

\[
\mathcal{L}(1) = 0 \quad (2.8)
\]

Hydrostatic equation:

\[
\frac{\partial p}{\partial z} = -\rho g \quad (2.9)
\]

Equation of state:

\[
\rho = \rho(\Theta, S, p) \rightarrow \rho(\Theta, S, z) \quad (2.10)
\]

Tracer transport:

\[
\frac{\partial}{\partial t} \varphi + \mathcal{L}(\varphi) = D_H(\varphi) + D_V(\varphi) \quad (2.11)
\]

\[
D_H(\varphi) = A_M \nabla^2 \varphi \quad (2.12)
\]

\[
D_V(\varphi) = \frac{\partial}{\partial z} \kappa \frac{\partial}{\partial z} \varphi, \quad (2.13)
\]

3-D primitive equations in spherical polar coordinates with vertical z-coordinate for a thin, stratified fluid using hydrostatic & Boussinesq approximations.

“The common parameterization hypothesis about turbulent processes is that they mix material properties, hence the most common operator form is eddy diffusion (e.g. by spatial Laplacians) with an eddy diffusivity as the free parameter.”

PARAMETERIZATIONS IN CESM1 POP2

• Vertical mixing (momentum and tracers)
  - surface boundary layer
  - interior

• Horizontal viscosity (momentum)

• Lateral mixing: mesoscale eddies (tracers)

• Overflows

• Submesoscale eddies (tracers)

• Diurnal cycle for short-wave heat flux

• Solar absorption
• Unresolved turbulent vertical mixing due to small-scale overturning motions parameterized as a vertical diffusion.

• Guided by study and observations of atmospheric boundary layer

\[ \partial_t X = -\partial_z \overline{w'X'} \quad \text{parameterize} \quad \overline{w'X'} = -K_x \partial_z X \]

where \( K_x \) represents an “eddy diffusivity” or “eddy viscosity” and \( X = \{ \text{active/passive scalars or momentum} \} \)
VERTICAL MIXING SCHEME: K-PROFILE PARAMETERIZATION (KPP)

• KPP is not just a vertical diffusion scheme because the scalars (Temp and Salinity) have non-local or “countergradient” terms $\gamma_x$

\[
\overline{w'X'} = -K_x (\partial_z X - \gamma_x)
\]

• KPP involves three high-level steps:
  1. Determination of the boundary layer (BL) depth: $d$
  2. Calculation of interior diffusivities: $\nu_x$
  3. Evaluation of boundary layer (BL) diffusivities: $K_x$

• Diffusivity throughout the boundary layer depends on the surface forcing, the boundary layer depth, and the interior diffusivity.

• KPP produces quite large diffusivities below the boundary layer, which mixes temp and salinity quite deep in times of very strong surface wind stress, such as strong midlatitude atmosphere storms.
VERTICAL MIXING SCHEME:
K-PROFILE PARAMETERIZATION (KPP)

1. BL depth $d$ is minimum depth where the bulk Richardson #
   ($Ri_b$) referenced to the surface equals a critical Richardson #
   ($Ri_{cr}=0.3$).

   $$Ri_b(d) = \frac{\left[ B_r - B(d) \right] d}{\left| V_r - V(d) \right|^2 + V_t^2(d)}$$

   $B_r$: near-surface reference buoyancy
   $V_r$: near-surface reference horizontal velocity
   $V_t(d)$: velocity scale of (unresolved) turbulent shear at depth $d$

   $Ri$ measures the stability of stratified shear flow. “Boundary layer eddies with mean velocity $V_r$ and buoyancy $B_r$ should be able to penetrate to the boundary layer depth, $d$, where they first become stable relative to the local buoyancy and velocity.”
VERTICAL MIXING SCHEME: K-PROFILE PARAMETERIZATION (KPP)

2. Calculation of interior diffusivities

\[ \nu_x(d) = \nu_x^s(d) + \nu_x^w(d) + \nu_x^d(d) + \nu_x^c(d) + \nu_x^t(d) \]

\( \nu_x \): interior diffusivity at depth \( d \) (below the boundary layer)

\( \nu_x^s \): (unresolved) shear instability

\( \nu_x^w \): internal wave breaking

\( \nu_x^d \): double diffusion

\( \nu_x^c \): local static instability (convection)

\( \nu_x^t \): tidal mixing

Superposition of processes sets interior vertical diffusivity, \( \nu_x \), below the surface boundary layer.
VERTICAL MIXING SCHEME: K-PROFILE PARAMETERIZATION (KPP)

Verification example at Ocean Weather Station Papa (50°N, 145°W):

Large et al (1994)

Figure 9. Time-depth sections of 4-day averages of observed temperatures in degrees Celsius (a) from ocean weather station (OWS) Papa during the ocean year March 15, 1961, to March 15, 1962 and (b) from the standard KPP simulation of OWS Papa.
The Gent–McWilliams parameterization: 20/20 hindsight

Peter R. Gent
National Center for Atmospheric Research, Boulder, CO, USA

Contents lists available at ScienceDirect

Ocean Modelling
journal homepage: www.elsevier.com/locate/ocemod
Why is GM needed?

Agulhas Retroflection

\[ O(1^\circ) \] models do not resolve the 1st baroclinic deformation radius away from the equatorial regions, and hence lack the mesoscale turbulence which mixes temperature, salinity and passive tracers in the real ocean.
Ocean Observations suggest mixing along isopycnals is $\sim 10^7$ times larger than across isopycnals.

- Early ocean models parameterized the stirring effects of (unresolved) mesoscale eddies by Laplacian *horizontal* diffusion with $K_H = O(10^3 \text{ m}^2/\text{s})$, whereas the vertical mixing coefficient $K_v = O(10^{-4} \text{ m}^2/\text{s})$.

- Horizontal mixing results in excessive diapycnal mixing, which degrades the ocean solution: e.g. Veronis (1975) showed that it produces spurious upwelling in western boundary current regions which “short circuits” the N. Atlantic MOC.

- Thus, was a recognized need to orient tracer diffusion in z-coordinate models along isopycnal surfaces, to be consistent with observed ocean mixing rates.
The GM Parameterization

\[
\frac{\partial T}{\partial t} + (u + u^*).\nabla T = \kappa \nabla^2 \rho T
\]

\[w^* = -\nabla.(\kappa \nabla \rho / \rho_z), \nabla \cdot u^* = 0.\]

GM (1990) proposed an eddy-induced velocity \( u^* \) in addition to diffusion along isopycnal surfaces.
7.1.3 The Gent-McWilliams Parameterization

The transport equation of tracer $\varphi$ is given by

$$\frac{\partial}{\partial t} \varphi + (\mathbf{u} + \mathbf{u}^\ast) \cdot \nabla \varphi + (w + w^\ast) \frac{\partial}{\partial z} \varphi = R(\varphi) + \mathcal{D}_V(\varphi),$$  \hspace{1cm} (7.2)

where the bolus velocity induced by mesoscale eddies is parameterized, from Gent and McWilliams (1990), as

$$\mathbf{u}^\ast = \left( \nu \frac{\nabla \rho}{\rho_z} \right)_z, \hspace{0.5cm} w^\ast = -\nabla \cdot \left( \nu \frac{\nabla \rho}{\rho_z} \right),$$  \hspace{1cm} (7.3)

where $\nu$ is a thickness diffusivity and subscripts $x, y, z$ on $\rho$ and tracers $\varphi$ denote partial derivatives with respect to those variables (this convention will be followed below). The Redi isoneutral diffusion operator (Redi, 1982) for small slope can be written as

$$R(\varphi) = \nabla_3 \cdot (\mathbf{K} \cdot \nabla_3 \varphi),$$  \hspace{1cm} (7.4)

where the subscript 3 indicates the three-dimensional gradient or divergence, i.e., $\nabla_3 = (\nabla, \frac{\partial}{\partial z})$. The symmetric tensor $\mathbf{K}$ is defined as

$$\mathbf{K} = \kappa_I \begin{pmatrix} 1 & 0 & -\rho_x/\rho_z \\ 0 & 1 & -\rho_y/\rho_z \\ -\rho_x/\rho_z & -\rho_y/\rho_z & |\nabla \rho|^2/\rho_z^2 \end{pmatrix},$$  \hspace{1cm} (7.5)

This tensor describes along-isopycnal diffusion that is isotropic in the two horizontal dimensions. The general anisotropic form of (7.5) is given in Smith (1999). The isopycnal diffusivity $\kappa_I$ is in general a function of space and time, and a parameterization for variable $\kappa_I$ will be described at the end of this section. In POP, we write the bolus velocity in the skew-flux form (Griffies, 1998):

\begin{align*}
\mathbf{u}^\ast & = \left( \nu \nabla \rho \right)_z, \\
\w^\ast & = -\nabla \cdot \left( \nu \nabla \rho \right),
\end{align*}
Baroclinic instability produces ACC eddies that try to flatten the isopycnals and produce a MOC that opposes the mean flow MOC.
Baroclinic instability produces ACC eddies that try to flatten the isopycnals and produce a MOC that opposes the mean flow MOC.
GM impacts

Gent et al. (JPO, 1995): Eddy-induced velocity \((v^*, w^*)\) acts to flatten isopycnals and minimize potential energy.

Temperature
- warm
- cold

Salinity
- salty
- fresh

Density
- light
- dense

Fig. 3. Initial states of (a) temperature and (b) salt [contour interval one-quarter that of (a)]. Both panels also show the streamfunction \(\kappa \rho / \rho\), for the parameterized eddy-induced transport velocity.

Fig. 4. Distributions of (a) temperature and (b) salt after an integration time of \(20 \Delta \tau / \kappa\). Both panels also show the streamfunction \(\kappa \rho / \rho\), for the parameterized eddy-induced transport velocity. Contour intervals are the same as in Fig. 3.

Fig. 5. Density distribution at various times of the integration. (a) Initial, (b) \(20 \Delta \tau / \kappa\), and (c) \(1000 \Delta \tau / \kappa\).
Impacts of GM

(a) Horizontal Diffusion, MOC (u)
(b) GM, MOC (u)
(c) GM, MOC (u+u*)

4° x 3° x 20L ocean model
Danabasoglu et al. (1994, Science)
Impacts of GM

Deep Water Formation

(a) Horizontal Diffusion

(b) GM

In (b), deep water is formed only in the Greenland/Iceland/Norwegian Sea, the Labrador Sea, the Weddell Sea and the Ross Sea.

4° x 3° x 20L ocean model

Danabasoglu et al. (1994, Science)
GM: the near-surface eddy flux

GM90 is valid only in the nearly adiabatic ocean interior. Therefore, it needs to be modified near the surface as the isopycnals are nearly vertical in the mixed layer.

The near-surface eddy flux scheme replaces the early approach of just tapering the GM coefficient to zero at the ocean surface.

Ferrari et al. (2008, J. Climate)

---

Fig. 2. A conceptual model of eddy fluxes in the upper ocean. Mesoscale eddy fluxes (blue arrows) act to both move isopycnal surfaces and stir materials along them in the oceanic interior, but the fluxes become parallel to the boundary and cross density surfaces within the BL. Microscale turbulent fluxes (red arrows) mix materials across isopycnal surfaces, weakly in the interior and strongly near the boundary. The interior and the BL regions are connected through a transition layer where the mesoscale fluxes rotate toward the boundary-parallel direction and develop a diabatic component.
GM summary

Mimics effects of unresolved mesoscale eddies as the sum of
- diffusive mixing of tracers along isopycnals (Redi 1982),
- an additional advection of tracers by the eddy-induced velocity $u^*$

Scheme is adiabatic and therefore valid for the ocean interior.

Acts to flatten isopycnals, thereby reducing potential energy.

Eliminates any need for horizontal diffusion in z-coordinate OGCMs
  $\Rightarrow$ eliminates Veronis effect.

Implementation of GM in ocean component was a major factor
  enabling stable coupled climate model simulations without “flux adjustments”.
GRAVITY CURRENT OVERFLOW PARAMETERIZATION

Based on Price & Yang (1998); described in Briegleb et al. (2010, NCAR Tech. Note) and Danabasoglu et al. (2010, JGR)
The Nordic Sea overflows (e.g., the Nordic Sea) are part of the prognostic model domain rather than just some marginal sea boundary conditions as in the MSBC, and the inflow into these marginal seas is accomplished by the prognostic flow in contrast with a parameterized inflow in the MSBC. Finally, treatment of the baroclinic and barotropic momentum and continuity equations is entirely new.

In our implementation here, we focus on the Nordic Sea overflows for two reasons: (1) there are considerably more observational estimates of the DS and FBC overflow properties than for the Ross and Weddell Sea overflows, thus making an assessment of the OFP in comparison with the observations more meaningful, and (2) both the DS and FBC directly affect the AMOC with potentially important impacts on climate. Therefore, this study concerns examining the impacts of the parameterized DS and FBC overflows on the ocean circulation and climate, focusing on the North Atlantic. We pay particular attention to the effects of these parameterized overflows on \( \text{DAMOC} \).

In the present work, FBC parameterization includes only the overflow branch between the Faroe Bank and Faroe Islands, carrying the largest fraction of the total estimated overflow transport across the Scotland-Iceland Ridge. The paper is organized as follows. Section 2 and Appendix A present the OFP and a summary of its implementation in the Community Climate System Model version 4 (CCSM4). An assessment of the OFP in comparison with available observations is given in Appendix B and summarized in section 3. The numerical model and experiments are described in section 4. The model results from both ocean-only and fully coupled climate simulations are presented in section 5. We use the ocean-only cases for verification of the OFP, while the coupled cases are used primarily to document climate impacts. Finally, a summary and discussions are given in section 6.

### 2. Overflow Parameterization

In this section, we present a brief summary of the OFP, noting differences with the PMO. Further details of the scheme are given in Appendix A, and a complete description can be found in Danabasoglu et al. [2010; available at http://www.cgd.ucar.edu/oce/about/staff/gokhan/]. Throughout the manuscript, the subscripts \( i, s, e, \) and \( p \) are used to denote interior, source, entrainment, and product water properties, respectively. Figure 1 is a schematic representation of the key parameters of a parameterized overflow: the latitude, \( \phi \), the sill depth, \( d_s \), the width of the channel at the sill, \( W_s \), the thickness of the overflow at the sill, \( h_s \), the depth of the entrainment at the shelf break, \( d_e \), the maximum bottom slope near the shelf break, \( \alpha \), the distance from the sill to the shelf break, \( x_{ssb} \), and the bottom drag coefficient, \( C_d \). Values for all these parameters as specified for both the DS and FBC are given in Table 1.

In Figure 1, the vertical cross section of the bottom topography is shown as it might be represented in a level coordinate model. With the usual prognostic, rather than parameterized, overflow, the model level corresponding to the sill depth (the green grid cell in Figure 1) is above the raised bottom topography. The other boxes (except the orange product box) represent the regions whose \( T \) and \( S \) are used to compute the necessary densities. All parameters shown in black are constants specified for a particular overflow (Table 1). See section 2 for further details.

**Figure 1.** A schematic of the Nordic Sea overflows. \( T, S, \rho, \) and \( M \) represent potential temperature, salinity, density, and volume transport, respectively. The subscripts \( s, i, e, \) and \( p \) refer to properties of the overflow source water at the sill depth, the interior water at the sill depth, the entrainment water, and the product water, respectively. The thick, short arrows indicate flow directions. The sill depth lies within the green box of raised bottom topography. The other boxes (except the orange product box) represent the regions whose \( T \) and \( S \) are used to compute the necessary densities. All parameters shown in black are constants specified for a particular overflow (Table 1). See section 2 for further details.
BOTTOM TOPOGRAPHY OF x1° OCEAN COMPONENT

[Map of bottom topography showing Denmark Strait (DS) and Faroe Bank Channel (FBC) overflows.]

Danabasoglu et al. (2010, JGR)

Determination of the product water injection site requires additional input from the OGCM; the potential temperature $T_n$ and salinity $S_n$ at each of the $N_p$ product injection sites. As $n$ increases from 1 to $N_p$, the associated depth $d_n$ increases and the location moves downslope unless there is a topographic cliff (e.g., DS levels 45 and 46; $n = 3$ and 4). The search for the injection sites begins with $n = N_p - 1$ and ends with $n = 1$. However, the first time the condition

$$\frac{T_p; S_p; d_n}{C26} > \frac{T_n; S_n; d_n}{C26}$$

is satisfied, the search stops and the product water is injected through side boundary conditions at the sites corresponding to the level of depth $d_n + 1$. In cases where this condition is satisfied for $n = N_p - 1$ the injection occurs at the deepest sites, as a dense bottom current that is free to flow without excessive entrainment, because of the relatively flat downstream topography shown in Figure 2. The product water injection occurs at the shallowest site when (7) is not satisfied.

[Map of product water injection sites into the OGCM domain.]
ATLANTIC MERIDIONAL OVERTURNING CIRCULATION (AMOC)

Danabasoglu et al. (2010, JGR) in Sv
TEMPERATURE AND SALINITY DIFFERENCES FROM OBSERVATIONS AT 2649-m DEPTH

\[ \text{mean} = 0.45^\circ C \]
\[ \text{rms} = 0.50^\circ C \]

\[ \text{mean} = -0.04^\circ C \]
\[ \text{rms} = 0.13^\circ C \]

\[ \text{mean} = 0.02 \text{ psu} \]
\[ \text{rms} = 0.03 \text{ psu} \]

\[ \text{mean} = -0.03 \text{ psu} \]
\[ \text{rms} = 0.03 \text{ psu} \]
HORIZONTAL VISCOSITY

Spatially uniform, isotropic, Cartesian, $\Delta=250\text{km}$ grid for illustration

\[
D(U) = A \ U_{xx} + A \ U_{yy} \quad D(V) = A \ V_{xx} + A \ V_{yy}
\]

Grid Re (Diffuse Noise) \quad \rightarrow \quad A > 0.5 \: V \Delta = 100,000 \: \text{m}^2/\text{s}

Resolve WBC (Munk Layers) \quad \rightarrow \quad A > \beta \Delta^3 = 80,000 \: \text{m}^2/\text{s}

Diffusive CFL \quad \rightarrow \quad A < 0.5 \Delta^2 / \Delta t = 8000,000 \: \text{m}^2/\text{s}

Realism (EUC, WBC) \quad \rightarrow \quad A \sim \text{physical} = 1,000 \: \text{m}^2/\text{s}

Smagorinsky \quad \rightarrow \quad A = C \Delta^2 \sqrt{(\partial_x U)^2 + (\partial_y V)^2 + (\partial_x V + \partial_y U)^2}
ANISOTROPIC HORIZONTAL VISCOSITY

\[
\partial_t u + \ldots = \partial_x (A \partial_x u) + \partial_y (B \partial_y u)
\]

\[
\partial_t v + \ldots = \partial_x (B \partial_x v) + \partial_y (A \partial_y v)
\]

Grid Re (Diffuse Noise) → Live with the “noise”

Resolve WBC (Munk Layers) → \( A = B = \beta \Delta^3 \), only near WBC

elsewhere:

Realism (EUC, WBC) → \( A = 300 \text{ m}^2/\text{s} \)

\( B = 300 \text{ m}^2/\text{s} \) in the tropics

\( = 600 \text{ m}^2/\text{s} \) polewards of 30°

Subject to diffusive CFL, but NO Smagorinsky